Static stochastic analysis in cylindrical panels' geometry

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Abstract. Cylindrical panels under uncertainties, described by a uniform probability density function, on parameters such as thickness and radius, are investigated when static loading is submitted. Firstly, an analytical approach - based on the equilibrium equations governed by Donnell's nonlinear shallow shell theory, Airy's stress function and standard Galerkin method - is taken to evaluate the effects of parameters uncertainties on the buckling load and post critical nonlinear equilibrium. Then, an approach based on the finite element method is considered to model the same geometry where the mesh is composed by shell elements and its convergence is conducted in terms of buckling load. The nonlinear equilibrium path is obtained through the modified Riks method and a perturbation parameter. In both methodologies, a set of deterministic samples simulates the stochastic system, which are evaluated from Chi-Squared hypothesis test. The uncertainty in the thickness results in a stochastic system where the nonlinear equilibrium path can be described as a uniform probability distribution. On the other hand, the radius shows a stretching along the curve where a two-component Gaussian mixture fits better the obtained response where the mean of axial load as respect to a certain displacement cannot be represented by the mean of its lower and upper boundaries.

Keywords: Stochastic, Cylindrical panel, Thickness, Radius, Uniform distribution

1 Introduction

The cylindrical panel is a geometry established as a circular sector from a cylindrical shell. It has several applications in civil, mechanical, naval, nuclear and aerospace engineering, such as component of planes, roofs, submarines, cooling towers and aquariums. Due to its slender aspect, cylindrical panels can lose its stability when submitted to static loading. Therefore, the influence of an initial geometric imperfection and lamina lay-up sequences on post buckling response are currently investigated in RamanaReddy, Gunda and Padal [1] through finite element analysis (FEA). Also, it turns relevant to evaluate the sensibility of the system's response to uncertainties parameters, which occur due to manufacturing issues, as presented in Palla and Silva [2].

Imperfect thin-walled isotropic cylindrical shells axially loaded are presented in Papadapoulous, Stefanou and Papadraksis [3], taking into account the combined effect of thickness, Young's modulus, geometric and boundary conditions on buckling load. The randomness are treated as non-Gaussian distribution (lognormal, beta, U-shaped beta and L-shaped beta) via FEA and Monte Carlo Simulation (MCS) methods. Considering a similar approach, Papadapoulos and Papadrakasis [4] analysis the same scenario but the uncertainties are taken as Gaussian assumptions. It is shown that the choice of probability density function affects significantly the buckling load and the lognormal and beta probability density functions fit better to experimental results.

In the present paper, cylindrical panels' stability is conducted, considering the effects of uncertainties in the thickness and radius parameters, which are described by a uniform probability density function. Two different approaches are presented. Firstly, an analytical approach, which requires Donnell's nonlinear shallow shell theory and a modal solution to transversal displacement field, proposed in Morais and Silva [5], Airy's stress function and a standard Galerkin method are necessary to describe the proposed problem. A FEA, implemented in a commercial software Abaqus® is the other approach evaluated in this work. A set of deterministic samples simulates the stochastic system, and the nonlinear equilibrium path's response is evaluated through Chi-Squared method.

2 Mathematical formulation and methodology

The geometry studied in this text is described in Fig. 1a where its geometric parameters are thickness h, radius R, length L and opening angle Θ. The thin cylindrical panel presents a linear, homogeneous and isotropic elastic material, which properties are described by Young's modulus E and Poisson's ratio ν. As for the boundary conditions, the geometry is simply supported, as shown in Fig 1b, and it is applied a static axial load uniform distributed in the panels edge, as illustrated in Fig. 1c.

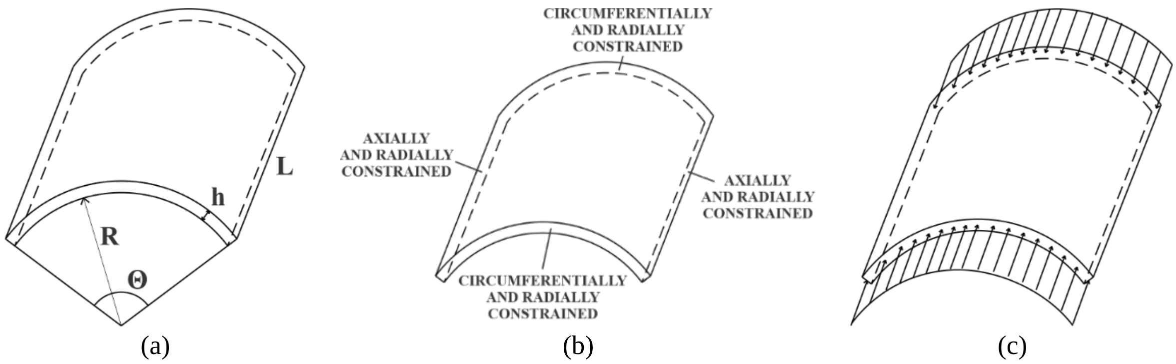

Figure 1. Cylindrical panel: (a) geometric parameters; (b) boundary conditions and (c) applied axial load

2.1 Analytical approach

The geometrically nonlinear strain-displacement relationships are based on Donnell's nonlinear shallow shell theory which is appropriate for shallow cylindrical panels where $h/R \leq 1/20$. Thus, the motion equations turn into a system of two partial differential equations as function of Airy's stress function $f(x,\theta)$ and the transversal displacement w:

$$D\nabla^4 w = \frac{1}{R} f_{,xx} + \frac{1}{R^2}\left[f_{,\theta\theta}\left(w_{,xx}\right) - 2f_{,x\theta}\left(w_{,x\theta}\right) + f_{,xx}\left(w_{,\theta\theta}\right)\right] + p_z$$
$$\frac{1}{Eh}\nabla^4 f(x,\theta) = \frac{1}{R^2}\left({w_{,x\theta}}^2 - w_{,xx}w_{,\theta\theta} - Rw_{,xx}\right), \tag{1}$$

where D refers to the cylindrical panel's flexural stiffness, which is expressed in eq (2), ∇^4 is the biharmonic operator in the cylindrical coordinate system, x and θ are the axial and circumferential coordinates, respectively, and p_z is the static loading.

$$D = \frac{Eh^3}{12(1-\upsilon^2)}. \tag{2}$$

Airy's strain function is a partial differential equation, whose solution is related to the applied load (homogeneous solution) and the transversal displacement field (particular solution). The transversal displacement w for cylindrical panel was proposed by Morais and Silva [5] and it is used in this work, attending the boundary conditions for a simply supported cylindrical panel and taking into account the main modal coupling that occur in this structural system:

$$w = C_1 \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi\theta}{\Theta}\right)$$
$$+ C_2\left[\frac{3}{4}\cos\left(\frac{2m\pi x}{L}\right) + \frac{1}{4}\cos\left(\frac{4m\pi x}{L}\right)\right]\left[\frac{3}{4} - \cos\left(\frac{2n\pi\theta}{\Theta}\right) + \frac{1}{4}\cos\left(\frac{4n\pi\theta}{\Theta}\right)\right], \tag{3}$$

in which C_1 and C_2 are the modal amplitudes of the transversal displacement field, m and n are the half-wave in axial and circumferential directions, respectively.

Then, after the analytical solution of Airy's stress function, the standard Galerkin method is applied to discretize the first equation of eq. (1), obtaining a nonlinear algebraic equation system. Since the modal amplitudes of displacement are coupled, Newton-Raphson's method is required to solve the discretized system to obtain the nonlinear equilibrium path.

2.2 FEA approach

In this approach, the cylindrical panel is composed by shells' elements, named as S4R, using the FEA software Abaqus®. This shell element is a 4-node, quadrilateral with reduced integration element where its mesh convergence is based on the buckling load value. The obtained FEA results are compared with analytical results. In FEA, the nonlinear equilibrium path is obtained through the modified Riks method and a perturbation parameter, whose use is responsible for destroying the bifurcation point corresponding to the buckling value, as similar as the geometric imperfection effects on post buckling equilibrium path. The middle node from the mesh was evaluated due its higher displacements because the first buckling eigenvalue leads to the buckling modes m=1 and n=1 in the numerical results.

2.3 Stochastic considerations

Deterministic samples of cylindrical panels are generated with its evaluated parameter established as any possible value between ±10% of the nominal value, being described by a uniform probability density function. The number of samples required is based on the mean and variance of critical load of the cylindrical panel. The stochastic system of nonlinear equilibrium paths are evaluated through sets of axial loads obtained in three different values of transversal displacements (0.01 m, 0.02 m and 0.03 m) and tested in the Chi-Squared method.

3 Numerical Results

The evaluated cylindrical panel has nominal parameters: thickness h = 0.01 m, radius R = 8.333 m, length L = 1.00 m, opening angle Θ = 0.12 rad, Young's modulus E = 210 GPa and Poisson's ratio ν = 0.3. As a result of the convergence study of FEA approach, the mesh has 870 shell elements (S4R). When it is investigated the random variable, which are thickness and radius, acting individually, the obtained buckling modes for both approaches are in disagree as respect to the half-waves number, as shown in Table 1:

Table 1. First buckling mode response for the approaches

h (m)	FEA	Analytical	R (m)	FEA	Analytical
0.009	m = 2 n = 1	m = 1 n = 1	7.500	m = 2 n = 1	m = 1 n = 1
0.00971			8.09982		
0.00972	m = 1 n = 1		8.09983	m = 1 n = 1	
0.011			9.166		

Since the first buckling mode is directly responsible for the nonlinear equilibrium path configuration, the random variables range are considered only inside the interval which both approaches have the same response m = 1 and n = 1. So, panels thickness can assume any value in h = [0.00972, 0.011] and radius in R = [8.09983, 9.166] according to a uniform distribution probability density function. The middle node coordinates is established as (r, θ, z) = (8.333, 0.062, 0.5). Firstly, the nonlinear equilibrium paths of stochastic system are obtained considering 1200 samples of its random variables.

3.1 Stochastic response on variables' individually assumption

The overall response of the stochastic system's nonlinear equilibrium path, displayed in Figure 2, when thickness is itself assumed as a random variable shows that for both approaches and for the three selected displacements evaluated the strength's values sets can be well represented by a uniform distribution as concluded in Table 2. Thus, this parameter does not imply such a nonlinear contribution in this problem and the mean's loading value of the interval can be assumed as the mean of its lower and upper boundaries, regardless the displacement evaluated. On the other hand, the radius' scenario does not follow the same. This parameter shows in Figure 2 a stretch along the system widely nonlinearly influenced in both approaches. This region does not fit properly according to a uniform distribution. There is also a difference between analytical and FEA procedures related to the nonlinearity segment along the path, which reveals perhaps the influence of the perturbation parameter required in FEA approach.

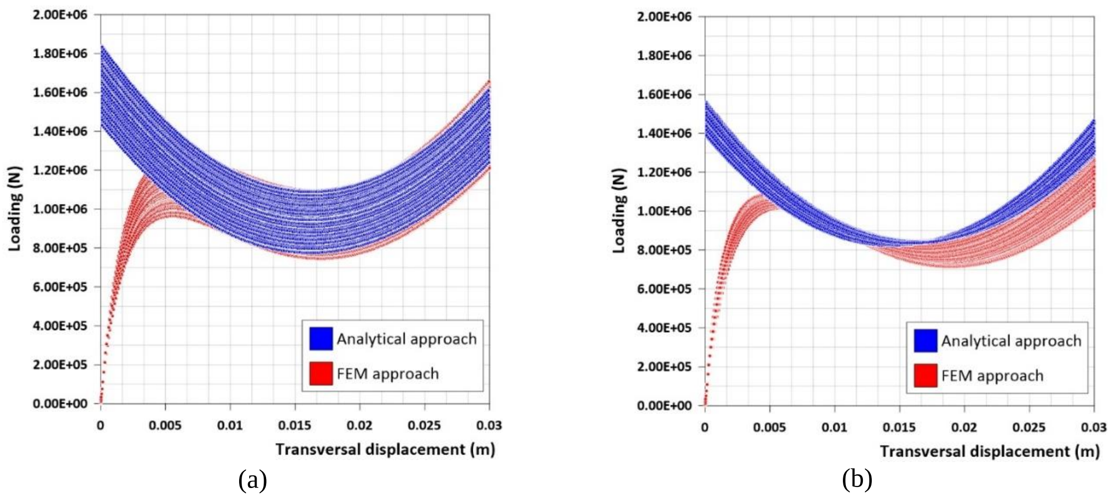

Figure 2. Stochastic system's nonlinear equilibrium path due to effect of: (a) thickness; (b) radius

Since the uniform distribution does not fit properly some radius' stretch effect, others are investigated, such as: Gaussian, Gamma, Lognormal, Two Gaussian Mixture Model, Largest Extreme Value, Smallest Extreme Value, Triangular, Rayleigh and Half Normal. The lowest chi-square χ^2 happens in both approaches with the two Gaussian model distribution. Table 2 shows the results of best distribution fit pro evaluated displacement.

Table 2. Best probability distribution fit

Parameter	Approach	d = 0.01 m (33%)	d = 0.02 m (66%)	d = 0.03 m (final)
Thickness h	Analytical	Uniform (χ^2 = 71.55)	Uniform (χ^2 = 72.90)	Uniform (χ^2 = 85.80)
	FEA	Uniform (χ^2 = 11.40)	Uniform (χ^2 = 12.33)	Uniform (χ^2 = 6.93)
Radius R	Analytical	Uniform (χ^2 = 68.13)	Two Gaussian mixture (χ^2 = 278.60)	Uniform (χ^2 = 122.96)
	FEA	Two Gaussian mixture (χ^2 = 105.39)	Uniform (χ^2 = 67.87)	Uniform (χ^2 = 38.67)

3.2 Stochastic response on combined variables' assumption

When both variables are evaluated together, some samples from the 1200 generated through the FEA approach have their buckling mode different from half-waves m = 1 and n = 1, as shown in Figure 3. Therefore, these 151 samples are not accounted to evaluate the nonlinear equilibrium path stochastic system's response. Despite that the convergence remains. Table 3 shows that in this case the mean of sets of axial load cannot be represented as the mean of its lower and upper boundaries. Furthermore, the two approaches disagree with each other on the best probability density function - two Gaussian mixture fits better analytical procedure and triangular FEA.

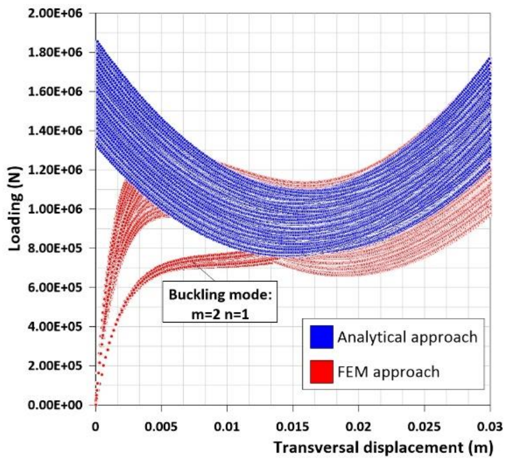

Figure 3. Stochastic system's nonlinear equilibrium path due to effect of thickness and radius combined

Table 3. Best probability distribution fit

Parameter	Approach	d = 0.01 m (33%)	d = 0.02 m (66%)	d = 0.03 m (final)
Thickness h and radius R	Analytical	Two Gaussian mixture (χ^2 = 51.35)	Two Gaussian mixture (χ^2 = 56.15)	Two Gaussian mixture (χ^2 = 42.16)
	FEA	Triangular (χ^2 = 82.45)	Triangular (χ^2 = 7.53)	Triangular (χ^2 = 9.40)

4 Conclusions

When variables are individually assumed as random parameters, in thickness' case both approaches lead to a uniform probability density function as suitable. On the other hand, the radius' scenario shows a stretching along the curve where a two-component Gaussian mixture fits better the obtained response, but this region does not situate the same in the analytical procedure and FEA one. Finally, the stochastic response on combined variables' assumption is also not suitably represented as a uniform probability density function, so the mean of sets of axial load cannot be represented by the mean of its lower and upper boundaries. A two-component Gaussian mixture represents better the sets of axial load in an analytical approach. A Triangular fits instead it better in FEA one.

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References

[1] D.V. RamanaReddy, J.B. Gunda and K.T.B. Padal, Post-buckling Behaviour of Imperfect Cylindrical Panels Subjected to Axial Compressive Load: Experimental vs. Theoretical. *International Conference on Advancements in Aeromechanical Materials for Manufacturing*, 2016.
[2] A.E.F. Palla and F.M.A. Silva, Physical and geometric parameters uncertainty effect on the nonlinear dynamics of cylindrical panels. *XLI Ibero-Latin American Congress on Computational Methods in Engineering*, 2020.
[3] V. Papadapoulos, G. Stefanou and M. Papadrakasis, Buckling analysis of imperfect shells with stochastic non-Gaussian material and thickness properties. *International Journal of Solids and Structures*, v. 46, p. 2800-2808, 2009.
[4] V. Papadapoulos and M. Papadrakasis, The effect of material and thickness variability on the buckling load of shells with random initial imperfections. *Computed Methods in Applied Mechanics and Engineering*, v. 194, p. 1405-1426, 2005.
[5] J.L. Morais and F. M. A. Silva, Influence of modal coupling and geometrical imperfections on the nonlinear buckling of cylindrical panels under static axial load. *Engineering Structures*, v. 183, p. 816-829, 2019.