Comparative study for evaluating the gradient of the failure function of wells drilled on salt rocks

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Abstract. This paper presents a comparative study of strategies for obtaining the gradient of the failure function used in the First Order Reliability Method (FORM) applied to oil wells drilled on salt rocks. The evaluation of the mechanical behavior of complex structures is commonly performed using the finite element method. In these cases, several challenges are found when it is desired to carry out a structural reliability analysis, since the function that defines the structural behavior is numeric and each query requires running a finite element analysis. This demands a high computational cost, making it practically impossible to perform the reliability analysis from classical simulation methods. On the other hand, to use transformation methods it is necessary to evaluate the gradient of the failure function, which is also a numerical expression, requiring the use of alternative techniques to perform its calculation efficiently. In this context, this work presents the comparison between several numerical methods to obtain this gradient. Several methodologies such as the response surface method (RSM), the adaptive response surface method (ARSM) and the finite differences method (FDM) are applied to a model of an oil well drilled on salt rocks. The main contribution of this paper is to check the efficiency of some procedures for obtaining the gradient of the failure function in the evaluation of the structural behavior of oil wells drilled on salt zones.

Keywords: Gradient of the failure function, FORM, salt rocks

1 Introduction

In the last decades, with the fast development of computational mechanics, structural analysis has developed considerably, mainly in the evaluation of the mechanical behavior of complex structures. It benefited greatly from advances in computer hardware and processing. On the other hand, as structures become more complex, the demand for its safety and reliability is increased.

In this context, structural reliability is used in order to develop verification procedures aimed at ensuring that structures will have acceptable safety and economic performance. However, this structural analysis becomes complex when numerical methods are necessary to describe structural behavior. Time and computational resources needed to perform reliability analysis using classic simulation methods are highly prohibitive. On the other hand, to use transformation methods such as First Order Reliability Method (FORM) it is necessary to evaluate the gradient of the failure function, that is a non-trivial task since also requires to run of numerical simulations.

Several strategies have been used to evaluate the gradient of the failure function, such as the response surface method, the adaptive response surface method, and the finite difference method. However, although they are classical methods, the comparison of efficiency between them is not quite explored in the literature.

In this scenario, this paper presents a comparative study of strategies for obtaining the gradient of the failure function used in the FORM for structural reliability analyses where the structure is simulated by finite element method. An oil well drilled in salt rocks is used in this study. The main contribution of this work is to compare efficiency of some procedures for obtaining the gradient of the failure function for oil wells drilled on salt zones.

2 Structural Reliability

According to Beck [1], the main objective of structural reliability is to measure the failure probability of structures P_f and, consequently, analyze their safety level. To achieve this goal, the structural reliability is based on the existence of a limit state function $G(\vec{X})$, where $\vec{X}$ is the vector of random variables of the problem. This function is defined so that $G(\vec{X}) < 0$ indicates the structure failure and $G(\vec{X}) \geq 0$ indicates its survival.

A commonly used reliability method to obtain the P_f is the First Order Reliability Method (FORM) [2]. This method consists of performing transformations on non-normal random variables to uncorrelated standard normal random variables, also transforming the limit state function to standard normal space.

The first transformation reduces the original probability distributions to equivalent standard normals, using the transformation presented by Hasofer and Lind [3]. The second transformation eliminates the correlations by the Cholesky decomposition. For more details on the method and the transformations used, see Melchers and Beck [4]. An important point refers to need to evaluate the gradient of the failure function and for the case of numerical failure functions it is necessary to use strategies to perform the analysis. Figure 1 presents a scheme of the values assumed by the random variables of the problem in the methods adopted in this work. These methods and how each one uses these values are aspects discussed below.

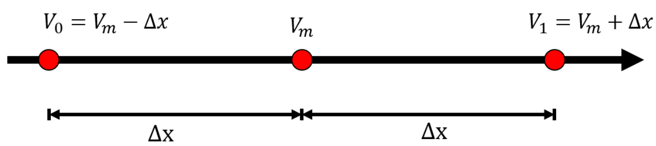

Figure 1. Schematic representation of the values assumed by the random variables

2.1 Response surface method (RSM)

According to Beck [1], this method consists of fitting a more simple function $\overline{G}(\vec{X})$, which is able to adequately approximate the original failure function. With this approximate function, the finite element model is completely abandoned and the analysis is carried out from the adjusted model. Thus, the gradient is easily obtained by the fitted function. The surface used in this work is a polynomial with a complete quadratic degree in all variables. To interpolate the function, a set of previously simulated scenarios is required. These scenarios are composed by the combination of three values, as in figure 1, where V_m is the mean value and Δx is three times the standard deviation of the variable. The fitted surface must be representative on full domain queried on the FORM. Furthermore, the number of points used to fit the surface is 3^n, where n is the number of variables.

2.2 Adaptative response surface method (ARSM)

The adaptive response surface method has many similarities with RSM, however, at each iteration of the FORM a new surface is generated around the point of the current iteration, in order to introduce more precision, but with higher computational cost. The surface used in this work is a polynomial with a complete quadratic degree in all variables and it is generated around the point using the combination of three values, as in Figure 1, where V_m is the mean value and Δx is 0.5 times the standard deviation of the variable. For each interaction, a new surface is fitted and just need to describe the surroundings of the current iteration point and, therefore, the points used on the surface may be closer together, that is, a smaller Δx. On the other hand, in this method it is necessary to simulate 3^n points at each iteration of the FORM, where n is the number of variables.

2.3 Finite difference method

In this method, the gradients are estimated a second-order centered finite-difference approach, choosing points near the iteration point and using a step Δx of ± 0.01. In this method no meta-model is used, that is, no function is adjusted to emulate the structural behavior of the oil well. For this, when it is necessary to evaluate the failure function, the corresponding oil well is numerically simulated and the gradient at a specific local is evaluated from discrete points. A great advantage of this method is that it is only necessary to simulate points in the directions of the variables. No crossed combination is realized. Therefore, at each FORM iteration it is only necessary to simulate $2n + 1$ points, where n is the number of variables.

3 Application – Oil wells drilled on salt zones

Oil wells drilling salt formations, very common on pre-salt fields, are often affected by problems arising from salt creep. Creep is slow time-dependent deformation process that, even under constant stress, can generate excessive displacements towards the closure of the well. It may trap the drilling column during operation or compromise the structural integrity during production. Therefore, it is essential to evaluate radial displacements in wells considering the creep phenomenon.

In this context, a well drilled in salt formations is the case study for the proposed comparison in this work. The structural behavior of the salt rock is evaluated using a in-house developed simulator that employs the finite element method following the formulation presented by Araújo [5]. Its results are used in the construction of the response surfaces (RSM and ARSM) from the methodology presented by Santos et al. [6]. In the next sections, the study case is defined and the random variables are presented.

3.1 Wellbore configuration

The chosen scenario is presented by Costa et al. [7] due to the good detailing of the data needed to reproduce the oil well behavior. Figure 2 shows the scheme of the adopted numerical model.

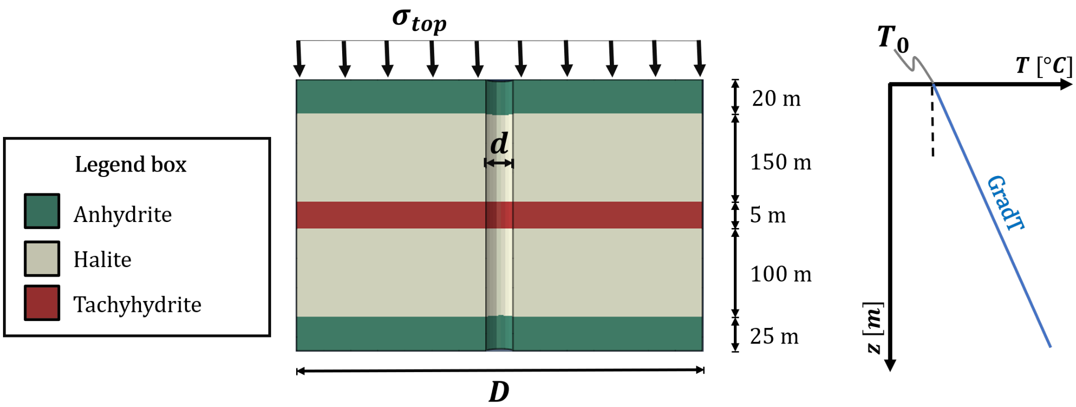

Figure 2. Schematic representation of the adoped numerical model

The adopted well has a diameter $d = 12.25$" and the rock massif is simulated with an outer diameter $D = 24.72$ m. The stress acting at the top of the massif is $\sigma_{top} = 107.63$ MPa and the temperature is $T_0 = 140$°C, with a geothermal gradient $gradT = 33.33$°C/km. More detailed data of the boundary conditions and other properties of the model are presented in Costa et al. [7]. The finite element model uses a mesh with 20 elements in the radial direction with length following a geometric progression with ratio 1.45. In the longitudinal direction, discretization is performed every meter in anhydrite and halite rocks and every half meter in tachyhydrite.

In addiction, based on this maximum radial displacement value, as shown by Santos and Ramos Jr [8], a failure function for oil wells drilled in salt rocks is presented in eq. (1):

$$G(\overrightarrow{X}) = \delta r_{max} - Z(\overrightarrow{X}) \tag{1}$$

The definition of the maximum acceptable radial displacement δr_{max} is based on the minimum cement thickness of 0.75" and, for a casing of 9.625" diameter, $\delta r_{max} = 0.5625$", as suggested by Costa et al. [7].

3.2 Random variables

In this work, four random variables are adopted: the temperature at the top of the massif; the geothermal gradient; stress at the top of the massif; and the density of the salt. As for characterization, due the dificulty to obtain these information with precision, the variables related to temperature are characterized with normal distribution, as shown by Fossum and Fredrich [9]. The stress at the top of the massif is adopted following a log-norm distribution, based on Li et al. [10]. The density of salt is characterized with the study presented by

Hamrouni et al. [11]. The statistical information of random each variable used is summarized in Table 1, including the mean, the coefficient of variation (COV) and the probability distribution function (PDF).

Table 1. Statistical characterization of random variables

Variable	Unit	Mean	COV [%]	PDF
T_0	°C	140	10	Norm
$gradT$	°C/km	33.33	20	Norm
σ_{top}	MPa	107.63	5	Log-norm
ρ_{salt}	kg/m³	2000	10	Norm

The statistical parameters adopted by the authors are chosen based on the aforementioned works. For more realistic analyses, accurately statistical data associated to Brazilian oil wells must be collected.

4 Results

For the parameters and values adopted in this work, Table 2 presents the design points (most likely failure point) obtained via FORM using RMS, ARMS and DFM. All methods leaded to points pretty close to each other, demonstrating that, for this scenario, they have a great similarity in the results, which is indicative of a good accuracy.

Table 2. Design points

Method	T_0 [°C]	$gradT$ [°C/km]	σ_{top} [MPa]	ρ_{salt} [kg/m³]
RSM	142.432	33.421	110.981	2006.974
ARSM	143.260	33.460	111.551	2008.696
DFM	143.391	33.472	111.534	2007.771

As can be seen in this table, the design points are very close to the mean value point. This indicates a high probability of failure of the structure. These probabilities are shown in Table 3.

Table 3. Failure probability (P_f) and number of simulations (N_{sim})

	RSM	ARSM	FDM
P_f [%]	25.359	21.808	21.817
N_{sim}	81	134	72

For the scenario adopted, the probability of the well closure to exceed the acceptable value is greater than 21%, which is a very high value. This characterizes a low level of safety of the structure regarding the established criteria. As each numerical simulation have the same computational cost, ARSM has the highest demand, because it needs more simulations, as seen in Table 3. This is justified because, on each iteration, several scenarios are simulated nearby of the evaluated point. DFM highlight out since it achieves failure probability similar to ARSM with lower computational coast, characterizing itself as a very efficient methodology for the problem addressed.

FORM is also able to provide the sensitivities of the variables. These factors obtained using RMS, ARMS and DFM are presented in Table 4.

Table 4. Reliability sensitivity analysis [%]

Method	T_0 [°C]	$gradT$ [°C/km]	σ_{top} [MPa]	ρ_{salt} [kg/m³]
RSM	6.860	0.039	92.824	0.277
ARSM	8.941	0.060	90.687	0.312
DFM	9.685	0.071	89.994	0.249

The sensitivities quantify the influence of the randomness of each parameter in the process of obtaining the failure probability of the structure. These factors are calculated from the cosines of the normal vector to the failure function at the design point on standard normal space. As shown in Table 4 the values obtained are very concordant with each other and, for all methods, the most influential variable is the stress at the top (σ_{top}), followed, in this sequence, by the temperature at the top (T_0), by the density of the salt (ρ_{salt}) and, finally, by is the geothermal gradient ($gradT$) with an almost negligible sensitivity.

5 Conclusions

This work presented a comparative study of several methodologies to obtain the gradient of the failure function in the first order reliability method (FORM) applied to oil wells drilled on salt rocks. From the results achieved, it can be concluded that, for the values and parameters considered, to carry out the reliability analysis using RSM, ARSM and DFM, the results obtained are quite similar. The DFM presented a computational cost considerably lower than the other two methods, therefore, is the most appropriate methodology to carry out this type of analysis. Sensitivity analysis was also performed, verifying that the different strategies are coherent with each other. The most important variable in the evaluation of the failure probability being the stress at the top of the layer with about 90% of the sensitivity and as a lesser variable influence the geothermal gradient with less than 0.1%.

Finally, it is noteworthy that the methodology applied in this work, for the oil well and the chosen variables, can be expanded: a) using complex models; b) to further variables of oil wells; c) to other structural problems that have their mechanical behavior evaluated using numerical methods; or d) using more accurate statistical data, allowing for even more realistic assessments.

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