Program for finite element analysis of grillages with circumferential arc-shaped and straight elements

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Abstract. This paper presents the development of a program for structural linear-elastic analysis of grillages with circumferential arc-shaped elements and regular straight bars under concentrated and distributed loads. It was based on the Finite Element Method, and the code was written in FORTRAN (FORmula TRANslation). The force method was used initially to calculate the flexibility matrix and then by inversion the stiffness matrix and load vectors were determined. Results of deflections, rotations, bending, torsion and shear, from numerous examples were checked against some of those available in the scientific literature, in order to validate the code. A comparison was also made with the validation results and those calculated by the virtual work method, or extracted from two similar softwares where the curved bars needed to be approximated by various small straight elements. The program developed was shown to give closer values to the exact solution, with a much simpler input data (just angles and radiuses), than the other softwares.

Keywords: Grillages, circumferential arcs, Finite Element Method, program.

1 Introduction

Structural analysis is the stage of a construction design in which the object's behaviour is idealized and expressed in terms of stress field, strain and deflections, as pointed by Martha [1]. It is one of the most important parts of the whole project, since it simulates the real structure through some geometric and constitutive parameters to perform its calculation and analyses.

Currently, it is very unusual and considered rather malpractice to perform some tasks related to this structural analysis phase without the assistance of a software (CAD – Computer-aided design). This issue can get particularly aggravated when some of the elements in the design does not have a straight axis, such as arcs, which is considered one of the greatest tensional invention of classical art, as stated by Torroja [2].

Silva & Souto [3] remarks that throughout history, arcs have been employed to fulfil mainly four objectives: shelter, transit, conduction, and containment. Circular arcs are also commonly used as beam elements to support balconies in some buildings, working as grillages, which are tridimensional structures subjected to loading perpendicular to their plane. The grillage system is frequently used in foundations, shipbuilding, and bridges, as commented by Moses & Onoda [4].

There are not many free softwares (initially developed for academic purposes) that deal with grillage analysis in Brazil, and even fewer (if any) that include curved bars in their codes. Good examples of the former are the VGPlan 0.2 [5] from University of São Paulo, and LESM 2.0.1 [6], from Pontifical Catholic University of Rio de Janeiro. Neither of those programs contain curved elements.

In this context, this paper presents the development of a software called PAE-Grelhas 1.0 for linear analysis of grillages with both circumferential arc-shaped and straight elements. It is based on the Finite Element Method, written in Fortran, with its graphical user interface (GUI) in the works similarly to that of Oliveira et al. [7].

2 Methodology

Initially, a code for linear analysis of grillages with only straight bars subjected to distributed and concentrated loads were developed as the baseline program. That was chosen based on the fact that its stiffness matrix and load vector are already well known and documented, as can be seen, for example, in Martha [1]. The output displays, besides the expected vertical displacement, rotations, shear, bending, torsion and support reactions, all the local and global vectors and matrices to facilitate calculations' check. That feature is optional in the final version of PAE-Grelhas 1.0 (with the curved element), though it seems to be a very attractive functionality, especially for academia.

The stiffness matrix for the circular arc-shaped bar was determined by inverting the flexibility matrix found for the generic arc shown in Fig. 1 when the force method is applied. A similar methodology was used for the load vector with uniformly distributed loads (UDLs), and it can be easily extrapolated for other types, such as linear ones. A detailed presentation of those integrals can be found in the appendices of Sawko [8]. Their implementation was pretty straight forward once those matrices were checked against some simple examples.

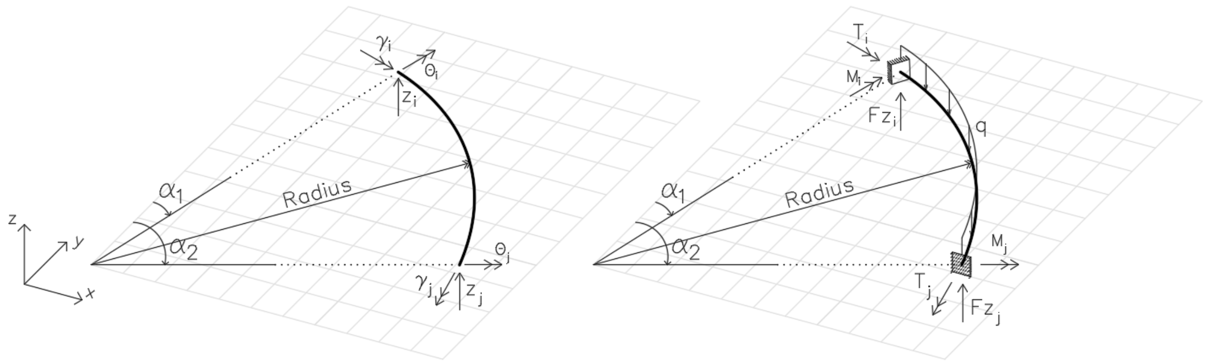

Figure 1: Generic circumferential arc-shaped element

The final program requires an input data (a *.txt* file for now) where all the information concerning the grillage to be analysed is gathered. In this module, nodal position, concentrated loads, support parameters, elements, angles, radius, material and geometric properties, and uniformly distributed loads are defined.

With the input data, the code is structured as presented in the flowchart in Fig. 2 and summarized below following the direct stiffness method procedure:

- Reading and storage of input variables;
- Calculation of each elements' length and orientation (rotation matrices);
- Assemblage of all local stiffness matrices and load vectors (UDL's nodal equivalent forces and moments);
- Assemblage of global stiffness matrices and load vectors;
- Inclusion of nodal concentrated loads in the global load vector;
- Consideration of boundary conditions in both global stiffness matrix and load vector;
- Calculation of displacement vector;
- Determination of nodal internal forces and support reactions;
- Output.

As previously stated, the output is another *.txt* summarizing all the input information, matrices, vectors, and calculated data.

Once the code was completed, it was thoroughly debugged with examples from the scientific literature (Wang [9]), real designed grillages, and exercises developed by the authors. Those problems were calculated using the virtual work method along with the force method. In order to stablish a comparison, two others academic softwares that perform linear analysis of grillages (VGPlan 0.2 [5] and LESM 2.0.1 [6]) were used where the curved bars needed to be approximated by various small straight elements. The following section will discuss some of the validation aforementioned as well as present a few results where PAE-Grelhas 1.0 accuracy can be verified.

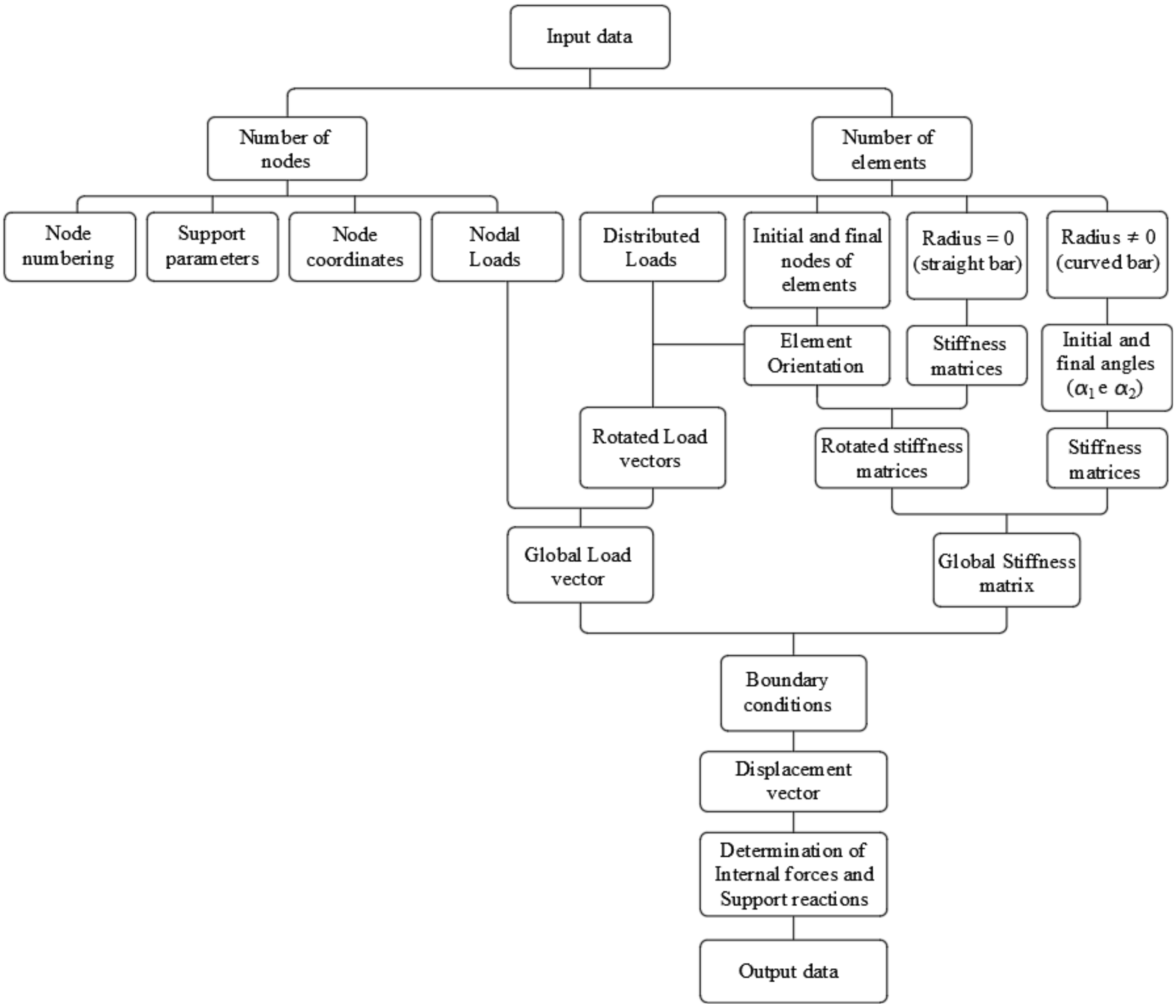

Figure 2: Flowchart of program processing logic.

3 Results

Figure 3 shows a grillage structure that was chosen to represent the validation since it includes two circumferential curved bars subjected to UDLs, which is the distinguishing feature available in PAE-Grelhas 1.0. Table 1 gives more information on the properties assumed for the problem, which was created by the authors. The input data is shown in Fig. 4, and the curved elements only require an initial and final angle, and their radiuses (refer to Fig. 1). For the calculation with other softwares, both arcs which have a similar geometry (90º with a radius of 2.0 m and 2.5 m) needed to be discretized using a few straight bars (12 and 15, respectively). Each of those elements had around 26 cm.

Table 2 presents the results for the proposed structure with: an analytical solution (from virtual work method), used as the main reference, calculated with all possible decimal places; a solution from LESM 2.0.1 [6]; a solution from VGPlan 0.2 [5]; and a solution from PAE-Grelhas 1.0.

Table 1: Properties

Parameters	Notation	Value
Young's modulus	E	$2.5 \cdot 10^7$ kN/m^2
Poisson's ratio	ν	0.25
Shear modulus	G	$1.0 \cdot 10^7$ kN/m^2
Moment of Inertia (Bending)	I_f	$7.2 \cdot 10^{-3}$ m^4
Moment of Inertia (Torsion)	I_t	$7.526 \cdot 10^{-3}$ m^4

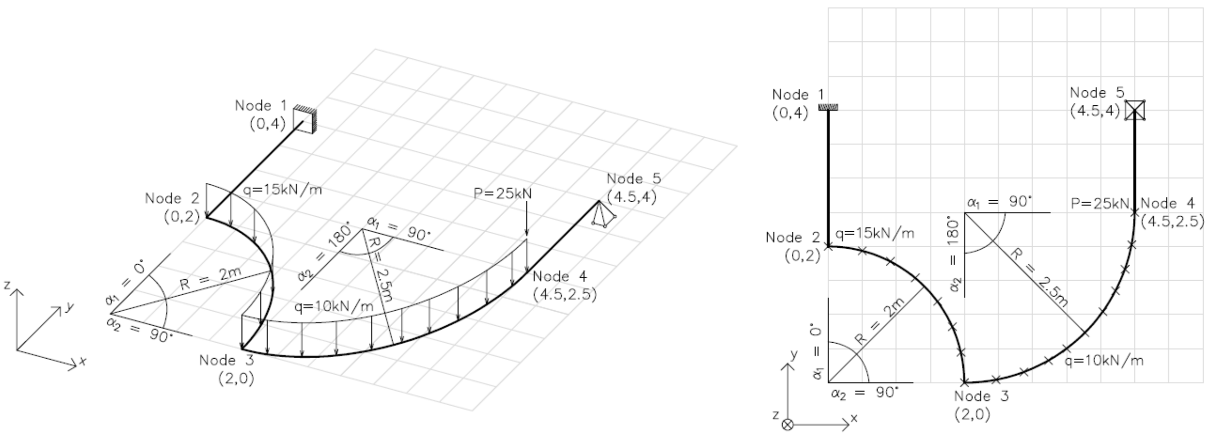

Figure 3: Grillage structure used for validation

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****--------------------------------- ARQUIVO DE ENTRADA DE DADOS ----------------------------------****
****----------------------------------------- PAE-Grelhas ------------------------------------------****

|Nº NÓS| & |Nº ELEMENTOS|

 5          4

 |NÓ|   |COORDENADA X|    |COORDENADA Y|     |Fz|        |Mx|        |My|   |RESTRIÇÃO Desloc_z|   |RESTRIÇÃO Rot_x|   |RESTRIÇÃO Rot_y|

  1         0.0              4.0           0.0         0.0         0.0             1                     1                  1
  2         0.0              2.0           0.0         0.0         0.0             0                     0                  0
  3         2.0              0.0           0.0         0.0         0.0             0                     0                  0
  4         4.5              2.5         -25.0         0.0         0.0             0                     0                  0
  5         4.5              4.0           0.0         0.0         0.0             1                     0                  0

|ELEMENTO|   |NÓ INICIAL (i)|   |NÓ FINAL(j)|      |Raio|     |Alfa1|     |Alfa2|     |E|       |v|       |If|        |It|       |Qz|

  1              1                  2              0          0          0        2.5e7     0.25     7.2e-3    7.526e-3    0.0
  2              2                  3              2          0         90        2.5e7     0.25     7.2e-3    7.526e-3    -15
  3              4                  3            2.5         90        180        2.5e7     0.25     7.2e-3    7.526e-3    -10
  4              4                  5              0          0          0        2.5e7     0.25     7.2e-3    7.526e-3    0.0
```

Figure 4: Input data in PAE-Grelhas 1.0

Table 2: Comparison of different solutions

Node	Analytical Result	LESM	Error (%)	VGPlan	Error (%)	PAE-Grelhas 1.0	Error (%)
Support Reaction Node 1 - Desloc_z (kN)	70.758538	70.855052	0.136399	70.855263	0.136697	70.757505	0.001461
Support Reaction Node 1 - Rot_x (kN·m)	-287.400421	-287.748078	0.120966	-287.747986	0.120934	-287.400434	0.000004
Support Reaction Node 1 - Rot_y (kN·m)	-130.681148	-130.806872	0.096207	-130.808014	0.097080	-130.676493	0.003562
Support Reaction Node 5 - Desloc_z (kN)	40.635259	40.654819	0.048136	40.654243	0.046717	40.636296	0.002550
Vertical translation Node 5 (m)	-0.0198971	-0.0199370	0.200307	-0.0199357	0.193773	-0.0199022	0.025417

As can be seen in Tab. 2, all softwares performed reasonably well with the grillage in Fig. 3. However, besides been the one with the smallest error, PAE-Grelhas 1.0 has the easiest input parameters, requiring only two angles and the radius for each arc, while all the other solutions are discretization-dependable and have onerous input. It is worth mentioning that PAE-Grelhas 1.0 was initially expected to give the exact solution. The error displayed in Tab. 2, as small as it was, needed to be considered and investigated. It is believed that when solving the complex trigonometric integrals for the arc's flexibility matrix (refer to Sawko [8]), the software used for those calculations applied some numerical integration that led to an approximated general expression.

The authors believe that while this issue does not invalidate the results from the program, considering that it has overperformed the other softwares in various other examples for typical civil construction parameters, an implementation of a numerical integration such as Gauss quadrature with a specific number of points should provide a more "controlled" approximation.

4 Conclusions

The program developed in this work, entitled PAE-Grelhas 1.0, enables a linear-elastic analysis of grillage structures with circumferential arc-shaped elements and regular straight bars under concentrated and distributed loads, providing the internal forces, vertical translation, rotations, and support reactions, with a much simpler input data (just angles and radiuses) than similar available softwares, serving as another tool in the teaching/learning process as well as research in related fields.

Improvements over this first version are already under way, including a numerical integration with Gauss quadrature for the arc's flexibility matrix, a graphical interface similar to Oliveira et al. [7], and interpolation functions in order to obtain the whole deformed shape for curved elements.

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