Optimizing the volume of reinforced concrete floors using grid analogy

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Abstract. One of structural engineering's main goals is to model a structure able to withstand the loads to which it is subjected, but consuming the least amount of material possible, which is the concept of structural optimization. This article aims to use an algorithm in Python 3.0 capable of optimizing the sizes of beams and slabs from a conventional building system made of reinforced concrete, so that the Serviceability Limit State of excessive deflection is satisfied in the most economical way. For this purpose, the floor is modeled by beam Finite Elements to represent both the structural beams and the slabs, in a Grid Analogy Method. For the optimization process, it is necessary to use a library able to solve constrained minimization of multivariate scalar function problems, in order to do this, the SciPy library is used. This framework is applied to typical sizing examples of reinforced concrete floors and results are compared in terms of displacements and volume of concrete.

Keywords: Reinforced concrete, structural optimization, Grid Analogy Method.

1 Introduction

Every project made and every decision taken by a civil engineer is based on the balance between safety and economy. The idea of building using the least amount of material guides the construction industry in all aspects, which makes optimization its main objective. However, before the creation of computer programs, due to the fact that all the projects were manually made, optimizing was extremely difficult.

Nowadays, engineering software allow designers to develop projects in a faster way and, consequently, to test various arrangements in order to obtain the most efficient. Programs focused on structural design are very common and are able to process the structural analysis and design, but not all of them use the same method to calculate the internal forces and deflections in structural elements.

Using grid analogy as a method to carry out structural analysis of slabs has begun since before the advent of computers, being Marcus (1932 *apud* Stramandinoli [1]) the first one to use it for this purpose. Since then, this method has been widely used.

Lightfoot (1959 *apud* dos Reis [2]) used computer programs to study buildings' floors by grid analogy, making him able to solve more complex problems like large size slabs, with or without beams, with irregular shapes and openings.

Associating an algorithm that calculates forces and deflections with a library able to solve constrained minimization of multivariate scalar function problems can optimize structural projects faster and more precisely than the usual way.

2 Grid Analogy

This matrix analysis method consists in modeling slabs as a group of orthogonal bars, parallel to the slabs' axes, and subjected to external loads perpendicular to the plane to which the bars belong. Each bar is divided by their joints into smaller parts called members.

The grid model uses as reference a global right-handed spatial coordinate system, the slab belongs to the horizontal XZ plane and the loads are parallel to the Y axis. Besides, each member has its own xyz local coordinate system, whose origin is in one of its joints, x axis is positively oriented from the origin joint to the final, y in the upward vertical direction and z perpendicular to the xy plane obeying the right-hand rule.

2.1 Degrees Of Freedom

The joints of the grid can translate in the global Y direction and rotate about any axis that belongs to the XZ plane. This rotation can be decomposed, due to its low value, to X and Z axes. However, because of the limitations imposed by the structural system, some of these displacements are restricted. Those restrictions are known as boundary conditions and the remaining displacements are the degrees of freedom of the system. The process to globally number the degrees of freedom occurs in a way that their number follows the order of their respective joint, which, in each joint, the first one to be indicated is the translation in Y, followed by the rotation about X and, lastly, about Z. Locally, the degrees of freedom are numbered following the same idea, but with only two joints, which the first one is the origin of the member and the second one is the end, that results in 6 degrees of freedom for each member.

2.2 Physical and Geometrical Properties

The physical properties are related to the material that composes the grid, so the members' secant and transverse modulus of elasticity are the same as the concrete used in the floor. While the geometrical properties depend on the member's cross-section dimensions and can be calculated by the following equations:

$$I = \frac{b\times h^3}{12} \tag{1}$$

$$J = 2\times I \tag{2}$$

2.3 Local Stiffness Matrix and External Loads Applied Directly on The Member

In order to uncouple bending and torsional moments, the member's cross-section must be symmetric with respect to the axis parallel to the load's direction and, besides that, it is assumed that those sections are free to warp out of their plane. Using the Displacement Method, also known as Stiffness Method, the member stiffness relations are obtained, which can be represented by the following matrix:

$$k = \frac{E\times I}{L^3}\begin{bmatrix} 12 & 0 & 6\times L & -12 & 0 & 6\times L \\ 0 & \frac{G\times J\times L^2}{E\times I} & 0 & 0 & -\frac{G\times J\times L^2}{E\times I} & 0 \\ 6\times L & 0 & 4\times L^2 & -6\times L & 0 & 2\times L^2 \\ -12 & 0 & -6\times L & 12 & 0 & -6\times L \\ 0 & -\frac{G\times J\times L^2}{E\times I} & 0 & 0 & \frac{G\times J\times L^2}{E\times I} & 0 \\ 6\times L & 0 & 2\times L^2 & -6\times L & 0 & 4\times L^2 \end{bmatrix} \tag{3}$$

2.4 Internal Forces Resulted from External Loads Applied Directly on The Member

External loads can be applied on members or joints. For the ones applied directly on members, they are analyzed locally and the internal forces and moments caused by them are calculated considering a fixed beam. Those forces form a vector Q_f in which they are numbered accordingly with their respective degree of freedom.

2.5 Coordinate Transformation, Global Stiffness Matrix and Global Forces Vector

To calculate the relations between the global and local coordinate systems, it is considered an arbitrary member oriented by an angle θ between the positive direction of the global X axis and its local x axis. Thereby, is obtained a coordinate transformation matrix T.

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 & 0 & 0 \\ 0 & -\sin\theta & \cos\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix} \quad (4)$$

The global stiffness matrix is obtained based on the relations of the local stiffness matrix, those relations are transformed into the global coordinate system and allocated according to the corresponding global number of the degree of freedom. For joints that are part of 2 or more members, the principle of superposition effects is used, in other words, the global stiffness is the sum of all joint's local stiffness.

To calculate the global forces vector, the internal efforts that result from external loads applied directly on the member are transformed to the global coordinate system and distributed in accordance with their respective degree of freedom. Subsequently, it is summed to the loads acting directly on the joints.

2.6 Displacements and Internal Forces

To obtain the displacements in each degree of freedom, the equilibrium is used through the following formula:

$$[u] = [K]^{-1} \cdot [F] \quad (5)$$

With the displacements in each joint, it is possible to obtain the internal efforts in each member. For this, it is necessary to locally analyze those members and the respective displacements of their joints, transformed back into the local coordinate system. The internal forces can be calculated by the following equation:

$$[Q] = [k]\cdot[u] + [Q_f] \quad (6)$$

3 Optimization

The optimization process is made using a function of SciPy Library [5] able to solve constrained minimization of multivariate scalar function problems. For this, some parameters must be defined.

3.1 Design Variables and Objective Function

Design variables are the parameters that can be changed and whose optimal values are the result of the optimization process. For problems involving the Serviceability Limit State of excessive deflection in reinforced concrete floors, those variables are the slab's thickness and the beams' width and height.

The optimization is guided by the objective function, which is analyzed to obtain its maximum or minimum value. For the problem in question, the main target is to reduce the cost to its minimum, that is, to consume the least quantity of concrete. So, the objective function can be expressed as the volume of concrete used in the floor.

$$V = L_x \times L_z \times e + \sum_i^n [b_i \times (h_i - e) \times l_i], \quad (7)$$

where L_x, L_z are the slabs lengths, e is its thickness, n is the number of beams, b_i, h_i their cross-section dimensions and l_i their lengths.

3.2 Constraints

In order to use criteria usually used in structural projects, the constraints are based in NBR 6118:2014 [3]. According to the item 13.2.2, the dimensions of beams' cross-sections must be, at least, 12 centimeters.

$$b_i \geq 12cm \quad (8)$$

$$h_i \geq 12cm \quad (9)$$

For slabs, the item 13.2.4 specifies that its minimum thickness is 8 centimeters.

$$e \geq 8cm \quad (10)$$

According to table 13.3, maximum deflection for beams is its length divided by 250 and, for slabs, it is its smallest span divided by 250.

$$\delta_{max,\ beams} = l/250 \quad (11)$$

$$\delta_{max,\ slabs} = l_x/250 \quad (12)$$

4 Conclusion

To obtain the program's results, it was used a rectangular slab with 4 meters of width, 6 meters of length and 12 centimeters of height, in which was applied a load of 2kN/m². In order to verify the grid analogy results, the analysis was made, initially, with simple supports in all sides and after with fixed supports. The reference values were calculated by Bares' method (Carvalho [4]) and the convergence was verified increasing the number of members per meter.

To simulate the example with simple supports on the program, it was made a floor with only one slab, whose dimensions are the same as the analyzed slab, and 4 beams in its borders, those beams have a very high bending inertia and null torsion inertia. The same process was done to fixed supports, but with beams with very high bending and torsion inertia.

The table 1 shows the reference values, the values obtained with the program with 20 members per meter and the percentage differences between them. The small percentage differences between them show that the grid analogy method implemented in the program is satisfactory.

Table 1. Displacement values

Case	Bares' method (m)	Program (m)	Percentage difference (%)
Simple supports	0.00085686	0.00095203	11.11
Fixed supports	0.00024247	0.00025715	6.05

The values of displacements can be seen in figure 1 for simple supports case and in figure 2 for fixed supports case. The green dots refer to the program values, whose convergence is noticeable, while the red dot represents the value calculated by Bares' method.

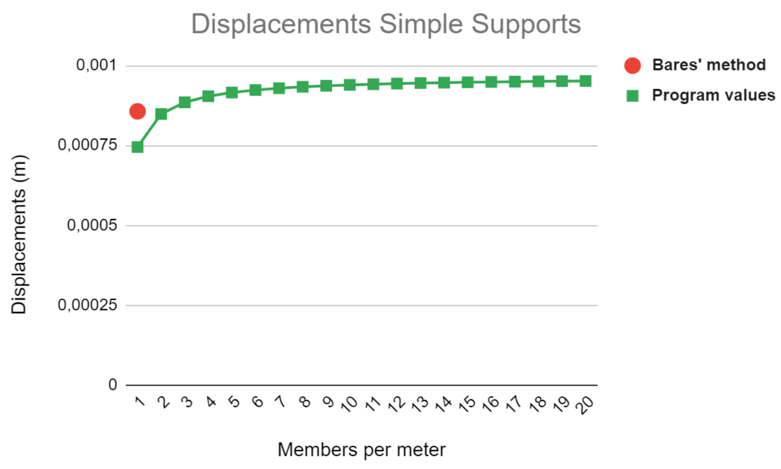

Figure 1. Displacements to slab with simple supports.

Figure 2. Displacements to slab with fixed supports.

The optimum slab's dimension obtained was $e = 8cm$, which is equivalent to the minimum value. The values related to the beams with 6 meters of length were $b = 12cm$ and $h = 22.40cm$, while the ones corresponding to the beams with 4 meters were $b = 12cm$ and $h = 12cm$.

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