Analysis of the calibration of the constants of the Modified Wöhler Curve

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Abstract. This analysis is based on the critical plane approach combined with the Modified Wöhler Curve, using uniaxial fatigue results, of ASTM743 CA6NM steel, subjected to loading ratios $R = (0, -1, \frac{-1}{3}, \frac{-2}{3}, \frac{1}{3}, \frac{2}{3})$. In this scenario, we show the indicated procedures to calibrate as constants of the Wöhler Curve. We postulate that, by incorporating the effects of multi-axiality, the critical plane approach parameters can be used successfully in estimating fatigue life under different test conditions. Thus, unlike the classical approach, the use of this enhancement has the advantage of allowing the construction of the verses relationship, regardless of the type of test used for calibration. To define the new relationship, a calibration strategy considering the Susmel and Lazzarin critical plane model is presented.As a result, it was found that from the constants A and b curve by the SN curve, we can represent an intermediate curve by means of these constants versus the compressibility ratio. This relationship is influenced by the loading rate of $R\rho$. This new parameter can be used as a multiaxial fatigue criterion for life prediction. Regarding the parameter A, for this material presented, for the conditions of $R = (-1, \frac{-2}{3}, \frac{1}{3}, \frac{2}{3})$, converged with the Susmel and Lazzarin hypothesis, the curve evolved according to the variation of the compressibility relation, changing a linear behavior. However, for the conditions of $R = (0, \frac{-1}{3})$, there is inconsistency in the available data about the theory. The tensions increased proportionally to the increase of rho, diverging from the assumed hypotheses. The b parameter behaved similarly to the A parameter.

Keywords: Wöhler curve, calibration, critical plane, constants.

1 Introduction

In an attempt to develop theories that can be used in general and complex situations of multiaxial fatigue, several models were proposed to solve the problem. In general, the models presented can be classified as empirical, based on the stress tensor invariants, mean stresses, critical planes, etc. Critical plane models are a good alternative in terms of life prediction accuracy. In this sense, this work will focus on the criteria associated with critical planes. This class of approaches considers that fatigue cracks originate in certain planes of materials where the combinations of shear and normal stresses or strains are more severe [1]. These criteria can predict the fatigue strength of the material and the location of the crack beginning and its orientation [2]. The physical interpretation of fatigue damage is based on the theory of cyclic deformation in crystals. This theory is also used to highlight stress components that can be considered significant for nucleation and growth in the first stage of crack propagation. Fatigue life estimates are performed using a modified Wöhler curve that can be applied to components subject to varying loads. Wöhler curves are functions that report fatigue strength to the macroscopic maximum amplitude of the shear stress. The plane where the maximum shear stress amplitude is observed is considered coincident with the one where a microcrack starts and, therefore, critical. These curves take into account the normal stress component in this plane as well as the phase angles [3].

1.1 The Susmel and Lazzarin model

The Susmel and Lazzarin Criterion is based on the Modified Wöhler Curve Method (MCWM), it considers a flat material subjected to a complex system of forces, that results in a multiaxial state of stress causing fatigue damage in the process zone Fig. 1, due to the state of alternating stress, there is the formation of persistent sliding bands and parallel to a material plane (θ, ϕ) [2]. Due to a series of cycles, the stress concentration causes the onset of a microcrack mainly due to shear stresses [4]. The fatigue crack initiation process is influenced by the maximum amplitude of the shear stress, for an xyz coordinate system. This critical plane are identified by its spherical coordinates (θ^c, ϕ^c) and defined by Eq. (1):

$$\tau_a = \underset{\theta,\phi}{max}\{\tau(\theta,\phi)\} = \tau(\theta^c,\phi^c). \tag{1}$$

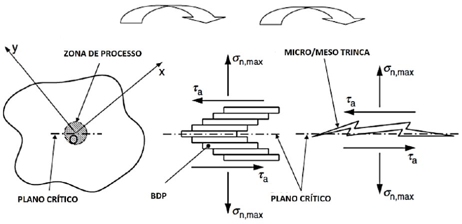


Figure 1. Persistent slip bands [5]

The authors indicate a criterion based on Modified Wöhler Curves (CWM) to estimate fatigue strength under multiaxial load conditions, considering that fatigue damage is based on the deformation of a crystal made of a given material [6]. This criterion is defined by the factor related to the degree of multiaxiality of the stress, for a given material, which can be evaluated through the applied stress in the proportion, defined by Eq. (2):

$$\rho = \frac{\sigma_{n,max}}{\tau_a} = (\theta^c,\phi^c). \tag{2}$$

1.2 Modified Wöhler Curve (CWM)

The Modified Wöhler Curve is a curve constructed based on the shear stress amplitude and fatigue life Fig. 2, in log-log basis, where the ordinate represents the shear stress amplitude and the abscissa to fatigue life [5]. The two curves in the diagram represent the fatigue limits according to the torsion and tension / compression tests, for the load ratio $R = 0$ and $R = -1$ and the compressibility ratio $\rho = 0$ and $\rho = 1$. The authors proposed the construction of an intermediate curve for this method, this curve depends on a simple linear equation, where interpolating these curves it is possible to obtain its constants [2, 3]. The authors hypothesized that this intermediate curve would have a linear behavior and that the slope of the curve varied according to the value of the compressibility ratio ρ. And they have provided that the calibration functions can be represented by Eqs. (3 - 5). Experimental evidence shows for a given material, when $\tau_{A,Ref(\rho)}$ decreases, the ratio ρ increases, as shown in Fig. 2.

$$k_\tau(\rho) = a\rho + b, \tag{3}$$

$$\tau_{A,Ref(\rho)} = \alpha\rho + \beta, \tag{4}$$

$$N_{f,e} = [\tau_{A,Ref(\rho)}]k_\tau(\rho). \tag{5}$$

where k_τ = curve slope coefficient, α and β is material constant, $\tau_{A,Ref(\rho)}$ fatigue limit expressed in terms of amplitude e $N_{f,e}$ is the fatigue life. Fatigue life can be estimated in terms of the compressibility ratio of the stress field in the fatigue process zone, measured in terms of ρ [7].

2 Methodology

The calibration of these constants is built through an intermediate curve that relates the shear stress amplitude with fatigue life, being controlled by the compressibility factor, ρ, similar to the fatigue curve, but with $\sigma_m \neq 0$.

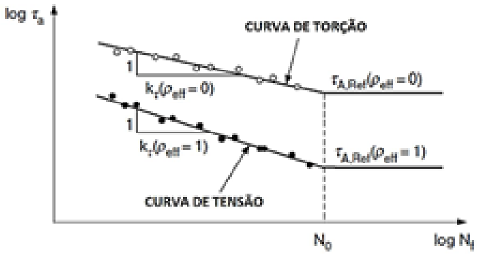


Figure 2. Modified Wöhler curve diagram under loading, reverse-compression, and torsion [2].

2.1 Calibration of the constants A and b of the Modified Wöhler Curve

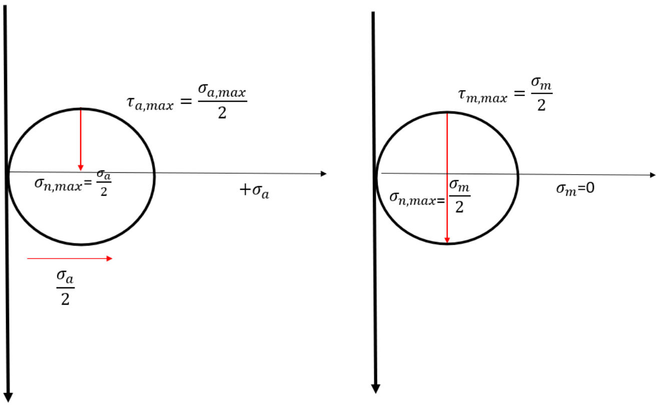
In this case, the calibration will be performed based on experimental results of uniaxial tensile tests, and with load ratios $R = (0, -1, \frac{-1}{3}, \frac{-2}{3}, \frac{1}{3}, \frac{2}{3})$. Based on the experimental results presented in the literature [8], the curves $(S - N)$ were constructed, and through these curves the constants A and b were determined, according to Eq. (6) . The methodology used for this calibration is described in this section.

$$S_a = A \cdot N^b. \tag{6}$$

where A = constant of the $(S - N)$ curve obtained through the intercepting of the curve, b = exponent of the curve obtained by slope the curve, σ_a, alternating stress amplitude and N = number of cycles to failure (Life). Consider the experimental results to obtain the influence of the mean stress, on fatigue strength. It is important to determine these tensions, according to Eq. (7):

$$\sigma_m = \left(\frac{1+R}{1-R}\right) \cdot \sigma_a. \tag{7}$$

After adjusting the $(\sigma_a - N)$ curve and obtaining the mean stresses, the shear stress amplitudes were determined in the critical plane through the relation Eq. (1). Consider a specimen subjected to a cyclic tensile load with a constant loading rate. The state of stress in plane O, is analyzed based on Mohr's circle, represented by Figure 5a and 5b. Circle A, represents the stress state of the maximum alternating normal stress Fig.5a. Circle B, represents the mean normal stress Fig. 5b.



(a) (b)

Figure 3. (a) Mohr circle for traction loading considering alternating stress. (b) Mohr circle considering average stress: experimental data.

From the analysis of circle A Fig. 5a, in the plane $45°$, it is verified that the maximum amplitude of the alternating shear stress, $\tau_{a,max}$, and the normal alternating component stress, $\sigma_{a,max}$, at the critical point, can be determined, according to Eqs. (8 and 9):

$$\tau_{a,max} = \left(\frac{\sigma_{a,max}}{2}\right), \tag{8}$$

$$\sigma_{n,max} = \left(\frac{\sigma_a}{2}\right). \tag{9}$$

Using the analogous relation to circle A, in circle B , we have the mean amplitude of the shear stress, $\tau_{m,max}$, related to the mean stress component, σ_m, and the maximum total normal stres $\sigma_{ntotal,max}$ at the critical point, these stress can be determined according to Eqs. (10 - 12):

$$\tau_{m,max} = \left(\frac{\sigma_m}{2}\right), \tag{10}$$

$$\sigma_{n,max} = \left(\frac{\sigma_m}{2}\right), \tag{11}$$

$$\sigma_{ntotal,max} = \left(\frac{\sigma_a}{2} + \frac{\sigma_m}{2}\right). \tag{12}$$

By obtaining the maximum shear stresses determined by the analysis of the Mohr circle, it is possible to create an intermediate curve, built from the relationship between the R and ρ ratios. It is possible to determine R, based on the influence of the components of the alternating stress components σ_a and the mean stress, σ_m, in this plane, according to Eq. 13. The Eq. 13, can be rewritten according to Eq. 14:

$$R = \frac{S_{min}}{S_{max}}, \tag{13}$$

$$R = \frac{S_{min}}{S_{max}} = \frac{\sigma_m - \sigma_a}{\sigma_m + \sigma_a}. \tag{14}$$

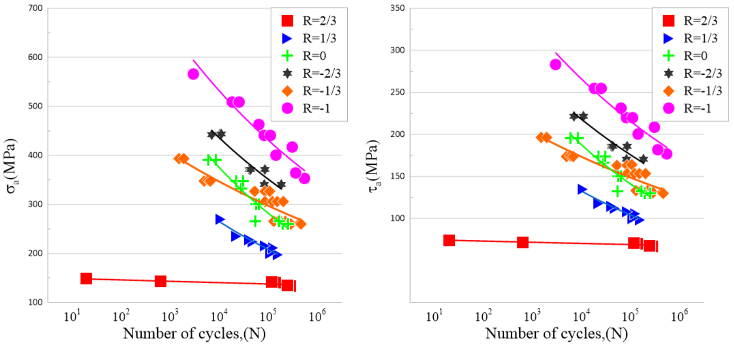
As the curve depends on the relation R / ρ, for a specific test it is possible to determine the ρ, for any R. A Equation (15), can be transcribed as Eq. (16). With the function $\rho = f(R)$, it is possible to apply a new evaluation to the MCWM, proposed in this work. And construct a new modified Wöhler curve from the curve constants $(\tau_a - N)$ as a function of the ratio ρ.

$$\rho = \frac{\sigma_{n,max}}{\tau_{a,max}} = \frac{\left(\frac{\sigma_a}{2} + \frac{\sigma_m}{2}\right)}{\tau_{a,max}}, \tag{15}$$

$$\rho = 1 + \left[\frac{1+R}{1-R}\right]. \tag{16}$$

3 Results and discussions

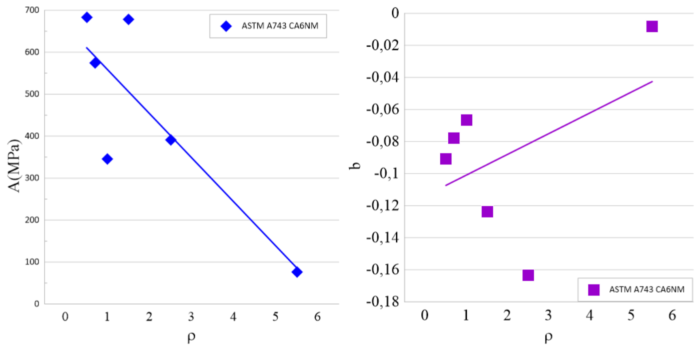
In this work, experimental data from fatigue tests performed on ASTM743 CA6NM steel, extracted from the literature [15] will be analyzed. To obtain these data, the graphs presented by [15] were scanned and electronically processed by the WebPlotDigitizer application.To avoid repetition, only the most important data is presented.To avoid repetition, only the most important data is presented.



(a) (b)

Figure 4. (a) S-N curve under the effect of average tension. (b) Representation of the Wöhler Curve.

In Figure 4a the experimental data of fatigue versus life in relation to R are presented, and their respective curves $\boldsymbol{SN}$. Analyzing quantitatively, considering the coefficient of determination (R^2) the data obtained presented $R^2 = 50\%$, indicates that the experimental data obtained a conservative representation in relation to the curve. The data significantly fit the curve. Despite this adjustment, the results are relatively scattered for $R = 0$ and $R = -1/3$ where the curves intersect.Figure 4b represents the Modified Wöhler Curves , represented by the maximum shear stresses in the critical plane, obtained according to Eq. (8) versus fatigue life. The results of the approaches $(\boldsymbol{S} - \boldsymbol{N})$ and $((\boldsymbol{\tau_a} - \boldsymbol{N})$, obtained a similar behavior, only the ratios $R = 0$ and $R = \frac{-1}{3}$, showed a small dispersion and intersected at the point of 10^5 cycles. The data showed a conservative behavior considering the CWM model.



(a) (b)

Figure 5. Comparison of the parameters:(a) Parameter $(A - \boldsymbol{\rho})$. (b) Parameter $(\mathbf{b} - \boldsymbol{\rho})$.

The figures 5a and 5b show the comparison of the parameters of the constants A and b versus ρ. The relations of ρ were calculated for $R = (0, -1, \frac{-1}{3}, \frac{-2}{3}, \frac{1}{3}, \frac{2}{3})$. Analyzing the parameters individually from the quantitative point of view, we can verify that the parameter A, represented by the intersection of the curve, Fig. 5a,considering the coefficient of determination $R^2 = (70.81\%)$, this indicates that the parameter $(\boldsymbol{A} - \boldsymbol{\rho})$ presented the experimental data consistently. The parameter b, represented by the slope of the curve ,Fig. 5b, considering the coefficient of determination $R^2 = (21.41\%)$ of the curve, indicates that the parameter $(\boldsymbol{b} - \boldsymbol{\rho})$, did not present the experimental data consistently, being o $R^2 < 50\%$.For the parameter A, steel ASTM743 CA6NM, presented a conservative behavior, for the ratios of $\rho = (1, \frac{1}{2}, \frac{1}{5}, 5)$ followed by the conservative direction of the MCWM theory, the stresses of A decreased proportionally to the increase of ρ and the curves presented linear behavior as expected. There was non-conservative behavior for $\rho = (2, 3)$. These indicated divergences from the MCWM theory. The tension increased in proportion to the increase in ρ. The b parameter presented results similar to the A parameter, as shown in Fig. 5b.

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4 Conclusions

This work demonstrates that multiaxial fatigue models can be used to calibrate the Modified Wöhler Curve. In this scenario, we show the procedures used to calibrate the constants A and b of the curve, using the Modified Wöhler Curve method combined with the use of critical planes. As a consequence, it was found that from these constants, we can represent an intermediate curve, through the relation of constants versus ρ, and that this relation is influenced by the load relation R . Based on the results obtained, it is concluded that: Considering the hypothesis of Susmel and Lazzarin [11] that the shear stress decreases proportionally with the increase of the compressibility ratio and that the Modified Wöhler Curves present a linear behavior. Regarding the parameter A, for this material presented, for the conditions of $\rho = (1, \frac{1}{2}, \frac{1}{5}, 5)$, converged with the hypothesis by Susmel and Lazzarin, the curve evolved according to the variation of the compressibility ratio, presenting a linear behavior. However, for the conditions of $\rho = (2, 3)$, there is inconsistency in the data presented about the theory. The tensions increased proportionally to the increase of ρ, diverging from the presented hypotheses. The b parameter behaved similarly to the A parameter. From these observations, it is important to carry out additional research with other materials and variable load ratios to better evaluate the presented parameters.

Authorship statement. The authors Simelia S. Santos and Jorge Luiz de A. Ferreira confirm that they are solely responsible for the authorship of this work, and that all material included here as part of this work is the property (and authorship) of the authors, or has the permission of the authors. owners to be included here.

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