Reliability-based topology optimization using evolutionary methods for three-dimensional structures analysis

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Abstract. This paper addresses structural topological optimization considering the reliability analysis using the Evolutionary Structural Optimization (ESO) and Smoothing-ESO (SESO) methods, whose heuristics allow that, at each iteration, whichever elements of the mesh whose contribution to the structure stiffness is considered inefficient is removed / added during the optimization procedure. A code in Python was developed based on the minimization of compliance to determine the optimal three-dimensional topologies of an L-shaped structure. The reliability analysis is performed using First Order Reliability Method (FORM), which aims to obtain a model with lesser volume and greater stiffness than the deterministic topological optimization (DTO) analysis.

Keywords: Reliability-Based Topological Optimization, FORM, ESO, SESO

1 Introduction

The three-dimensional (3D) elastic analysis in the Topology Optimization (TO) field using the Finite Element Method (FEM) is highlighted with the works developed by Zhou and Wang [1], who present a three-dimensional TO program using the MATLAB program. Its code is referred to as the 177-line program and is a successor to the 99-line program of Sigmund [2]. Liu and Tovar [3] propose a 169-line MATLAB code incorporating efficient strategies of three-dimensional TO using a modified SIMP model with 8-node hexahedral regular elements with 3 translational degrees of freedom at each node.

Zuo and Xie [4] present a 100-line Python code for three-dimensional TO. The code used is a program that uses the Abaqus Scripting Interface, which provides convenient access to finite element analysis and was developed to minimize compliance, with a volume constrain, using Bidirectional Evolutionary Structural Optimization (BESO). Vogiatzis et al. [5] developed a MATLAB code using the Level Set Method (LSM), which can be integrated into the TO procedure and can convert the design into an STL file (STereoLithography), which is the format for 3D printing. Simonetti et al. [6] present the application of the SESO method to perform optimization procedures to maximize the stiffness of three-dimensional structures.

In the last two decades, Reliability-Based Topology Optimization, (RBTO), which integrates the concept of structural reliability in the design of continuous structure topology, has been investigated by many authors, such as: Allen et al. [7], Bae et al. [8], Kim et al. [9], Liu et al. [10], Moon et al. [11], Patel et al. [12], Kim et al. [13] and Wang et al. [14]. However, there are few works that couple RBTO methods to three-dimensional structures, highlighting López et al. [15], who approach the RBTO combining the methods of Sequential Optimization and Reliability Assessment (SORA) with an external optimization software. Awruch et al. [16] propose the RBTO with the BESO optimization procedure, Liu et al. [17] use the Segmental Multipoint Linearization (SML) method for a more accurate estimate of the failure probability gradient. Eom et al. [18] perform the RBTO for 3D structures using Bidirectional Evolutionary Structural Optimization (BESO) and the Standard Response Surface Method (SRSM) to determine a limit state function.

Although the RBTO is a rapidly expanding field of research, the relationship of TO with probability constraint is still very challenging, due to the difficult task of evaluating the probability of failure in a direct estimate. This difficulty motivated the development of several methods of uncertainty propagation, such as the Monte Carlo Simulation (MCS) and the First and Second Order Reliability Methods (FORM/SORM). In general, RBTO approaches based on FORM estimate the reliability of the structure (using a reliability index) within the topological optimization algorithm and are referred to as nested-loop (or double-loop) methods. However, these nested loop approaches generate a high computational cost, and the convergence stability is poor, Zhao et al. [19].

The RBTO analysis includes the calculation of failure probabilities, uncertainties related to external forces and manufacturing errors to find the optimal setting of structure, that is, the one with the greatest stiffness and less volume, capable of meeting the expected performance for the defined conditions. According to Lopez and Beck [20], reliability is the degree of subjective probability that a system will not fail, within a specified period and respecting its operating conditions.

Evolutionary methods consider the impact that the finite elements of the mesh have on the final structure and its stiffness. For example, there is the ESO method, which at each iteration, removes some inefficient elements from the initial block and checks, based on the minimization of compliance, if the structure characteristics are adequate for the defined conditions. The SESO method, in addition to removing the elements, returns some of them to the structure, smoothing the optimization procedure.

The use of the ESO method with first-order reliability methods was demonstrated by Kim et al. [21]. Using the hybrid method proposed by Kharmanda [22], it is possible to avoid the high computational cost and the weak convergence stability associated with the application of the estimate within the optimization algorithm with nested loops. In this article, the reliability analysis is performed before the evolutionary method, seeking to determine optimal structures that have adequate rigidity and shape for the design they are intended for, with as less volume as possible.

In this article, the programming language used was Python, and its NumPy, SciPy and PyVista libraries. According to Millman and Aivazis [23] this language has been widely used for its expressive syntax and its rich collection of built-in data types. With it, it's possible to quickly and conveniently manipulate n-dimensional arrays. As an example, for the application of the above-mentioned methods, the L-shaped structure model was selected. The results found are consistent with the literature.

This article is organized as follows: in section 2, the Evolutionary Methods, ESO and SESO are presented. In section 3, the general formulation of the RBTO and the FORM. In section 4, the Numerical Example, L-shaped structure.

2 Evolutionary Methods

2.1 ESO e SESO

The ESO method was developed by Xie and Steven [24] and works by changing the topology of a structure with gradual and systematic removal of finite elements from the mesh. These removed elements correspond to regions that do not contribute to the good performance of the structure. According to Simonetti et al. [25], there is empirical evidence that the optimal solutions found with the ESO method are similar to those obtained by the classical optimization methods.

For the application of this technique, a finite element mesh that circumscribes the entire structure, called extended fixed domain, is defined including boundary conditions in forces, displacements, cavities and other initial conditions. Then, the parameters of interest for optimization are evaluated. In this article, the maximization of the stiffness of the structure, minimizing compliance, was evaluated. In the procedure presented, the compliance of each element can be assessed as follows:

$$U_e < RR_k * (U_i^{max}) \tag{1}$$

where U_e is the strain energy of the element, U_i^{max} is the maximum strain energy of the structure in the effective iteration k e RR is the Rejection Ratio of the k-th equilibrium state. The RR value is defined according to eq. (2):

$$RR_{k+1} = RR_k + ER \tag{2}$$

SESO is based on the ESO philosophy and applies a weighting to the constitutive matrix so that the element that would be eliminated, is maintained and receives a smoothing. This smoothing is processed through the application of a degradation in the value of its initial stiffness in such a way that it remains in the design domain and that, naturally, during the removal process, its influence can contribute and determine its permanence or its definitive withdrawal from the design domain. Thus, the elements located near the limit to the left of this maximum strain energy are kept in the structure, defining a smoother removal heuristic. The classic procedure of ESO and SESO is shown in Fig. 1.

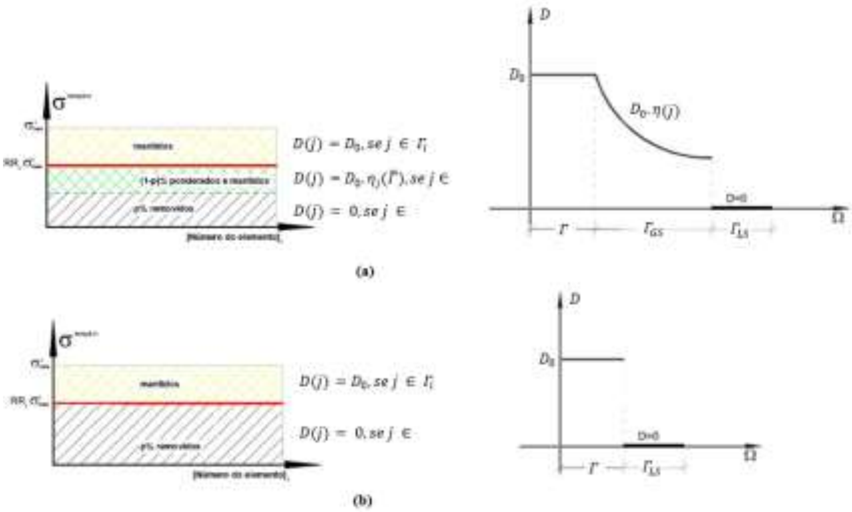

Figure 1. Classic procedure (a) SESO and (b) ESO

3 General Formulation for RBTO

Deterministic optimization designs do not consider the uncertainties of the variables involved in the optimization procedure, such as the geometry, strength and material properties of the structure being designed. As the objective of optimization processes is to determine how the material must be distributed to have an optimal structural response to predefined conditions, the effects of uncertainty related to the variables involved in the design must be considered. RBTO addresses this need and finds an optimal setting under probabilistic constraints, so that it becomes reliable for these uncertainties. The formulation for an RBTO problem using ESO/SESO is described below:

$$\begin{aligned} \text{Minimize:} \quad & \mathrm{C}(x_i) = U^T K U \\ \text{Subject to:} \quad & P_S(\mathbf{X}) = \mathrm{P}\left[G\left(x_i, \mathbf{X}_j\right) \geq 0\right] \geq P_t \\ & \beta(\boldsymbol{u}) = \beta_t \\ & \mathrm{K}(x_i, \boldsymbol{X}_j, \boldsymbol{u})\mathrm{U}(x_i, \boldsymbol{X}_j, u) = \mathrm{F}(\boldsymbol{X}_j, \boldsymbol{u}) \\ & V(x_i, \boldsymbol{X}_j, \boldsymbol{u}) = \sum_{i=1}^{NE} x_i V_i(x_i, \boldsymbol{X}_j, \boldsymbol{u}) - V^* \leq 0 \\ & x_i = 1 \ or \ x_i = 10^{-9} \\ & i = 1, \ldots, n \ and \ j = 1, \ldots, m \end{aligned} \tag{3}$$

where x_i is the finite element, which are the design variables, $\mathbf{X}_j$ is the j-th random variable, C is the objective function, P_S is the probability of success, P_t is the target probability of success G is the state function limit, *n* is the number of variables and *m* is the number of uncertain variables. To control the topologies obtained by the RBTO model, the reliability index $\beta(\boldsymbol{u})$, see Kharmanda and Olhoff [26], is introduced with a normalized vector $\boldsymbol{u}$ which, in the case of the normal distribution, is calculated as follows:

$$u = \frac{X_j - m_{X_j}}{\sigma_{X_j}} \tag{4}$$

Therefore, the probability of failure can be obtained by integrating the joint probability density function $f_X(\mathbf{X})$, as seen on Eq. (5).

$$P_f = \mathrm{P}\left[G\left(x_i, \mathbf{X}_j\right) \leq 0\right] = \int \dots \int_{G(x_i, X) \leq 0} f_X(\mathbf{X}) dx \tag{5}$$

where P_f is the probability of failure, whose integration can be represented as Fig. 2.

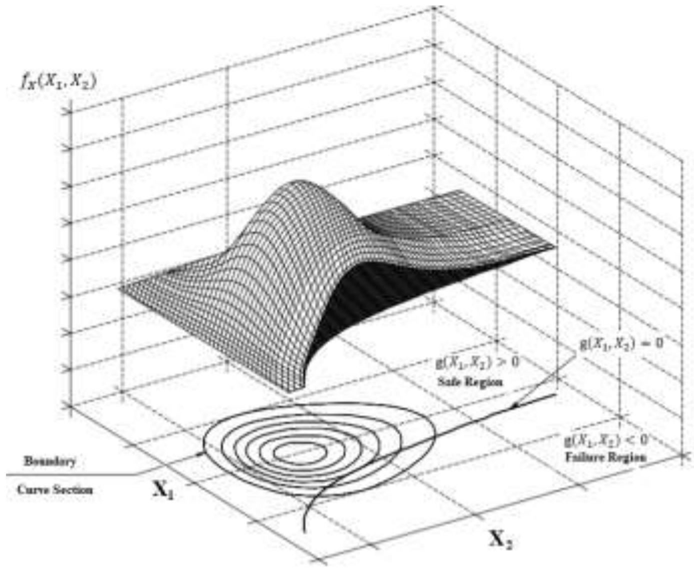

Figure 2. Probability Integration

As this integral is difficult to solve, the FORM method was used for reliability analysis. With this method, the approximate value of the probability of failure can be calculated using $P_f \approx \Phi(-\beta)$, where $\Phi(.)$ is the standard cumulative distribution function, β is the reliability index that can be determined by finding the shortest distance to the fault surface in the standard normal distribution space, i.e, finding the most probable failure point (MPP). Thus, the reliability index is evaluated according to Eq. (6).

$$\begin{aligned} \text{Minimize:} \quad & \beta_i = \sqrt{\textstyle\sum_{i=1}^{m} u_j^2} \\ \text{Subject to:} \quad & G(x_i, \mathbf{u}) \leq 0 \\ & \beta(u) \geq \beta_t \end{aligned} \tag{6}$$

with β_t being the target reliability index.

3.1 First Order Reliability Method - FORM

The FORM allows to incorporate the probability distribution functions and the correlation between the problem's random variables into the analysis. It is called FORM because the performance function G(x) is approximated by the first-order Taylor expansion. According to Lopez and Beck [27], FORM is considered a transformation method, as its solutions are mapped to a Gaussian normal distribution. In this distribution, the MPP is also the closest point to the origin. The reliability index (β), Hasofer and Lind [28], can be evaluated as the distance between the MPP and the origin, within the Gaussian normal distribution, see Fig. 3. The FORM has the advantage of its low computational cost.

In this method, all the statistical information relative the design random variables are used, this includes non-normal marginal distributions as well as correlation coefficients between pairs of variables, and the integration domain is approximated by a linear function. Du and Chen [29] summarize FORM procedures as a transformation from an X space to a U space, with the Rosenblatt transformation from physical space to standard normal distribution space, followed by MPP search in U space, and the computation of the reliability index (β), and finally the reliability R = ϕ(β) is calculated.

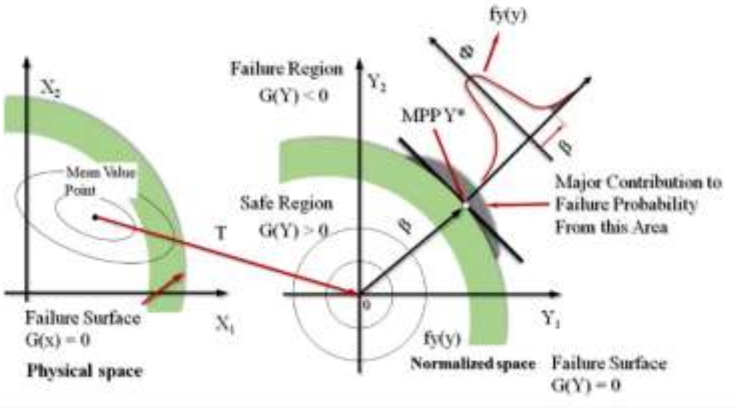

Figure 3. Transformation of physical space T to normalized space, by the FORM Method

4 Numerical Example

In this section, a numerical example is presented, in which the optimal structure were achieved with a RBTO analysis using evolutionary methods ESO and SESO. Due to a bidirectional removal heuristic of SESO, the performance index gradually increases when efficient elements are added to the design domain and inefficient elements are removed, as shown in the graphs.

4.1 L-Shaped Structure

In this example, the topology optimization procedure is investigated on an L-shaped structure, whose initial design domain and boundary conditions are shown in Fig. 4a. A vertical downward force of $1\ kN$ is applied at the center of the free edge. The design domain is discretized by 40x40x20 hexahedral finite elements of 8 nodes with 3 degrees of freedom at each node. The initial design parameters are given in Table 1, where *nelx*, *nely* and *nelz* represent the geometry of the structure, F, the external load, and V, the volume, that are considered random variables, μ is the Poisson coefficient and its modulus of longitudinal elasticity of the material, which in this example are considered constant. The reliability index (β) used was 3.0, with a failure probability equal to 1.358E-3.

Figures 4b and 4c show the optimization history in iterations 10, 15 and optimal topologies achieved in iterations 56 and 58 of ESO/SESO, respectively. Furthermore, it is noted that the performance index can monitor the optimization procedure, capturing the oscillations that occur in compliance.

With the objective of analyzing the influence of the target reliability index on the optimal settings, Table 2 shows the efficiency of the proposed approach comparing the variation in the number of iterations to achieve convergence in the reliability calculation procedure versus the number of iterations involved in the TO procedure. It also shows that finite element analysis with this formulation has a lower computational cost than that achieved with the traditional RBTO.

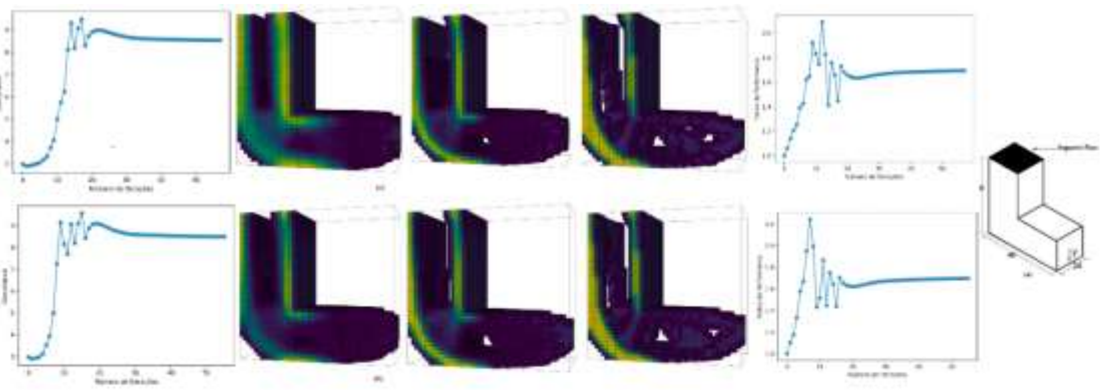

Figure 4. (a) Design domain and boundary conditions (b, c) SESO – Compliance and performance index by iteration and topologies in iterations 10, 15 and optimal topology.

Table 1. Parameters used in the RBTO analysis of the L-Shaped Structure.

Parameters	Distribution Type	Mean (μ)	Standard deviation (σ)
nelx(mm)	Normal	40	4
nely(mm)	Normal	40	4
nelz(mm)	Normal	20	2
E(GPa)	Constant	1	0.1
V	Normal	0.21	0.021
ν	Constant	0.15	0
F(kN)	Normal	1	0.1

Table 2. Efficiency of the proposed approach for different reliability index.

Reliability index (β_t)	Failure probability	Number of iterations (reliability)	Number of iterations (optimization procedure)	Number of iterations (classic RBTO methods)
0.8	2.12424E-1	0	52	0
1.8	3.6081E-2	1	54	54
2.8	2.570E-3	1	56	56
3.8	7.2348E-5	2	54	108

5 Conclusions

In this article, the Python programming language was used to find optimal structures, using the reliability method coupled with evolutionary optimization methods ESO and SESO. The results found show that the inclusion of uncertain variables in the TO procedure can be important for the search for elastic structures with greater rigidity and smaller volume. The use of evolutionary methods ESO and SESO associated with RBTO via FORM proved to be valid, with structures comparable to those found in the literature.

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