Stationary Evaluation of CPTu Data in Brazilian Marine Clay

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Abstract.
Structural integrity is an important premise in oil well design. In this context, soil characterization helps the well design team to assess soil resistance and design conductor and surface casing strings that supports all construction and operation loads. Piezocone Penetration Test (CPTu) is commonly used as it is simple and provides comprehensible soil data. Using CPTu dataset, it is possible to characterize geomechanical behavior using statistics presented in the literature. Application of statistics procedures in parameter characterization must be done using samples that attend stationarity criteria i.e. constant mean and variance of the fluctuating component. Present work applies a modified Bartlett Statistics in experimental marine soil CPTu data to evaluate sample sizes that satisfy stationarity criteria. Initally, trends in parameters are calculated using linear regression analysis and layer classification is applied in detrended parameters. Subsequently, autocorrelation samples are evaluated and fit into models. These autocorrelation models estimate the scale of fluctuation which measures high statistical correlation domain and Bartlett profiling is then applied. Bartlett's peak value is compared with critical modified Bartlett Test formula evaluated by Phoon et al. [1] to assess if stationarity of the sample is within 5% confidence level. Results show sample intervals that achieve stationarity.

Keywords: Stationary evaluation, CPTu, Geomechanics.

1 Introduction

The oil and gas industry is concerned with structural integrity while keeping the expenses at an affordable level. Also, environmental and human safety are important premises that are addressed in many normative codes. Focusing on well structural integrity, analysis of the soil-casing coupling is significant for conductor and surface casings as they serve as foundation element, supporting severe loads from the well construction and throughout its entire life cycle. In this context, soil characterization has great importance.

Piezocone Penetration Test (CPTu) is one of the most used methods to characterize geomechanical behavior, being widely employed in conductor and surface casing design. CPTu data comprises information about tip resistance (q_T), pore pressure (U_2) among other parameters. Design parameters e.g. undrained shear strength, are evaluated from this dataset. However, measurements in marine clay soil present high levels of uncertainty due to soil inherent variability. Although the random field analysis provides a concise description of spatial variation, it poses considerable practical difficulties for statistical inference. Weak stationarity is a necessary requirement for subsequent statistical analysis on any layer of a soil record to be valid (Phoon et al. [1]). Since weak stationarity of geotechnical parameters is confirmed, subsequent statistical analyses are attainable, such as the generation of histograms and application of goodness-of-fit tests.

Hence, present work evaluates stationarity intervals of geotechnical parameters found in CPTu data using the modified Bartlett test, which is employed to reject the null hypothesis of weak stationarity for spatially correlated data. To carry out this test for stationarity, the autocorrelation function and the scale of fluctuation of the soil layers must be estimated. The autocorrelation function (ACF) is a simple but robust approach to estimate the scale of fluctuation, using the method of moments and fitting a plausible theoretical autocorrelation model (ACM) to evaluate the scale. However, it is very important to ensure that the trend function is removed from each layer before estimating such statistics. In general, the detrending process is not unique. Different trend removal procedures will in most cases result in different values of the random fluctuating components and different shapes of the autocorrelation function.

2 Materials and Methods

2.1 Soil classification

The CPTu is widely used for demarcating layer boundaries and identifying soil types because it provides almost continuous profiles and it is highly repeatable. According to Phoon et al. [1], both layer demarcation and soil classification are interlinked. A properly demarcated soil layer should consist of the same material type, and the CPTu data should therefore appear as a fairly distinct cluster on the soil classification chart. However, inherent soil variability usually complicates the interpretation and it would be quite useful to supplement these existing procedures using a statistical approach.

As soil samples are collected during CPTu, many methodologies have been developed in the past decades to help characterize soil using data obtained in these tests. Some methods such as Marr and Endley [2] and Robertson [3] use charts to classify soil behavior using parameters from CPT tests. Although very useful and proven to be precise for many soil conditions, these methods are difficult to implement in a programming routine. Then, Jefferies and Davies [4] developed a classification methodology using the following normalized geotechnical parameters: The normalized tip resistance (q_T), the normalized pore pressure ratio (B_q) and the normalized friction ratio (F_r). This methodology is based on the material classification index (I_c)

$$I_c = \sqrt{\{3 - log[q_T(1 - B_q)]\}^2 + [1.5 + 1.3log(F_r)]^2}. \tag{1}$$

Table 1 presents the soil classification given by the index. Since it does not depend on the use of a chart, this classification can be easily implemented.

Table 1. Soil classification according to the material index.

Soil classification	Zone	Material Index I_c
Gravelly sands	6	$I_c < 1.25$
Sands – clean sand to silty sand	5	$1.25 < I_c < 1.90$
Sand mixtures – silty sand to sandy silt	4	$1.90 < I_c < 2.54$
Silt mixtures – clayey silt to silty clay	3	$2.54 < I_c < 2.82$
Clays	2	$2.82 < I_c < 3.22$
Organic soils - peats	1	$I_c > 3.22$

To carry out the modified Bartlett test for stationarity, the autocorrelation function and the scale of fluctuation of these layers must be estimated. However, it is very important to ensure that the trend function is removed from each layer before estimating such statistics ([5]). The real value of a geotechnical property $\xi(z)$ may be decomposed into a smoothly varying trend function $t(z)$ and a fluctuating component $w(z)$ representing the inherent soil variability: $\xi(z) = t(z) + w(z)$, where z is the depth coordinate. In this paper, trend will be removed using linear regression analysis of the q_T profile.

2.2 Autocorrelation Function and Models

In finite-scale models, the scale of fluctuation (δ) is a concise indicator of the spatial extent of strongly correlated domains. According to Uzielli et al. [6], a simple but robust approach is to estimate the sample autocorrelation function (ACF) by using the Method of Moments to parameterization, fit a plausible autocorrelation model (ACM) and calculate the scale.

As suggested by Box et al. [7], ACF is calculated for lag distances $\tau_j = j\Delta_z$ corresponding to $j = [1, n_d/4]$, where n_d is the number of data points in a given profile and Δ_z is the sampling interval. The jth coefficient of the Method of Moments approach for the autocorrelation function of the fluctuating component $w_i = w(z_i)$ is given by

$$R(\tau_j) = \frac{\sum_{i=1}^{n_d-j} w_i \times w_{i+j}}{\sum_{i=1}^{n_d-j} w_i^2}. \tag{2}$$

Several types of ACM are employed in the geotechnical literature to fit the ACF and four different kinds of ACM were tested for CPTu data. Coefficient of determination, R^2, was calculated to evaluate best fit. To increase reliability of the estimated δ, only ACMs producing $R^2 > 0.9$ were accepted and fit procedure was applied only in samples which exceeded Bartlett's r_B limit

$$r_B = \frac{1.96}{\sqrt{n_d}}. \tag{3}$$

This guideline has been used by Spry et al. [8] and is motivated by the well-accepted fact that the estimated autocorrelation coefficients become less reliable with increasing lags and are deemed not significantly different from zero inside the range $\pm r_B$ ([6]). The analytical expressions of the four used ACMs and the formulas relating the scales of fluctuation to the model parameters are shown in Table 2.

2.3 Modified Bartlett Test

The modified Bartlett Test was created by Phoon et al. [1] in order to develop a new and more discriminating statistical test that can detect the stationarity of a soil profile. A continuous Bartlett statistic profile is first generated by moving a sampling window over the soil profile. The sampling window is divided into two equal segments and the sample variance (s_1^2 or s_2^2) is calculated from data points lying within each segment [1]. The proposed new test statistic is the peak value of the Bartlett statistic profile. The critical value (B_{max}) of this modified Bartlett test statistic at 5% level of significance can be determined from the simulated soil profiles.

According to Phoon et al. [1], considering the case of two sample variances, s_1^2 and s_2^2, the Bartlett test statistic reduces to

$$B_{stat} = \frac{2.30259(m-1)}{C}[2log(s^2) - (log(s_1^2) + log(s_2)^2)]. \tag{4}$$

where m is the number of data points used to evaluate s_1^2 or s_2^2 and $C = 1 + \frac{1}{2(m-1)}$. Total variance, s^2, is defined as $s^2 = \frac{s_1^2+s_2^2}{2}$.

In order to ensure that the modified Bartlett test is sufficiently general for practical applications, Phoon et al. [1] select a range of realistic values for the sampling length T, the scale of fluctuation δ and for the sampling segment size (W) in the computation of the Bartlett statistic profile. For concise presentation, these profile factors are presented in the following dimensionless form:

$$k = \frac{\delta}{\Delta_z}; I_1 = \frac{T}{\delta} = \frac{n\Delta_z}{k\Delta_z} = \frac{n}{k} \text{ and } I_2 = \frac{W}{\delta} = \frac{m\Delta_z}{k\Delta_z} = \frac{m}{k}. \tag{5}$$

where k represents the number of points in one scale of fluctuation, n is the total number of sample points in a soil layer of length T, I_1 represents the normalized sampling length, m is the number of sample points in half of the sampling window and I_2 represents the normalized segment length.

Phoon et al. [1] select the range of values for the above dimensionless profile factors (k, I_1, I_2) following two practical considerations. First, to ensure that the sample variances in each segment can be estimated with reasonable accuracy, the sample size in one m segment must exceed 10 (Lacasse and Nadim [9]) and, secondly, Bartlett statistic is computed by comparing the sample variances in two adjacent segments forming the sampling window. Hence, the total number of points n in the soil record must exceed 2 m, or equivalently, $I_1 > 2I_2$. In this context, Phoon et al. [1] say that the number of points k in one scale of fluctuation is ranged between 5 and 50. The value of the normalized sampling length is also set between 5 and 50. Hence, n can vary between 25 and 2500. The normalized segment length is chosen as $I_2 = 1$ (for $k \geq 10$) and $I_2 = 2$ (for $5 \leq k < 10$) in order to ensure that $m > 10$.

Once the Bartlett statistic profile is obtained, the peak value (B_{max}) can be identified. The probability distributions of B_{max} for different values of the dimensionless factors are rather complicated and cannot be conveniently parametrized using standard probability distribution functions for practical use. As geotechnical data may follow different correlation models, the rejection criteria will depend on the ACM. Phoon et al. [1] developed a rejection criteria for each autocorrelation model, in order to increase the reliability of the test. These criteria are summarized in Table 2. It should be noted that the test statistic corresponding to these rejection criteria is the peak value of the Bartlett statistic profile, B_{max}. The null hypothesis of stationarity in the variance is rejected at 5% level of significance if B_{max} is larger than B_{crit} .

Table 2. Autocorrelation models and relations between scale of fluctuation, characteristic model parameters and critical Modified Bartlett Test statistic at 5% level of significance ([1]).

Autocorrelation model	Equation	Scale of fluctuation	Rejection criteria
Single exponential	$R(\tau) = exp(-\lambda\|\tau\|)$	$\delta = \frac{2}{\lambda}$	$(0.23k + 0.71)ln(I1) + 0.91k + 0.23$
Cosine exponential	$R(\tau) = exp(-b\|\tau\|)cos(b\tau)$	$\delta = \frac{1}{b}$	$(0.28k + 0.43)ln(I1) + 1.29k - 0.40$
Second-order Markov	$R(\tau) = (1 + d\|\tau\|)exp(-d\|\tau\|)$	$\delta = \frac{4}{d}$	$(0.42k - 0.07)ln(I1) + 2.04k - 3.32$
Squared-exponential	$R(\tau) = exp[-(a\tau)^2]$	$\delta = \frac{\sqrt{\pi}}{a}$	$(0.73k - 0.98)ln(I1) + 2.35k - 2.45$

3 Results

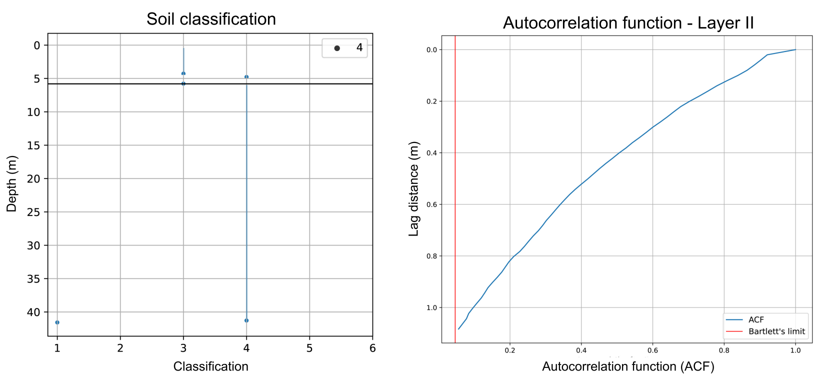
Due to the company's privacy policy, all data was decharacterized. This study considers one CPTu borehole. Its maximum penetration depth is 40.8 m.

Statistical analysis is performed in the following way: The borehole is subdivided into layers according to the Jefferies and Davies [4] methodology described in Section 2; the trend is removed from each section using linear regression; an appropriate theoretical model is fit to the sample autocorrelation function and estimate the scale of fluctuation and, finally; the Bartlett statistic profiling is performed on the detrended residues of the cone tip resistance (q_T) of the borehole.

Moreover, as described above, to increase the reliability of the estimated δ, the ACMs were fit only to the initial part of the sample ACF with coefficients exceeding Bartlett's limit. Figure 1 shows both the soil classification chart according to Jefferies and Davies [4] and the ACF for the Layer II limited by its respective Bartlett's limit. Note that ACF for the Layer I was not plotted due to the size of the layer, reflecting a small amount of data when compared to Layer II.

Table 3 shows the results of the modified Bartlett Test. It is noticeable that $B_{max} < B_{crit}$ in Layer I and this section is likely to be sufficiently stationary. In the second layer $B_{max} > B_{crit}$, which rejects the null hypothesis. In this case, Phoon et al. [1] suggest to divide the interval into two parts at the point of exceedance (22.96 m), which can be interpreted as a secondary boundary. After that, all the steps are repeated and, in this new case, the sublayers were found to be stationary, since $B_{max} > B_{crit}$ for II (A) and II (B). Moreover, the values of the coefficient of determination (R^2) of the the fitted autocorrelation models $R(\tau)$ are quite good. Figure 2 shows Bartlett statistic profile of layers I, II and sublayers II (A) and II (B). After that, the dimensionless profile factors ($k, I1, I2$) were calculated following eq. 5. Thus, the peak Bartlett value (B_{max}) was compared with the critical value (B_{crit}) for each layer, following the equations on Table 2. According to Phoon et al. [5], if $B_{max} < B_{crit}$, the null hypothesis (H_0) of weak stationarity cannot be rejected at the 5% level of significance and the section is likely to be sufficiently stationary for robust statistics to be evaluated.

Figure 1. Layers classified according to Robertson methodology and ACF of the borehole (Layer II).



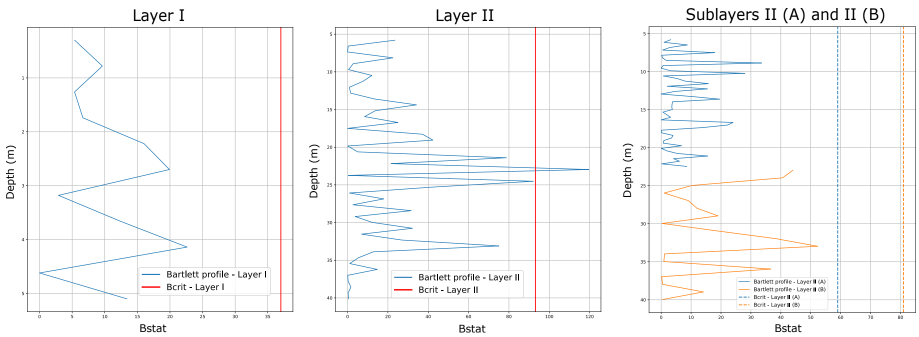
4 Conclusion

For the case study presented herein, the modified Bartlett test proved to be useful and accurate in the identification of stationarity. One of the possible reasons for the test to be successful is the homogeneity of the parameters. It is important to emphasize that the layers subdivision cannot be carried out indefinitely because the length of layer must be longer than five times the scale of fluctuation ($I_1 > 5$). This fact, however, implies that

Table 3. Summarized results of the modified Bartlett test for primary and secondary layers of the borehole.

Borehole											
Layer	Depth (m)	$R(\tau)$	R^2	δ (m)	r_B	k	I_1	I_2	B_{max}	B_{crit}	H_0
I	0.3 - 5.8	Single exponential	0.951	0.480	0.118	24	11	1	22.62	37.00	Accepted
II	5.8 - 41.57	Cosine exponential	0.989	0.788	0.046	39	45	1	119.63	93.11	Rejected
II (A)	5.8 - 22.96	Second-order Markov	0.999	0.360	0.066	17	50	1	33.59	59.02	Accepted
II (B)	22.96 - 41.57	Single exponential	0.974	1.00	0.064	50	18	1	52.33	81.02	Accepted

Figure 2. Bartlett statistic profile of Layers I, II, II (A) and II (B).



the designer chooses a limit depth interval. A limitation of the test is the application of the criterion itself, which was developed using marine clay from another region. Despite being a statistical approach, the suitability of the criterion to Brazilian soil is open to discussion.

All things considered, the modified Bartlett test provides some advantages as a consistent measure that is not affected by the uncertanties of subjective interpretation and it is sufficiently discriminative to decide if a section is stationary, especially when visual clues are ambiguous.

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