Peridynamic Prognostic Tool Potentiality Measured by the Finite Elements Method

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Abstract. At different times and in different applications, humanity needed knowledge about flaws in materials, such as cracks in structures and components, but the influence of such flaws was not clearly known. At the end of the 20th century, a theory called Peridynamics emerged, first introduced by Stewart Silling[1], as an extension of the standard theory of solid mechanics, a different way of seeing what happens internally to the material (a non-localtheory), using Newton's second Law and making use of displacement integrals, to solve problems in structures with discontinuities, such as cracks. In parallel, there is the Finite Elements Method (FEM) like mencioned for J. P. Dias [2], which is already well established and widespread both in academia and industry, which can also be used to investigate the behavior of solid elements that have discontinuities. Thus, the work in question hás the intuition to compare two different programs, one written using the FEM and the other the Peridynamic theory, to achieve results of the effectiveness and convenience of Peridynamics, since the FEM already presents several references proving its results.

Keywords: Peridynamics, Finite Elements Method, Discretization, Damage, Prognostic.

1 Introduction

Deterioration, leading to defects, is a common occurrence in equipment or mechanical systems. In an industrial environment the consequences from these occurrences can be, among other things, unexpected breaks, sudden stops in production, heavy losses of money and accidents. In order to prevent such situations and to preserve the reliability of a certain mechanical component or system during its lifetime, accurate health monitoring techniques are required. Such techniques can be found in well established tools, like prognostics and heath monitoring (PHM), with which crack propagation, the cause of fracture in components, can be studied and predicted. This field of study leads to computational models, commonly represented by mathematical models that are based either on a physical description or on data measurements of the damage. The Finite Element Method (FEM) based models are the predominant ones in this case, being able to characterize crack features, such as direction, propagation speed, branching, etc. Although the FEM based models had a good evolution on understanding the cracks behavior, there are still some problems mainly involving mathematical formulation and computational effort. However, a new potential tool called Peridynamic (PD) has recently being developed and has shown more success on predicting the crack features previously mentioned.

2 Methodology

As mentioned before, the peridynamic theory is a non-local approach that uses Newton's second law with displacement integrals. The key point in this approach is that in a mesh, discretized material points can interact either with their nearest neighbors or with another points inside a region defined by a radius named horizon, δ.

To measure the potentiality of peridynamics as a prognostic tool, a complex computer program written in Matlab by Túlio V. B. Patriota [3] was studied. Given the difficulties to obtain analytical solutions for peridynamic problems, numerical simulations are one of the best options to understand a given problem or even to be used in practical applications. However, a simplified idea of the peridynamic theory is shown below.

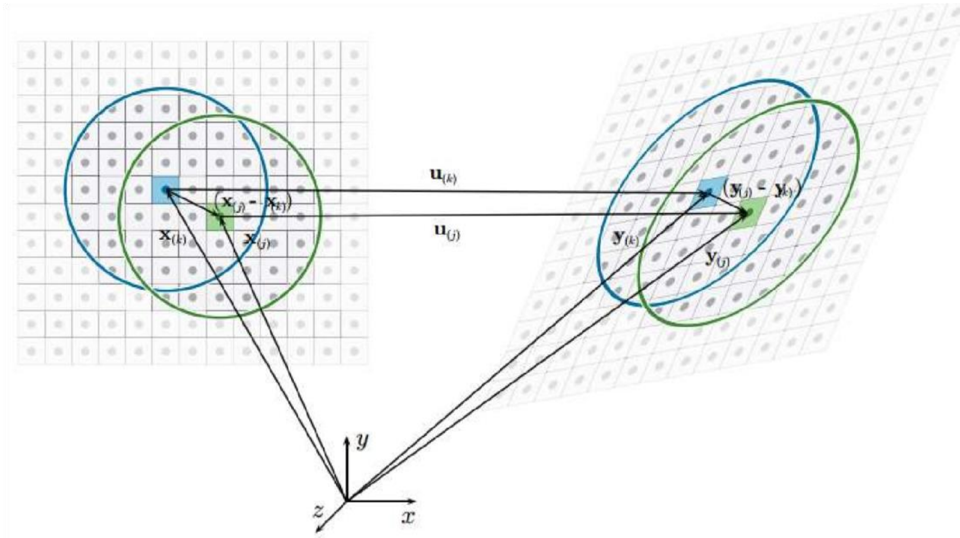

Figure 1: Peridynamic model method presented by M. Speronello [4]

Each material point have a force interaction with other material points, and when they are subjected to a external load, an unbalance occurs, leading to a deformation on the structure and a material point displacement. Figure 1 has two variables, $x_{(k)}$ and $x_{(j)}$, being the starting positions of the material points in two different horizons, $y_{(k)}$ and $y_{(j)}$, and are the same material points after an infinitesimal time interval when the body is subjected to a external load, and at last, $u_{(k)}$ e $u_{(j)}$ are the material point displacement. The analysis is time-based until a point leaves the horizon of another. When this occurs, the interaction force becomes null, characterizing the material failure, as mentioned by E. Madencie E. Oterkus [5].

Newton's second law principle applied to the variables above will result on Equation 1:

$$\rho(x)\ddot{u}(x,t) = \int_{H_x} f(t,\eta,\xi)\, dV_x + b(x,t) \quad (1)$$

In Equation 1, $\rho(x)$ is the material density, $\ddot{u}(x,t)$ is the aceleration as a function of space and time, $f(t,\eta,\xi)$ is a interaction force between the nodes or material points, dV_x is the volume of each node in the neighborhood and $b(x,t)$ is the prescribed body-force density Field.

It's important to know that the integral of the interaction force element takes place over the neighborhood H_x. It's also important to notice that there are constitutive relationships involving the interaction force between nodes. This component $f(t,\eta,\xi)$ is a function with more parameters correlated and can be written as Equation 2:

$$f(t,\eta,\xi) = R(t,\eta,\xi)cS(\eta,\xi) \quad (2)$$

There is something interesting and important on Equation 2. Na analogy can be done with Hooke's Law, where a force applied to a spring is proportional to its displacement and the spring constant. Similar to that, the micro-modulus c that represents the elastic stiffness of the Bond between nodes, is a dependent parameter of the material and $S(\eta, \xi)$ is the bond stretch. The term $R(t,\eta, \xi)$ is a Heaviside function that assumes the values 1 or 0 depending on a failure criteria.

Another important details in this function: t is the time, η is the relative displacement between two points from two different horizons and ξ is the relative position between two points from two different horizons in a undeformed state.

In this work, until this moment, it was noticed that this last relationship is extremely important to get results. Based on the numerical implementation quoted above, it's necessary for the peridynamic approach that the term inside the integral on Equation 1 does not become null. Differently from another approaches, where the interaction forces between nodes eliminate each other according to Newton's third Law (action-reaction law), in peridynamics this term must have a non-null resultant.

On the oder hand, with another approach, the finite element method has his principles based on a integral equation defining a complex domain, a body, that can be decomposed in finite integrals that define simpler domains. In this way, if it's possible to calculate all this integrals, their somatory is equal to the total complex domain, as presented by A. F. M. Azevedo [6]. Mathematically, in a brief manner, the FEM consists on solving a set of linear systems, as represented on Equation 3.

$$[A][X] = [B] \tag{3}$$

On Equation 3, X is the incognita vector of the problem, being able to represent each node displacements, A is the matrix with all the parameters representing the element stiffness and B is a vector with all the applied forces on the nodes of each element.

As a partial result, a computer program in Octave has already been written by the authors. This computer program was based on the FEM theory and it was used to simulate an example from the book "Peridynamic Theory and its Applications", from E. Madenci and E. Oterkus [5]. The example is outlined on Figure 2.

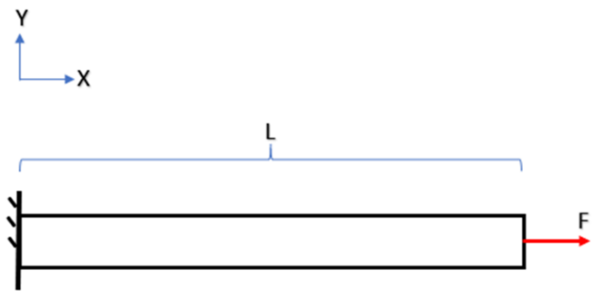

Figure 2. Constrained bar with axial load.

With all the material information present in the example, it was possible to obtain displacement results on five different points of the bar. When plotting the results obtained from the simulation against the provided results from the book, it was possible to obtain the graphic on Figure 3.

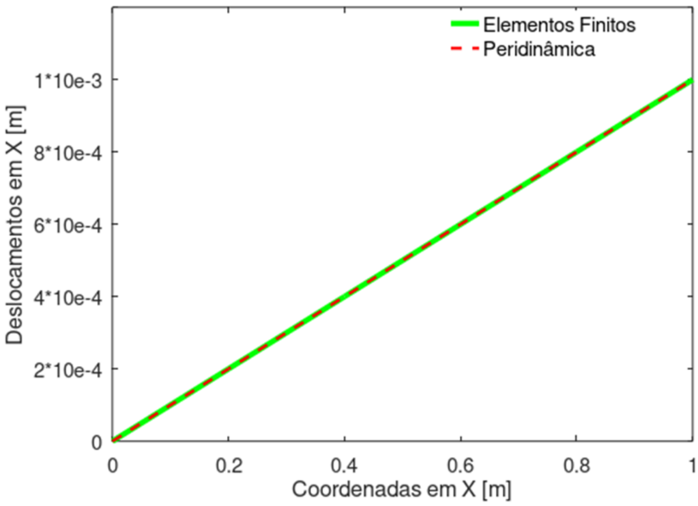

Figure 3. Displacement against coordinates on X axis.

3 Conclusions

The finite element method is already very consecrated, both in academia and in industry, and its function in this work was to validate the peridynamic method, because convergent numerical values were expected. Proof of that was the result obtained on Figure 3, even with the simplicity of the simulated example. Also important to note, a limitation in this work involving the study of the peridynamic theory and the comparison of its results was in the computer program written in Matlab by Túlio V. B. Patriota [3]. The limitation was the spatial dimensions; the program can only simulate 2D problems.

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