A SIMP-based algorithm to maximize natural frequencies in two-dimensional structures

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Abstract. This paper aims to study the topology optimization of two-dimensional structures based on their dynamic response by implementing a computational algorithm that optimally distributes the material in a project domain based on the SIMP approach (Solid Isotropic Material with Penalization). It allows for a well-defined result and therefore a layout that can be manufactured since it penalizes intermediate densities thus eliminating them from the final result. The model uses the Finite Element Method (FEM) for spatial discretization and to evaluate the objective function, the restraints, and the sensitivities through iterations. Since it does not depend on the initial representation, the solution of a topology optimization problem can be represented with a high degree of geometric complexity, requiring a certain level of refinement of the finite element mesh. This refinement can cause sub-regions of the domain resembling a checkerboard pattern, which is avoided when a sensitivity filter is used. The optimization objective function is taken as the lowest eigenfrequency of the structure, as to avoid resonance, a common problem in civil and mechanical structures.

Keywords: Topology Optimization; SIMP; dynamic analysis; checkerboard patterns; FEM.

1 Introduction

Structural optimization aims to determine an optimal solution, defined in a design domain, that favors mechanical performance while satisfying all the constraints imposed by the model. Technological advances and research in numerical methods have boosted the implementation of computational algorithms in the elaboration of structural projects, providing results of high reliability and simplifying the steps of verifying calculation criteria and also of model constraints. Moreover, there is a search for the conception of slender vertical structural arrangements that must be able to withstand the loads imposed on them with permissible displacements. Therefore, it is evident the need to improve the methodology for the elaboration of projects of less flexible structural arrangements or the use of vibration control devices that bring the rigidity of the system, used in mitigating the dynamic response of buildings subject to the action of winds, the occurrence of earthquakes or vibrations induced by man. Structural topological optimization consists of determining the best material distribution within a predefined design domain, under given boundary conditions, in order to minimize or maximize, that is, to extremize the objective function and meet the constraints of the model. The design variables (density, geometric characteristics of the microstructure, etc.), related to the distribution of material, are updated according to the mechanical response expected of the structure based on parameters such as flexibility, natural frequencies, stress, etc. From an initial assumption of density distribution within the design domain, an iterative process is used to update the value of densities from finite element analyses. Filters can be applied to ensure solutions and avoid numerical instabilities, such as the checkerboard pattern. This work aims to present the theory of topological optimization applied to the maximization of stiffness in beams by dynamic analysis. The formulation of the model proposes the maximization of the critical natural frequency in order to avoid the resonance phenomenon when the structure is subject to low magnitudes of excitation frequencies.

2 Numerical instabilities

The optimization process is subject to many numerical instabilities both because of the numerical analysis (*e.g.* through finite elements) and because of the variable updating processes. This works focuses on two of such instabilities: the checkerboard pattern and mesh dependence.

2.1 Checkerboard pattern

This numerical instability is characterized by the formation of alternating solid and void elements in the topology, in a similar way to a checkerboard. The most widespread technique to avoid checkerboard instability is the sensitivity filter proposed by Sigmund e Bendsøe [1]. This technique acts on the sensitivity analysis, when a spatial filter is applied after calculating the derivative of the objective function with respect to the design variable, avoiding sudden variations to the densities field without overly increasing the computational cost.

2.2 Mesh dependency

Mesh dependency refers to the problem of obtaining qualitatively different solutions when modifying the mesh. Mesh refinement aims to achieve greater clarity and better definition of contours, however, refining a mesh in a topological optimization problem can increase its complexity [2]. A treatment to reduce the space of allowable models and consequently have a mesh-independent problem is to include in the problem some local or global restriction on the variation in the space. This inclusion may be done by adding constraints (perimeter, local gradients, etc.) in order to reduce the parameter space, or by applying filters during implementation as the mesh independence filter. A solution to prevent instability is to update the densities of the elements in the mesh according to the weighted average of the densities of the elements in a fixed vicinity of minimum radius as shown in Fig. 1.

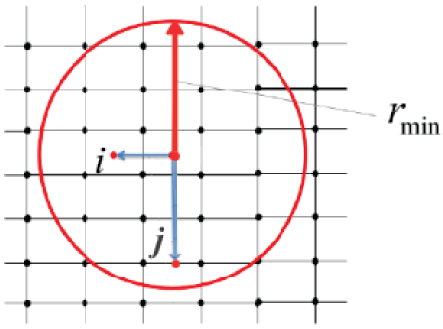

Figure 1. The mesh independence filter

3 Formulating a structural optimization problem

Structural optimization aims to achieve the best performance for a structure, and can be defined by the objective of the problem described by a function, design variables, and a set of constraints used to impose certain limits on design variables. The goal is a scalar response that directs structural optimization to better configuration of design-independent variables. Structural design variables include points that define structural characteristics such as dimensional parameters or density of material in a structural region [2].

3.1 Mathematical representation of a structural optimization problem

An optimization problem in general form can be defined as searching for the maximum (or minimum) value of the function $f(x)$, in which $x = (x_1, x_2, x_3, \ldots, x_n)$ is the vector of variables in the solution space R_n, and expressed by:

$$\textbf{Maximize: } f(x)$$

$$\textbf{Subject to: } g_j(x) = 0 \qquad j = 1:M$$

$$h_k(x) = 0 \qquad k = 1:N$$

where $g(x)$ and $h(x)$ are respectively the equality and inequality restrictions, M is the number of equality constraints and N the number of inequality constraints [3].

In general, design variables x includes structural characteristics, such as number of members, cross sections, or material. The objective function $f(x)$, depending on the method adopted, consists either in an evaluation of the cost of the structure (minimization problem) or its own performance (rigidity, service life, etc.). Constraints, in structural problems, include criteria such as equilibrium, maximum displacements and manufacturing restrictions, as well as specific criteria of each methodology, such as desired volume fraction or minimum material density.

3.2 SIMP Model

The term "SIMP", standing Solid Isotropic Material (or Microstructure) with Penalization, bases its topological representation on discretizing the design domain into finite elements and assigning each of these elements a variable representing its "density", which will affect the effective modulus of elasticity of that element. Therefore, the resulting solution can be represented not only with solid and void regions, but with intermediate densities. For an element,

$$E = \rho E_0 \tag{1}$$

where E is the effective modulus of elasticity for that element, ρ its density and E_0 the modulus of elasticity of the solid material. To move intermediate elementary densities ($0 < \rho < 1$) towards a $0/1$ solution, a power-law approach is used, penalizing intermediate densities, as in equation:

$$\rho = (x_e)^p \tag{2}$$

where $p > 1$ is the penalization power and x_e is the design variable associated with element e's density. SIMP uses only one design variable per element, facilitating its implementation in practice, compared to other types of microstructures, and generates clearer solutions because of the power-law approach. This penalization is necessary as intermediate densities are generally not manufactured. Therefore, answers as close to null or unity as possible are required [1] e [4].

4 Discretization of a beam using the finite element method (FEM)

Figure 2 shows the discretization of a beam using FEM. On the left, the representation of a simply supported beam in two-dimensional domain Ω, with geometric and mechanical boundary conditions defined describing the continuous problem, and to the right, the problem discretized into a mesh of 6 bilinear finite quadrilateral subdomains, interconnected by nodal points located at the vertices of the elements.

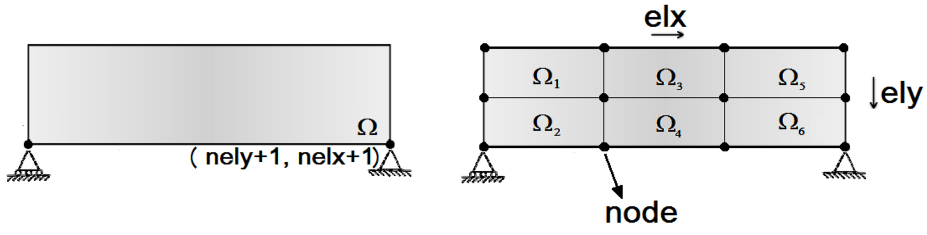

Figure 2. The discretization of a beam using FEM

Throughout this text, square cartesian elements are used for the discretization of the domain. Design variables x_e indirectly correspond to each element's density [8].

5 Topological optimization problem of flexibility minimization using FEM and SIMP

The topological optimization problem using FEM approach and SIMP method, where the goal is to minimize dynamic flexibility $W(x)$, that is, maximize the first or second smallest eigenvalue λ_1. This eigenvalue corresponds to the vibration frequency of the structure $\omega_1 = \sqrt{\lambda_1}$; [5] e [7].

$$\textbf{Maximize: } W(x) = \lambda_1 \tag{3}$$

$$\textbf{Subject to: } (K - \lambda_1 M)u_1 = 0$$

$$\frac{V(x)}{V_0} \geq V_f$$

$$0 < x_{\min} < x_e \leq 1, \qquad e = 1, \ldots, n$$

where x gathers the design variables x_e and the first restriction is the mathematical representation of the free-vibration dynamic problem. The volume inequality restriction is different from the usual maximum volume fraction and imposes a minimum final volume fraction V_f. This procedure is necessary since imposing an increment on the vibration frequency forces the solution to infeasible low densities.

The global matrices K and M, respectively of stiffness and mass, can be defined by:

$$\mathrm{K} = \sum_{e=1}^{n} (\mathrm{x}_e)^{\mathrm{p}} K_e \tag{4}$$

$$\mathrm{M} = \sum_{e=1}^{n} (\mathrm{x}_e) M_e \tag{5}$$

where K_e and M_e are respectively the stiffness and mass matrices of the element and $p > 1$ a penalization factor used as a solution of the intermediate densities [6].

5.1 Sensitivity analysis

The sensitivity of the objective function with respect to one of the design variables can be calculated by:

$$\mathrm{s_e} = \frac{\mathrm{d}\lambda_1}{\mathrm{dx_e}} = \frac{\mathrm{u}_{1,\mathrm{e}}^{\mathrm{T}} \left(p x_e^{p-1} K_0 + \lambda_1 M_0 \right) u_{1,e}}{u_1^T M u_1} \tag{6}$$

where $u_{1,e}$ are the displacements of the first vibration mode corresponding to the element, obtained from the Solution of the equilibrium system of equations [1] e [8].

5.2 Optimality Criteria

The Optimality Criteria (OC) method was chosen to update the design variables in the optimization procedure. It gathers the sensitivities of the objective function with respect to the design variables with the imposition of the volume fraction restraint such that the densities are updated as $x_{e,new} = B_e^{\eta} x_{e,old}$ and

$$\mathrm{s_e} = -\frac{\frac{\mathrm{d}\lambda_1}{\mathrm{dx_e}}}{\nu V_e} = \frac{p(x_e)^{p-1} \mathrm{u}_{1,\mathrm{e}}^{\mathrm{T}} K_0 \; u_{1,e}}{\nu V_e} \tag{7}$$

The Lagrange Multiplier ν can be found by an intersection algorithm so that the volume fraction restriction can be obeyed and the updated values $x_{e,new}$ must obey the minimum and maximum restrictions [1] e [6].

6 Results and discussions

A beam fixed on both sides, discretized in a mesh of 50 by 20 elements, was modeled in order to maximize the second eigenfrequency, subject to a volume fraction and a penalty respectively equal to 0.5 and 3. Figure 3 presents the result in the post-processing stage and Figure 4 displays the optimized result after applying the mesh independence filter and sensitivity analysis. Figure 5 presents the history of convergence of the critical natural frequency estimated at each iteration according to the updated densities of the discretized mesh elements associated with the second eigenvalue.

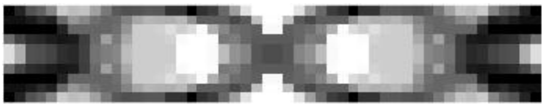

Figure 3. The result without the post-processing phase

Figure 4. The result in the post-processing stage

7 Final considerations

In this work, the topological optimization method is formulated based on the dynamic response of the two-dimensional structure subjected to volume restrictions, maximizing the natural or fundamental frequency, avoiding the possibility of resonance when the beam is subject to low amplitudes of vibration or excitation. The proposed formulations and the examples analyzed proved effective, similar to those found in the literature, therefore contributing to the validation of the model.

More work is needed in the application of the model, since the obtained geometries still present feasibility problems.

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References

[1] SIGMUND, O. & BENDSØE, M.P. - **Topology optimization, theory, methods and applications**. Springer, 2004.
[2] AREMU, A. *Topology optimization for additive manufacture*. Loughborough University, 2013.
[3] HAFTKA, Raphael T.; GÜRDAL, Zafer. **Elements of structural optimization**. Springer Science & Business Media, 2012.
[4] SIGMUND, O. *Design of material structures using topology optimization*. PhD thesis, Technical University of Denmark, 1994.
[5] RAO, S. S. **Vibrações Mecânicas**. São Paulo: Pearson Prentice Hall. 2008.
[6] MEIJBOOM, Marion. **Topology optimization of dynamic problems**. Eindhoven University of Technology. 2003.
[7] VENCESLAU, Sheyla M. Maia. *Modelagem matemática de sistemas vibratórios com aplicação de autovalores*. Master Thesis, Federal University of Sergipe, 2015.
[8] SIGMUND, O. - *A 99 line topology optimization code written in matlab*, 2001.