Dynamic instability of cable stayed masts

Antônio Vaz Eduardo Júnior[1], Zenón J. Guzmán N. del Prado[1]

[1]*School of Civil and Environmental Engineering, Federal University of Goiás*
Av. Universitária, Qt. 86, Leste Universitário, 74605-220, Goiânia, Goiás, Brazil
antoniovaz@discente.ufg.br, zenon@ufg.br

Abstract. Cable stayed structures are widely used in several engineering areas such as civil, mechanical, telecommunications and offshore engineering. These structures are light weight and efficient to carry both axial and lateral loads due to the stayed cables and can be characterized by large displacements and high load bearing ratios. As these structures can display large displacements which are associated to both nonlinear static and dynamic behavior, the research of its nonlinear response is of high interest by engineers and scientists. In this work, the dynamic instability of cable stayed masts is studied. For this, the mast is described as a simplified nonlinear one degree-of-freedom system subjected to a lateral harmonic load. The Hamilton principle is applied to obtain the nonlinear dynamic equilibrium equation which is solved using the 4th order Runge-Kutta method. A parametric detailed analysis is considered to obtain the time response, Poincaré maps and resonance . Obtained numerical results show the great influence of both cable tensioning and cable positioning on the non-linear behavior of the system and could be used as a tool for an analysis of the nonlinear dynamics of the structure previous to design.

Keywords: cable stayed structures, dynamic instability, nonlinear dynamics, Poincare maps, Resonance curves.

1 Introduction

Cable-stayed masts are structures widely used in several fields of engineering. In general, they are very light and slender structures, which makes them susceptible to non-linear behavior [1].

Cable-stayed towers consist of a column usually simply supported the base, and anchored to the side with various stays, usually steel cables, and are often used to support TV, radio and telecommunications signal transmission antennas [1].

Regarding cable-stayed masts, the oldest works are more related to the development of methodologies for determination of efforts, aimed at design.

In respect to the study of its stability and dynamic behavior, some works can be cited. For example, El-Ghazaly and Al-Khaiat performed, using ANSYS finite element software, a static and linear analysis of a tower 600 meters high. Madugula, on the other hand, made a comparative study between the natural frequencies of plane models, obtained in an experimental way, comparing them with those obtained through an analysis of finite element models [1].

In this work, the dynamic instability of cable stayed masts is studied. For this, the mast is described as a simplified nonlinear one degree-of-freedom system subjected to a lateral harmonic load. The mast is described as a rigid bar pinned at its base, supported by two pre-tensioned cables, fixed along its height, and under the action of a harmonic lateral load representing dynamic effects that can act on a real structure, such as wind, for example. The Hamilton principle is applied to obtain the nonlinear dynamic equilibrium equation which is solved using the 4th order Runge-Kutta method. A parametric detailed analysis is considered to obtain the time response, Poincaré maps, Resonance curves. Obtained numerical results show the great influence of both cable tensioning and cable positioning on the non-linear behavior of the system and could be used as a tool for an analysis of the nonlinear dynamics of the structure previous to design.

Numerical results are obtained by varying the geometric and mechanical parameters of the system, such as the position or height of the stays and the intensity of the harmonic load. The influence of these parameters allows to evaluate their effects on the dynamic behavior of the tower, such as natural frequency, periodicity, and displacements as well as resonance curves, Poincaré maps time responses and basins of attraction. These results allow the verification of possible optimal parameters regarding the stability of the set in the pre-design step and help to understand how these structures generally behave from the perspective of dynamics.

2 Mathematical formulation

Consider a rigid tower with density ρ, length L, cross section A, stayed by two pre-tensioned cables with axial stiffness k_1, k_2 located at a distance d_1 and d_2 from its base and fixed at a height h from its base, bears a fixed mass M at its top and is subjected to both axial P and lateral harmonic load $P(t)=P_o\cos(\omega t)$ as seen in Fig. 1. The effects of shortening and bending on the bar are disregarded. The cables are modeled as linear springs, whose rigidity is determined through the relationship between their elastic modulus, cross section and length.

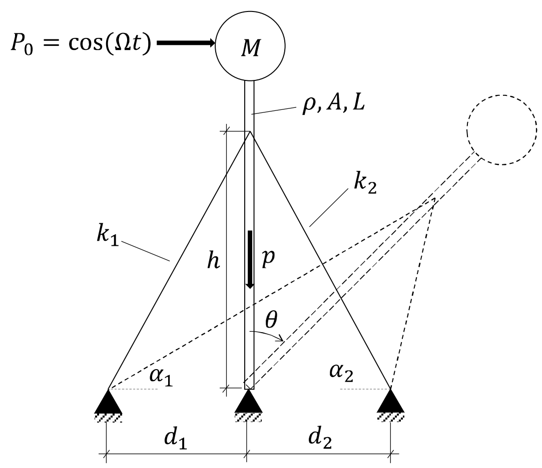


Figure 1. Model of a one-degree-of-freedom system for a cable-stayed mast

The internal strain energy considers the elongation and shortening of the stays during mast displacement, together with the elongation due to pre-stressing. The work due to the external forces acting on the system is the sum of the work done by the mast's own weight with the work done by the other loads (equipment weight and harmonic load) applied along it. The total kinetic energy of the system is obtained by integrating the kinetic energy of an elementary mass along the length of the bar. Thus, the Lagrangian function of the system is given by:

$$L = \left(\frac{1}{6}\rho A L^3 + \frac{ML^2}{2}\right)\dot{\theta}^2 - \frac{1}{2}\frac{{F_{01}}^2}{k_1} - F_{01}\gamma LB - \frac{1}{2}k_1\gamma^2 L^2 B^2 - \frac{1}{2}\frac{{F_{02}}^2}{k_2} - F_{02}\gamma LB - \frac{1}{2}k_2\gamma^2 L^2 B^2 + p\gamma L(1-\cos\theta) - P_0\cos(\Omega t)\, L sin(\theta)$$
$$B = \sqrt{\left(\frac{1}{\tan\alpha_{\mathrm{i}}} \pm \sin\theta\right)^2 + \cos^2(\theta)} - \sqrt{\frac{1}{\tan^2(\alpha_{\mathrm{i}})} + 1} \quad (1)$$

Where $\gamma=h/L$ and i=1, 2.

The equilibrium equation can be obtained by applying the Hamilton principle as:

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = 0 \quad (2)$$

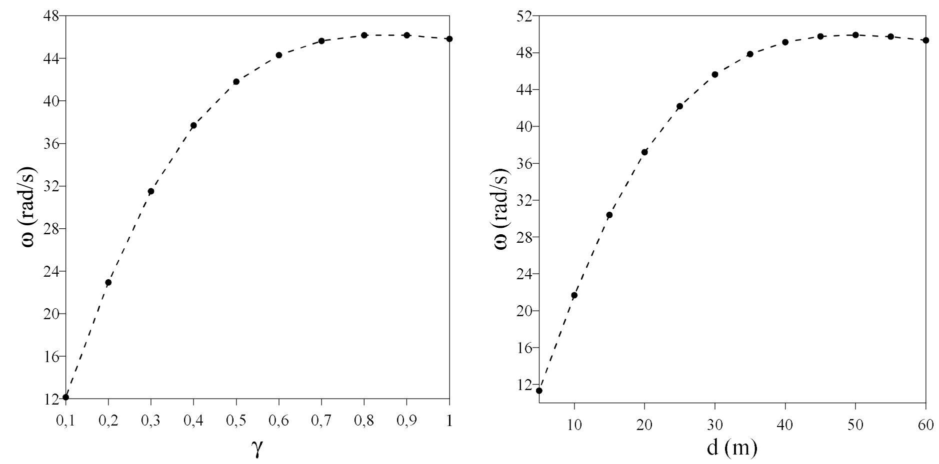
Thus, the differential equation of motion of the system is expressed by:

$$
\begin{aligned}
&F_{01}\gamma L\left(\frac{\cos\theta}{\tan(\alpha_1)\sqrt{\left(\frac{1}{\tan(\alpha_1)}+\sin\theta\right)^2+\cos^2\theta}}\right) \\
&-\frac{1}{2}k_1\gamma^2L^2\left[\frac{2\cos\theta\left(\sqrt{\left(\frac{1}{\tan(\alpha_1)}+sin\theta\right)^2+\cos^2\theta}-\sqrt{\frac{1}{\tan^2(\alpha_1)}+1}\right)}{\tan(\alpha_1)\sqrt{\left(\frac{1}{\tan(\alpha_1)}+\sin\theta\right)^2+\cos^2\theta}}\right] \\
&+F_{02}\gamma L\left(\frac{\cos\theta}{\tan(\alpha_2)\sqrt{\left(\frac{1}{tan(\alpha_2)}-\sin\theta\right)^2+\cos^2\theta}}\right) \\
&+\frac{1}{2}k_2\gamma_2^2L^2\left[\frac{2\cos\theta\left(\sqrt{\left(\frac{1}{\tan(\alpha_2)}-\sin\theta\right)^2+\cos^2\theta}-\sqrt{\frac{1}{\tan^2(\alpha_2)}+1}\right)}{\tan(\alpha_2)\sqrt{\left(\frac{1}{tan(\alpha_2)}-\sin\theta\right)^2+\cos^2\theta}}\right] \\
&+pyL\sin\theta-2\xi\left(\frac{1}{3}\rho AL^3+ML^2\right)\omega\dot{\theta}-\left(\frac{1}{3}\rho AL^3+ML^2\right)\ddot{\theta}=P_0\cos(\Omega t)\,L\cos\theta
\end{aligned}
\tag{3}
$$

3 Results

For the mast, a material with density is ρ=7860 kg/m³, length L=50 m and cross-sectional area A=2.028E-3 m² is considered. For the cables, a modulus of elasticity E=127.5 GPa and cross-sectional area Ac=4.870E-4 m² is considered been k_1=k_2=1.347E+6 N/m, the damping factor was considered as ξ = 0.001. These parameters were based on towers and cables available in the industry. For M, the mass value of the equipment installed at the top of the mast was considered between 100, 500, 1000 and 1500 kg. For the analysis of the mast behavior according to the intensity of the lateral load P_o, load values between 1.200E+3 and 1.200E+5 N were considered.

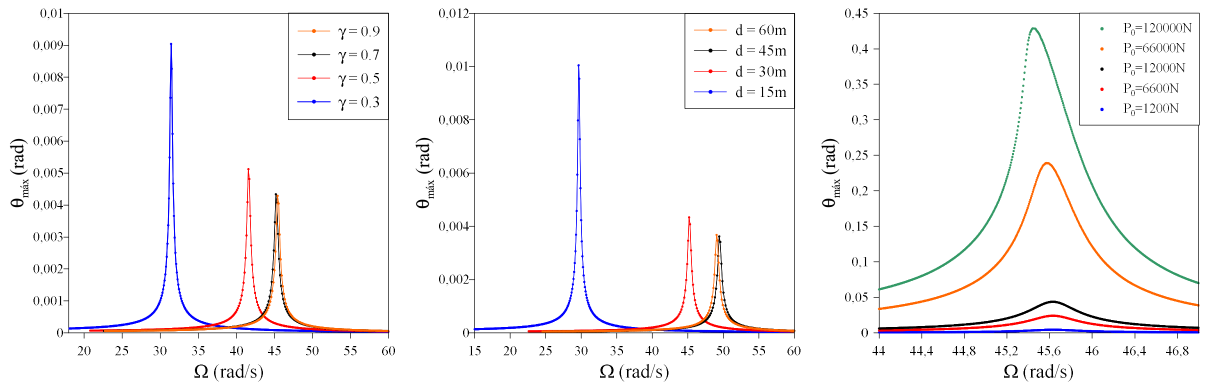
For a first analysis, the variation of the natural frequency of the system is evaluated as a function of its geometric properties. Figure 2 displays the variation of the natural frequency of the system as a function of γ parameter and the distance d from the base of the tower. As can be observed, as the value of both parameters is increased, the natural frequency of the system is also increased and showing a maximum value as a small decreasing for larger values of parameters.



(a) (b)

Figure 2. Variation of the natural frequency of the system. (a) Due to the relative height of stays positioning, (b) Due to the horizontal distance of stays fixation

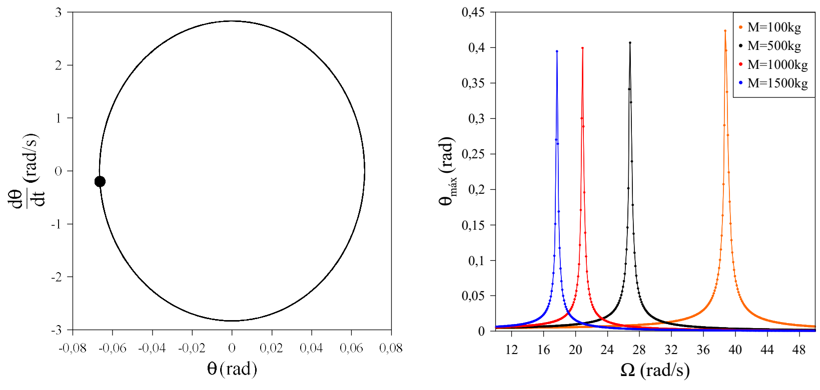
Now, the influence of parameters $\gamma = h/L$, d and P_o on the resonance curves for increasing values of the frequency of lateral load will be obtained, using a lateral load value of 1.200E+3 N. Figure 3 displays the resonance curves, Fig. 3(a) for increasing values of g parameter, Fig. 3(b) for increasing values of distance d and Fig. 3(c) for increasing values of load P_o. As can be observed, for all curves show low level of softening behavior, resonance curves of Fig. 3(a) and (b), are shifted due to the change of natural frequency which is influenced by the change of the geometry of the system. Also, resonance curves of Fig. 3(c) display increasing lateral displacement, which means increasing the level of nonlinearity, as the value of load is increased



(a) (b) (c)

Figure 3. Resonance curves of the system. (a) Due to the relative height of stays positioning, (b) Due to the horizontal distance of stays fixation (c) Due to the magnitude of the harmonic lateral load

Now, to show the characteristic of dynamical behavior, Fig. 4(a) phase portrait and Poincaré map and basin of attraction for a point in the resonance curve of Fig. 3(b) with d=30m meanwhile Fig. 4(b) shows the resonance curves for increasing values of the mass located at the top of the tower, as can be observed, the sitfness of the system decreases as the mass M increases, an expected result that can be verified by decreasing its natural frequency.



(a) (b)

Figure 4. Dynamic behavior of the system. (a) Poincaré mapping, (b) Resonance curves related to the increasing of the equipment mass installed at the top of the mast.

4 Conclusions

In this work, the dynamic instability of cable stayed masts is studied. For this, the mast is described as a simplified nonlinear one degree-of-freedom system subjected to a lateral harmonic load. The mast is described as a rigid bar pinned at its base, supported by two pre-tensioned cables, fixed along its height, and under the action of a harmonic lateral load.

Through the relationships between their natural frequencies and their geometric properties, it is verified that there are optimal positions for the fixation of the stays, both in relation to the height of the tower and the horizontal distance of fixation which generate the highest natural frequency of the system. For the results obtained, using a 50-meter-tall tower as a reference, it is observed that the optimal relative height for fixing the stays is about 70% of its height, as can be seen in Figure 2(a). If this value is exceeded, there is a reduction in the natural frequency of the system caused by the loss of stiffness.

Regarding the horizontal distance for positioning the stays, it is observed that the optimal distance for positioning is about 50 meters from the tower. Note that from this value, as can be seen in Figure 2(b), there is a decrease in the natural frequency of the system, also caused by the decrease in the stiffness of the tower under these conditions.

Both above considerations are also verified with the resonance curves shown in Figure 3(a) and (b). It is observed that, as the mentioned parameters move towards their optimal values, the resonance curves get closer to each other, presenting a slight tipping when these values are exceeded. This phenomenon is called softening and characterizes a decrease in system stiffness.

This behavior can be seen more clearly in the resonance curve present in Figure 3(c), which shows the behavior of the structure when the amplitude of the harmonic lateral load applied is increased. A considerable increase in the absolute maximum displacements of the tower is observed as the intensity of application of the harmonic load increases. The tipping of the curve to the left is characteristic of the softening phenomenon, which indicates a loss in the structure's rigidity when the loads acting on it are considerably increased.

Based on the above, it can be seen that cable-stayed masts are structures with typically non-linear behavior, requiring a more in-depth evaluation of their geometries, as well as the solicitations that act on them. Variations in its geometric parameters and acting loads considerably influence its behavior, as well as its properties, such as its natural frequencies (stiffness), for example. Therefore, the need for a deeper study of this type of structure becomes visible, in order to reach ideal parameters and dimensions, aiming safety, economy and durability.

Based on the Poincaré mapping shown in Figure 4(a), it can be seen that the dynamic behavior of the mast is substantially periodic, with a single period. This behavior can be explained by the dimensions of the tower and the magnitude of the loads considered, since low values for these do not explore in depth the non-linearity of the system.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] Pasquetti, Eduardo; Gonçalves, Paulo B. “Static and dynamic stability of guyed towers”. Rio de Janeiro, 2003. 99p. MSc Dissertation - Department of Civil Engineering, Pontifícia Universidade Católica do Rio de Janeiro.
[2] S.T. Thornton; J.B. Marion. *Dinâmica clássica de partículas e sistemas*. CENGAGE Learning, 2016.