Influence of Non-linear Damping on Non-linear Structures Vibrations

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Abstract. The study of linear and non-linear dynamic behavior of structures is an area that has received constant attention in recent decades, due to the fact that the structures have become increasingly light and slender, causing vibration problems to be taken into account during the project. Many studies have been developed to try to understand the nonlinear response of structural systems, especially chaotic vibrations considering different types of nonlinearities in systems with gain or loss of stiffness. In this sense, many studies have been developed considering linear damping, but in many applications, it is necessary to consider non-linear damping, such as drag forces or vibration isolation, which mainly affect the non-linear dynamic response of systems. This work aims to study the role of non-linear damping in non-linear oscillations of a discrete system with a degree of freedom. It is considered a discrete system of a degree of freedom and subjected to variable loads over time considering three types of damping (linear, quadratic and cubic). Resonance curves are obtained for various load and damping values in order to observe their influence on the dynamic instability of the system.

Keywords: nonlinear damping, nonlinear dynamics, structures.

1 Introduction

The study of linear and non-linear dynamic behavior of structures is an area that has received constant attention in recent decades, due to the fact structures have become increasingly light and slender, causing vibration problems to be taken into account during the project. Many investigations have been developed to try to understand the non-linear response of structural systems, especially chaotic vibrations considering various types of nonlinearities in systems with gain or loss of stiffness.

Among the various parameters of the structures, damping plays a fundamental role with regard to phenomena of loss of stability and in the order of magnitude of oscillations, but as the correct determination of damping is still a great difficulty, the right choice of this parameter is of great scientific interest.

In this sense, the study of the value of damping is justified, because it defines the limits of stability or instability. In addition, many studies have been developed considering linear damping, but in many applications, it is necessary to consider nonlinear damping, such as drag forces or vibration isolation, which mainly affects the non-linear dynamic response of systems.

In the literature it is possible to find several studies related to the effect of non-linear damping and non-classical damping on non-linear vibrations of dynamic systems, where chaotic, near-periodic vibrations, attraction, and escape basins as well as multiple resonance and coupling phenomena were studied [1, 2, 3, 4, 5, 6, 7].

This work aims to study the role of non-linear damping (linear, quadratic and cubic) in the non-linear oscillations of dynamic systems governed by the Duffing equation. It is intended to develop a study to evaluate the non-linear oscillations of rigid bars considering geometric nonlinearity as well as non-linear damping when subjected to variable external loads over time.

2 Mathematical Formulation

It is considered a rigid bar of length *L*, density *r* and cross section *A*, fixed at the base by an elastic rotational spring with linear stiffness (k_1),quadratic (k_2) and cubic (k_3), with linear damping (C_1),quadratic (C_2) and cubic (C_3), subjected to axial (P_2 sin(Ωt)) and lateral (P_1 cos(Ωt)) harmonics with frequency Ω acting at the top and with q rotation at the base as seen in Figure 1.

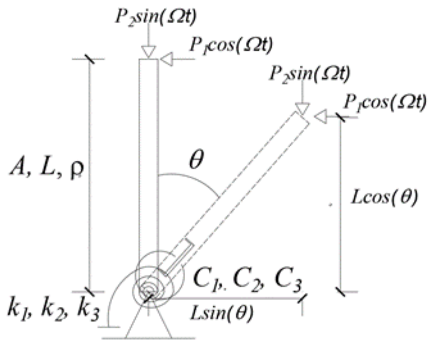


Figure 1 – Rigid bar model.

To obtain the dynamic equilibrium equation of the system the deformation energy *(U)* of the system is given by:

$$U = \frac{1}{2}k_1\theta^2 + \frac{1}{3}k_2\theta^3 + \frac{1}{4}k_3\theta^4 \tag{1}$$

The work (*W*) of external forces is given by:

$$W = -P_1 cos(\Omega t) L sin(\theta) + P_2 sin(\Omega t) L[1 - cos(\theta)] \tag{2}$$

The work of non-conservative forces can be written as:

$$W_{nc} = \frac{1}{2}C_1\dot{\theta}^2 + \frac{1}{3}C_2\dot{\theta}^3 + \frac{1}{4}C_3\dot{\theta}^4 \tag{3}$$

The kinetic energy*(T)* of the system is obtained from the kinetic energy of a different element of mass *dm*, located at a distance *r from* the base as well:

$$T = \int_M \frac{1}{2}\dot{r}^2 dm = \frac{1}{6}\rho A L^3 \dot{\theta}^2 \tag{4}$$

The Lagrangian ($\mathcal{L}$) function can be written as:

$$\mathcal{L} = T - U + W \tag{5}$$

Replacing the Eq. (1), (2) and (3) in eq. (4), one comes to:

$$\mathcal{L} = \frac{1}{6}\rho A L^3 \dot{\theta}^2 - \left(\frac{1}{2}k_1\theta^2 + \frac{1}{3}k_2\theta^3 + \frac{1}{4}k_3\theta^4\right) + P_1 cos(\Omega t) L sin(\theta) + P_2\, sin(\Omega t) L[1 - cos(\theta)] \tag{6}$$

Applying the Hamilton principle given by Eq. (7)

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} - \frac{\partial \mathcal{L}}{\partial \theta} = -\frac{\partial W_{nc}}{\partial \dot{\theta}} \tag{7}$$

You will find the non-linear dynamic equilibrium equation of the system given by:

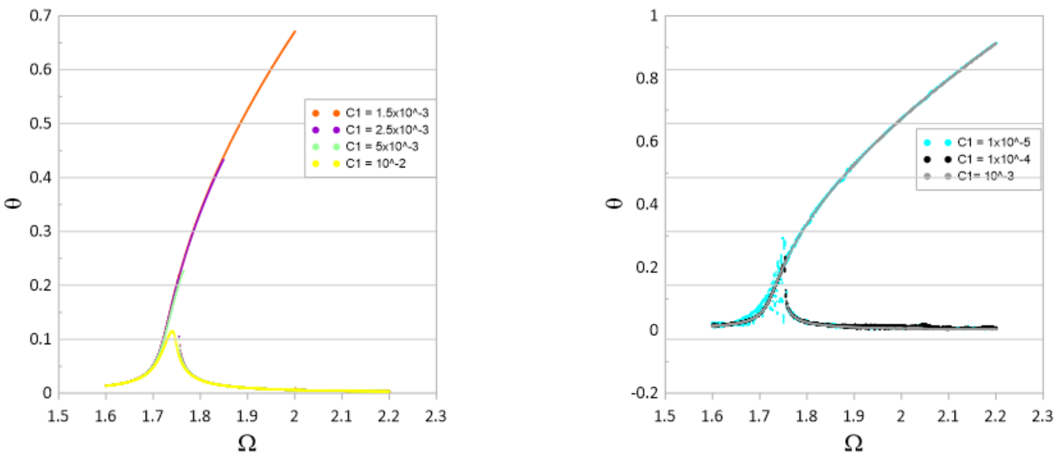
$$\frac{1}{3}\rho A L^3\ddot{\theta} + C_1\dot{\theta} + C_2\dot{\theta}|\dot{\theta}| + C_3\dot{\theta}^3 + k_1\theta + k_2\theta^2 + k_3\theta^3 = P_1 cos(\Omega t) L cos(\theta) + P_2\, sin(\Omega t) L sin(\theta) \tag{8}$$

3 Results

For the development of analyses of the dynamic behavior of the rigid bar of Figure 1, the following numerical properties were used: $k_1 = 1.0$; $k_2 = 0$; $k_3 = 1.0$; $L = 1.0$; $P_1 = 0.002$; $P_2 = 0$; $\rho = 1.0$. While the parameters C_1, C_2 and C_3 were varied for different numerical values to evaluate the influence of damping on the nonlinear dynamic behavior of the bar.

The resonance curves of the system were obtained by adopting as a parameter the frequency of lateral load, obtained by applying the crude force method and the numerical integration of the Runge-Kutta method, considering incremental values of linear, quadratic and cubic damping coefficients. Resonance curves aim to evaluate the influence of the type of damping on the dynamic instability of the system.

Figure 2 presents the resonance curves considering only incremental values of viscous linear damping C_1 varying its value in the range of 10^{-2} to 10^{-5}. As can be seen in Fig. 2 (a), for high damping values, the curve presents linear behavior but, as the damping value is reduced, the curves exhibit hardening behavior of resonance. For low values of the damping coefficient, the point of dynamic instability is increasing.



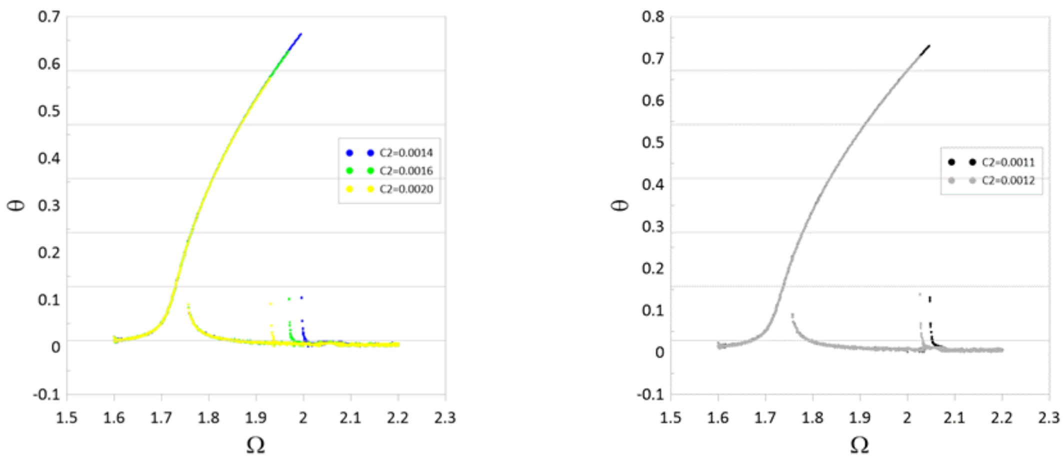
(a) (b)

Figure 2 - Parametric analysis of isolated linear damping.

Figure 3 presents the resonance curves considering only incremental values of quadratic damping with intensity variation of the damping parameter C_2 in the range of $2.0\text{x}10^{-3}$ to $1.2\text{x}10^{-3}$. As can be observed, all curves show hardening type behavior without *presenting* linear behavior and the point of dynamic instability (jump) is quite similar for all values *of* C_2. It is also possible to observe that, as the damping value is increased, there is the appearance of a new branch of instability around $\Omega = 2.0$, a phenomenon that was not observed when considering viscous linear damping.

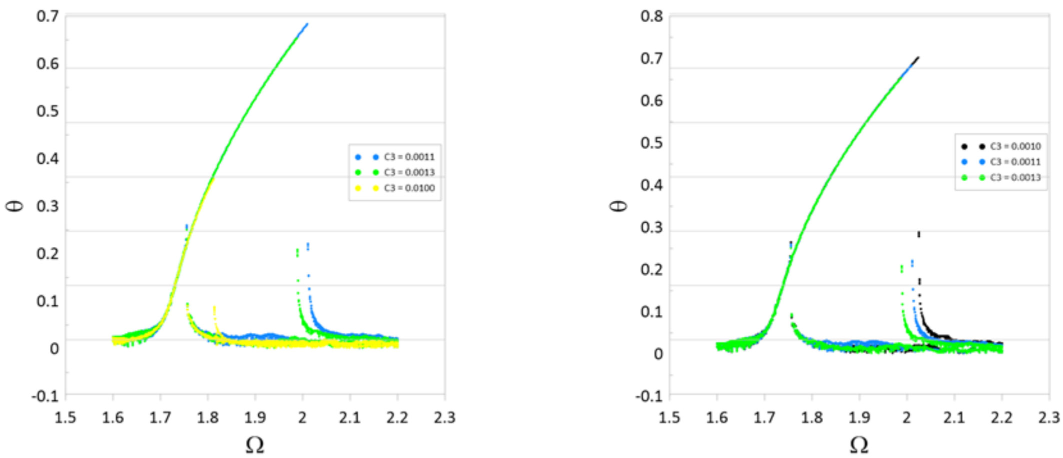
Figure 4 presents the resonance curves considering only incremental values of cubic damping with intensity variation of the damping parameter C_3 in the range of $1.0\text{x}10^{-2}$ to $1.3\text{x}10^{-3}$. As can be seen, the curves again exhibit hardening behavior, but this time it is possible to observe more clearly the effect of damping on the non-linear vibrations of the system. As the value of the cubic damping coefficient is reduced, the resonance curves present incremental values in the displacement and point of greater dynamic instability. It can also be observed that there is the emergence of a new branch around $\Omega = 2.0$ as well as in the region of small amplitude vibrations, the amplitudes are variable depending on the value of the damping.

The resonance curves presented describe the great influence of damping on nonlinear dynamic behavior, and allow us to observe that depending on the damping chosen, the system may present new branches of dynamic instability.



(a) (b)

Figure 3 - Parametric analysis of quadratic non-linear damping isolated.



(a) (b)

Figure 4 - Parametric analysis of cubic damping isolated for higher numerical values.

4 Conclusions

In this work, the influence of linear, quadratic and cubic damping on the nonlinear dynamic behavior of a system with geometric cubic nonlinearity was studied when submitted to harmonic load action.

Resonance curves were obtained for incremental values of damping coefficients and, in all cases, hardening behavior was observed, which is typical for structures with non-linear cubic stiffness.

The first immediate and natural effect observed is the abatement of the curve, as the damping factor increases, both for cases of vibration with linear damping and with non-linear. Thus, it is possible to observe that the value of the linear or non-linear damping coefficient is sufficiently significant to the point of changing the order of magnitude of the maximum deflections calculated for the structure.

When linear damping is considered, resonance curves are typical and for low damping values, the point of instability presents large amplitude vibrations. On the other hand, when quadratic or cubic damping is considered, the resonance curves present the emergence of a new branch of vibrations, suggesting a new bifurcation point in the region of small amplitude vibrations. Another characteristic due to nonlinear damping is that the region of small amplitude vibrations presents distinct amplitudes for distinct damping values.

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