Fracture mechanics of aircraft structures with riveted and adhesive stiffeners

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Abstract. A structural component is tolerant of damage if it can safely sustain critical length fractures until it is repaired or its economic life has expired. Reinforcers or stiffeners have the main function of improving the resistance and stability of these structures and providing a means of decelerating or stopping the propagation of fractures in nuclear containments, reactors, viaducts, tall buildings, aircraft, ship hulls, bridges and offshore structures. Analyzing the stress intensity factor and how the behavior of a sheet with and without stiffeners is different are some of the issues studied in this work. The stress-intensity factor (SIF), a parameter that describes the intensity of the singular stress field, has been used successfully to estimate fracture strength and fatigue crack growth rates in situations where the assumptions of linear elasticity are valid.

Keywords: Fracture Mechanics, Stress Intensity Factor, Finite Element Method, Quarter-Point Elements, Stiffeners.

1 Introduction

Repairs to an aircraft fuselage are generally varied, as the material's behavior changes according to the presence of a crack. Thus, there is no specific standard that is applicable in all cases, but the concept of guaranteeing the safety of the structure. Reinforced panels are basic structural components used in the aeronautical industry, which are designed to satisfy damage tolerance requirements. In the event that a fracture reaches a critical length, the applied loads must be safely distributed to the reinforcement. A large part of the commercial aircraft fleet in circulation has already exceeded its service life, and as a result they are prone to fatigue damage. Aircraft structures are made of thin reinforced metal plates and longitudinal and transverse stringers. Reinforcements can be secured to the sheet by means of rivets, adhesive, or alternatively machined to form an integral panel. Reinforcers provide an alternative solution to distribute the panel load around a fractured section. If a fracture reaches a critical length and starts to propagate, the load is transferred from the fuselage to the struts and the fracture can be contained [1]. The present work aims to study different types of rigid structural reinforcements in the fuselage of an aircraft, for different reinforcements, in addition to the variation of their location and size. It is intended to compare the stress intensity factor and show that, for cases with rigid reinforcement, the values are significantly lower when compared to the original structure when both cases are submitted to an initial crack. It will be considered that the plastically deformed region remains small in relation to the dimensions of the crack and the entire cracked body. The finite element method (FEM) has been used extensively for solving fracture mechanics problems, 3D Numerical simulations using FEM will be modeled in the Abaqus computer package. The simplicity of the SIF concept helps predict durability and damage tolerance, concepts used today to design critical components subject to fatigue and fracture. Analyzing the SIF and how the behavior of a plate with and without stiffeners differs are some of the issues addressed in this work.

2 Numerical Model

The FEM can be considered the most powerful numerical method for simulating fracture mechanics problems. The use of FEM to analyze fracture problems, however, involves a great difficulty that resides in capturing the high stress gradient near the crack and accurately reproducing the single stress field at the crack tip. This is the reason why numerous studies have been carried out over the last four decades on the use of FEM to obtain accurate and reliable models in fracture mechanics problems.

2.1 Finite Element Method

The formulation of the FEM can be made from the Principle of Minimum Total Potential Energy, the Weighted Residual Method or the Principle of Virtual Displacements. The FEM uses the concepts of “discretization” of the continuum and “interpolation matrix” which provide the displacements at a point inside the element as a function of its nodal displacements. The term discretization refers to a model with a finite number of unknown displacements at the model's nodes for the analysis of continuous means as opposed to an analysis with an infinite number of variables as performed by the Theory of Elasticity that uses continuous functions, that is, with an infinite number of unknowns as a solution [2].

2.2 Quarter Point Element

As conventional MEF elements cannot reproduce the crack tip singularity, to obtain accurate results, special elements around the crack tip were introduced, capable of reproducing the singularity in the deformations. These elements, called quarter point (QP), demonstrate that when the middle node near the tip of the crack is placed in the position of a quarter of the side, this element can obtain the necessary accuracy for modeling a fractured body in 2D or 3D. The QP elements are arranged around the crack tip in a rosette shape, the standard rosette is formed by elements that form 30º, 40º or 45° angles to each other, normally aligned with the crack [3].

2.3 Evaluation of the stress intensity factors

Consider a homogeneous body of linear or non-linear elastic material free of body forces and subject to a two-dimensional strain field. Suppose the body contains a notch, having flat surfaces parallel to the x-axis and a rounded tip indicated by the arc Γ [4]. The deformation energy density is defined as

$$W = W(x,y) = W(\varepsilon) = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \tag{1}$$

where ε_{ij} is the infinitesimal stress tensor. The expression of the energy release rate for a two-dimensional elastic case can be expressed as

$$J = \int_{\Gamma} \left(W dy - T \frac{\partial u}{\partial x} ds \right) \tag{2}$$

where Γ is the curve surrounding the tip of the notch, T are the components of the tensile vector, u are the components of the displacement vector, s is the area bounded by any closed path.

The J integral can be seen as a generalization of the potential energy release rate, using the concept of a path-independent integral, used to evaluate the energy release rate on crack growth. The original definition is as the rate of potential energy release of the system in relation to the variation of the crack length. It is important to emphasize that, for materials with linear elastic behavior, the elasto-plastic fracture parameter J is equivalent to the rate of potential energy release G [5]. The J-integral is related to the stress intensity factor; under plane stress conditions, the relationship is:

$$J^{I} = \frac{K_I^2}{E} \text{ and } J^{II} = \frac{K_{II}^2}{E} \tag{3}$$

2.4 Contour Integrals

Boundary integrals can provide data such as integral values J and K for linear homogeneous materials and for interfacial fractures situated at the interface between two homogeneous linear materials. Abaqus offers a few different ways to assess the contour integral, one of these approaches is based on the conventional FEM, which typically requires the user to mesh the fracture geometry, to explicitly define the fracture tip, specify the direction and the extent of the virtual crack. The SIF calculated by Abaqus are related to the energy release rate (Integral J) through

$$J = \frac{1}{8\pi} K^T B^{-1} K \tag{4}$$

where $K = [K_I, K_{II,} K_{III}]^T$ are the SIF, and is called the pre-logarithmic energy factor matrix.

For the case where there is an interfacial fracture between two different isotropic materials

$$J = \frac{1-\beta^2}{E^*}(K_I^2 + K_{II}^2) + \frac{1}{2G^*} K_{III}^2 \tag{5}$$

where

$$\frac{1}{E^*} = \frac{1}{2}\left(\frac{1}{E_1} + \frac{1}{E_2}\right) \tag{6}$$

$$\frac{1}{G^*} = \frac{1}{2}\left(\frac{1}{G_1} + \frac{1}{G_2}\right) \tag{7}$$

$$\beta = \frac{G_1(k_2 - 1) - G_2(k_1 - 1)}{G_1(k_2 + 1) + G_2(k_1 + 1)} \tag{8}$$

3 Numerical examples of SIF using the FEM

3.1 Crack symmetrical about stiffener

Two configurations of infinite stiffened panels are shown in Fig. 1, for which alternative results are known from the literature, were analyzed in order to check the accuracy of the proposed method. The results for the normalized opening mode stress intensity factor K/K_o are presented and compared to those given in ref. [6]. In both cases, the boundary element model is a square sheet of width L with L/a 40, riveted and adhesive attached stiffeners are considered and the stiffeners are submitted to proper end forces in order to satisfy the condition of strain compatibility (i.e., if no crack is present, there are no interaction forces between the sheet and the stiffeners), and the in-plane bending and shear stiffness are equal to zero.

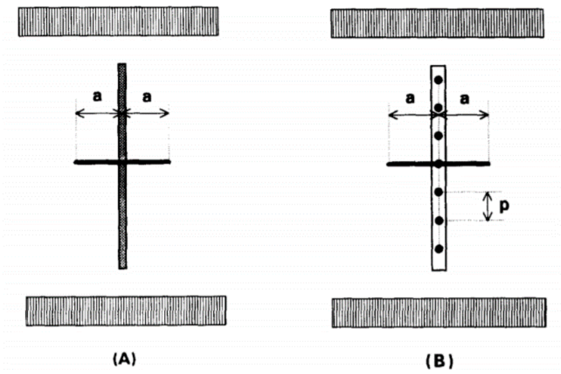

Figure 1: Crack symmetrical about a stiffener. Case (A): stiffener adhesive attached; case (B): stiffeners riveted attached [6].

$$\lambda = \frac{2E_s at}{AE_n} \quad (9)$$

In this example a sheet containing a crack of length 2a symmetrical about a stiffener subjected to uniaxial tensile stress is analyzed (Fig. 1). Broken stiffeners are modelled as two independent stiffeners, with the end forces applied only to the extreme point further away from the crack tip. The SIF obtained for those two models are presented in Fig. 4.

3.2 Stiffened panels made of 2024- T3

Riveted-stiffened panels made of 2024-T3 sheet material with either aluminum or steel stiffeners were modeled, the dimensions of the panel are shown in Figure 2. The stringers are fixed to the plate using equally spaced rivets. The plate and the stringers are subjected to uniaxial stresses σ and $\sigma\frac{E_R}{E}$, respectively. The plate contains an initial fracture, which extends equally on either side of a stringer or in the center of the space between the stringers. Rivet forces act symmetrically with respect to the crack. Rivet rows do not exert any collinear force with the fracture due to symmetry. According to the laws of balance, the forces of the rivet act in opposite directions on the plate and the spar [7].

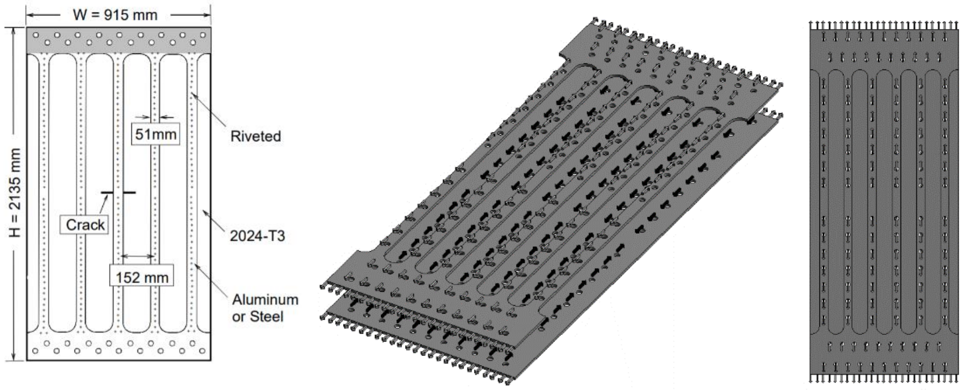

Figure 2: Distribution of stresses applied in a plate with stiffening stringers [8]

The equations needed to determine the unknown forces of the rivet (Q) were obtained by equating the displacement in the rivets with the displacement in the stringers. The displacement of the stringer in the y direction with uniaxial stress $\sigma\frac{E_R}{E}$ is simply given by:

$$V = \frac{\sigma y}{E} \tag{10}$$

The uniaxial stress subject to the stiffeners is given by

$$S_e = S\frac{Es}{E} \tag{11}$$

3.3 Rivet Forces

The displacements in the finite width stringer subjected to a pair of point forces in Figure 3 can be approximated by the displacements in the infinite sheet subjected to pairs of equal and uniformly spaced point forces.

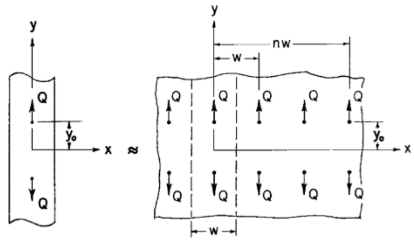

Figure 3 - Distribution of stresses applied in a plate with stiffening stringers (POE JR, 1971)

$$V = \frac{(1+v)(3-v)Q}{8\pi t_s E_S}\sum_{n=0}^{\infty}\psi_n \tag{12}$$

where

$$\psi_n = \left(1-\frac{2nw}{d}\right)\ln\left[\frac{\left(1-\frac{2nw}{d}\right)^2+\alpha_4^2}{\left(1-\frac{2nw}{d}\right)^2+\alpha_3^2}\right]+\left(1+\frac{2nw}{d}\right)\ln\left[\frac{\left(1-\frac{2nw}{d}\right)^2+\alpha_4^2}{\left(1-\frac{2nw}{d}\right)^2+\alpha_3^2}\right] + \left(\frac{1-v}{3-v}\right)\left\{\alpha_4\tan^{-1}\left[\frac{2\alpha_4}{\left(\frac{2nw}{d}\right)^2+\alpha_4^2-1}\right]-\alpha_3\tan^{-1}\left[\frac{2\alpha_3}{\left(\frac{2nw}{d}\right)^2+\alpha_3^2-1}\right]\right\} \quad (13)$$

and

$$\alpha_1=\frac{2(\mathrm{x}-\mathrm{x}_0)}{\mathrm{d}} \quad (14) \qquad \alpha_2=\frac{2(x+x_0)}{d} \quad (15) \qquad \alpha_3=\frac{2(y-y_0)}{d} \quad (16) \qquad \alpha_4=\frac{2(y+y_0)}{d} \quad (17)$$

For n=1, 2, 3….

4 Results and Discussions

The results obtained for the stiffeners in the example in Figure 1 are in agreement with those used as reference [6], with a difference lower than 4% for both reinforcement models. It is noteworthy that none of the models mentioned accurately represent the real distribution of stresses around the rivet hole as the values found for reference in the literature. With the first example it can be obtained that the riveted reinforcement is at least 10% more mechanically efficient compared to the adhesive reinforcement. These results demonstrate that for regions with a high stress concentration, riveted reinforcements are still the best option for structural repair.

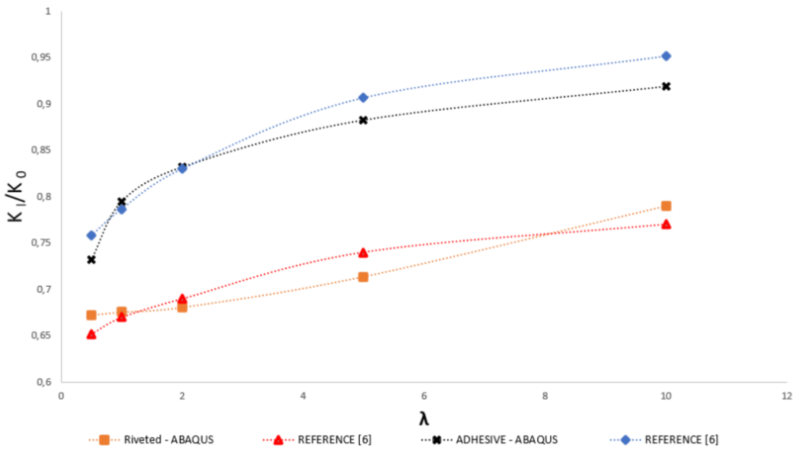

Figure 4: Normalized stress intensity factors for example 1

The last simulated model presented a discrepancy of up to 11% (Figure 5), with the values found in the literature [8]. These values are in agreement, as even the reference values had errors of the same magnitude when compared with the tests carried out in the laboratory for the studied plate because the equations used to determine the force of the rivet are empirical equations. These values can decrease by increasing the mesh refinement transition zone. The SIF results show that the stiffener of the studied panel is efficient, and that value of the SIF decreases rapidly as the fracture approaches one of the stringers.

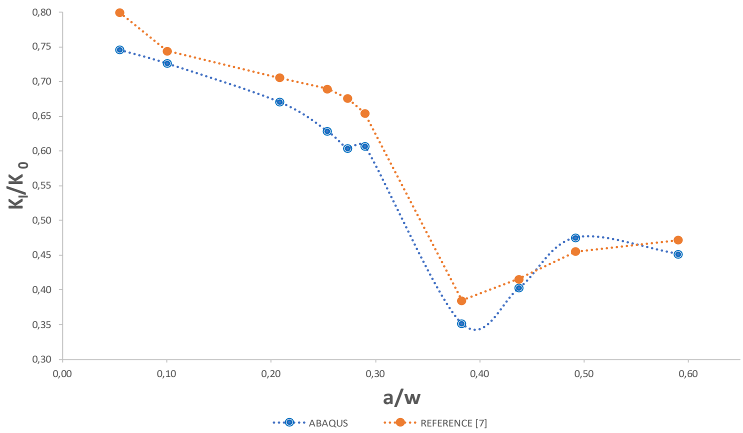

Figure 5: Normalized stress intensity factors for example 2

5 Conclusions

For these studied models it was possible to note that for small cracks, the SIF for a hardened sheet is essentially the same as that of an unhardened sheet. For longer cracks, the SIF is reduced by the spars. The lesser the tension intensity factor the greater the stiffness of the material that will be used as reinforcement, the smaller the spacing between the spars or for more spaced rivets. It can be concluded with the modeled examples that the FEM using QP concepts, can reliably describe a fractured body and perform the SIF calculation. The mesh of the FEM models is very important, but the refinement of the simulations will not always bring better results, since in many occasions a refined coarse mesh would bring convergent results when compared to extensively refined meshes. Still on the meshes, a well-made transition zone proved to be more effective and providing more accurate results than the refinement of the mesh itself, in addition to unstructured meshes, they obtained results closer to those found in the literature. In short, the ABAQUS and the FEM proved to be an excellent numerical tool, making it possible to carry out analyzes of three-dimensional stationary fractures based on FITs.

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