MODELING OF BEAMS WITH WEB OPENINGS USING THE BOUNDARY ELEMENT METHOD

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Abstract. This paper presents an alternative modeling of beams with openings in the web, using the 2D Boundary Element Method formulation (BEM) to analyze the stress and displacement distributions at internal points. The material is considered to be elastic linear. A "2D MEC" program was elaborated using Kelvin's fundamental solution with continuous and discontinuous linear elements for the boundary discretization, including the openings. The PYTHON programming language was used to develop the computer code, since it's an open-source platform. Some examples already evaluated in other scientific works are presented, to test the effectiveness of the implemented computational code, as well as the formulation used. The results found demonstrate that the Boundary Element Method, in a simple program, can be inserted in simple practical solutions as usual structural calculation.

Keywords: Beams with Web Openings; Boundary Element Method, 2D Analysis.

1 Introduction

During the design of a structure, all loads (permanent, variable and exceptional) capable of producing states of stress or deformation in the structural parts should be taken into consideration. The absence of compatibility between architectural, structural and complementary projects can often cause the need for holes and openings in the structural parts of a construction, interfering with the strength and durability of the element. Thus, without due verification, openings in the web (discontinuous region) result in non-compliance with the specifications of NBR 6118 [1], since changes in stress flow around these locations will not be taken into account.

NBR 6118 [1] describes that hole have a small dimension in relation to the structural part, while openings will have larger dimensions. Furthermore, holes with small distances between them will be considered as one opening. According Mansur et al. [2], openings in beams cause effects on their behavior. The corners of the openings suffer a concentration of stresses that may result in cracking of the structural part. However, if these occasional openings, their dimensions and locations are predicted and the reinforcement design is carried out, changes in the behavior of the structure may be minimized. The calculation of the necessary reinforcement to resist the bending moment is one of the most important points in the detailing of reinforced concrete parts and this design is done based on the ultimate limit, which occurs when the concrete does not meet the parameters of functionality and durability for which it meets the needs of the project and is classified as: Ultimate Limit State (ULS) and Service Limit State (SLS).

For analysis in question, the work makes use of the Boundary Element Method (BEM). The BEM is capable of generating an analysis of the structure's behavior, using the variables of points on the boundary (Brebbia [3]). It is applied to solve differential equations that define the equilibrium of a body, here considered elastic, linear and homogeneous. In developing the BEM formulation, the differential equations are transformed into integral equations applied to the boundary of the problem. The Boundary Element Method brings as interesting characteristics to lower the number of dimensions of the problem (for example, if the problem is 2D the discretization is one-dimensional, in its boundary), ensure accuracy in the expressions of displacements and stresses in internal points (Almeida et al. [4] and Fedelinski et al. [5]).

In this context, a "2D MEC" modeling program was created from Kelvin's fundamental solution using continuous and discontinuous linear elements for discretization of the element contour with the openings. The PYTHON programming language was used to develop the program. To prove the applicability of the code and its methodology, problems already solved in scientific works were used.

2 Boudary Element Method formulation

One wants to analyze a structural element defined by a domain **Ω** and its respective boundary Γ which presents two-dimensional elastic-linear, homogeneous and isotropic characteristics. In eq. (1) the equilibrium differential equations of the elastic problem are given, for *j = 1,2*, the stresses tensorσ_{ji} and body forces b_i, and in eq. (2) the boundary conditions are presented as a function of displacements (u) and surface tractions (p).

$$\sigma_{ji,j} + b_i = 0. \tag{1}$$

$$\begin{aligned} u &= \overline{u}, \text{em } \Gamma_1 \ , \\ p &= \sigma \cdot n = \overline{p}, \text{em } \Gamma_2 \ . \end{aligned} \tag{2a, b}$$

2.1 The numerical solution

For the numerical approach to the stated problem a boundary method is used. By performing some algebraic steps in eq. (1), results in the Somegliana Identity, described by the integral equation that governs the problem, eq. (3), thus defining a solution from the analysis of variables preponderantly on the boundary of the elastic body.

$$u_{lk}^i + \int_\Gamma p_{lk}^* \cdot u_k d\Gamma = \int_\Gamma u_{lk}^* \cdot p_k d\Gamma + \int_\Omega b_k \cdot u_{lk}^* d\Omega. \tag{3}$$

In eq. (3), use is made of weights for error minimization. For the BEM the chosen weighting function is the fundamental solution. In the present work, Kelvin's fundamental solution for two-dimensional elastic problems will be used, according to expressions (4a and b).

$$\begin{aligned} u_{lk}^* &= \frac{1}{8\pi\mu(1-\nu)}\left((3-4\nu)\, ln\left(\frac{1}{r}\right)\delta_{lk} + r_{,l}r_{,k}\right), \\ p_{lk}^* &= -\frac{1}{4\pi(1-\nu)r}\left(\frac{\partial r}{\partial n}\Big((1-2\nu)\delta_{lk} + 2r_{,l}r_{,k}\Big) + (1-2\nu)\big(n_l r_{,k} - n_k r_{,l}\big)\right), \end{aligned} \tag{4a, b}$$

where μ is the transverse modulus of elasticity and n is the vector normal to the boundary.

From studying equation (3) it is identified that its application can occur both in the domain and outside of it. Thus, we arrive at eq. (5), in a generalized form for any points.

$$c_{lk}^i u_{lk}^i + \int_\Gamma p_{lk}^* \cdot u_k d\Gamma = \int_\Gamma u_{lk}^* \cdot p_k d\Gamma + \int_\Omega b_k \cdot u_{lk}^* d\Omega \, , \tag{5}$$

where c^i_{lk} is a constant that depends on the position of the source point i, worth 1 for domain points, ½ for boundary points when smooth, and 0 for external points.

In order to apply the numerical method, it is necessary to discretize the problem in order to determine the points (nodes) where the solution equations will be written and its variables considered. Being the problem two-dimensional and observing the boundary, in this work linear elements are used for its approximate conformation, giving rise to the end points that will be used as functional nodes. Therefore, when discretizing equation (5), assuming the non-existence of body forces (b_k), eq. (6) is obtained.

$$c_{lk}^i u_{lk}^i + \left(\int_\Gamma p_{lk}^* \cdot \varphi \quad d\Gamma\right) \cdot u_k^j = \left(\int_\Gamma u_{lk}^* \cdot \varphi \quad d\Gamma\right) \cdot p_k^j. \tag{6}$$

By rewriting equation (6) in matrix format we have eq. (7), for this transformation we use the interpolating functions on the chosen elements and the nodal values of displacements and surface tractions.

$$[H]\{U\} = [G]\{P\}. \tag{7}$$

Therefore, the boundary conditions in eq. (7) are applied, columns and rows are exchanged, giving rise to a system of linear equations, in which the unknowns can be either displacements or surface tractions, according to eq. (8). When using a discontinuous linear element, with two functional nodes per element, you will have a system that reaches the order of 4n x 4n, where n is the number of elements.

$$[A]\{X\} = \{B\} \tag{8}$$

2.2. Opening Modeling

The introduction of web openings decreases the stiffness of the beams resulting in the possibility of strength problems in the element. The analysis of reinforced concrete beams, for example, with web openings is important for two main reasons: to determine an appropriate reinforcement arrangement for the regions surrounding the openings and to verify the possibility of reaching the ultimate load for beams with holes/openings.

In the Boundary Element Method, regions considered as cavities (empty), are analyzed from the same principle previously discussed, also with the fundamental problem, here used the Kelvin problem, with a boundary adjustment. Consider a two-dimensional, elastic-linear, homogeneous solid with the domain $\overline{\Omega}$ and boundary $\overline{\Gamma}$, and enclosing a cavity defined by the boundary Γ, Figure 1, with normal vector to its boundary, which will always point outwards, to get an integral representation for the boundary.

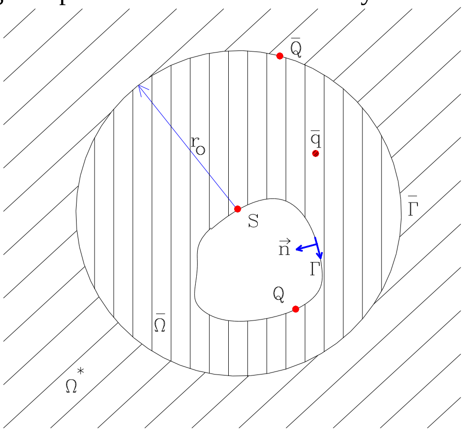


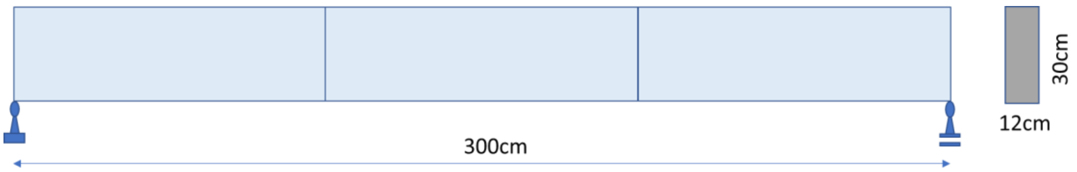
Figure 1. Infinite region with void - space of Kelvin's fundamental problem.
Source: Barbirato [6]

The body is defined by $\Gamma + \overline{\Omega} + \overline{\Gamma}$ and has a circle of radius r_o with center point defined at S belonging to the boundary Γ of the cavity and is contained in the plane Ω^* infinite (or semi-infinite). To obtain the integral representation, the corresponding portions of the domain are added to the identity $\overline{\Omega}$ and boundary $\overline{\Gamma}$ and we try to make the radius of the circle r_o tends to infinity, thus being a limit situation of the problem. According to Rocha [7] the integral representation determined for finite geometry problems will also be valid for infinite regions, thus being a possible formulation for the analysis of excavation and opening problems.

3 Application

Based on the formulation presented in the previous items, a computational program, written in free software language PYTHON, was developed to implement the discontinuous linear element, with the interactions of displacements and surface forces for the coupling of subdomains via the subregions technique and modeling of cavities (openings in the problem of this work). The "MEC 2D" code, therefore, requires a simple computer, a microcomputer, and performs processing at a higher speed than platforms whose codes are interpreted.

In order to evaluate the use of the Boundary Element Method to analyze the behavior of beams with openings in their web, the problem addressed in Simão [8] was used. The beams are made of reinforced concrete, with 300cm length between supports and rectangular cross section measuring 12cm x 30cm. A concrete material with characteristic compressive strength of 30MPa was considered. Following the models of Simão [8], four cases were processed: a) beam without web opening; b) with 1 opening of 15cm x 10cm distant 15cm from the beam side face; c) with 1 opening of 60cm x 10cm distant 15cm from the beam side face; and d) with 1 opening of 15cm x 10cm distant 60cm from the beam face. In the modeling for the use of the BEM, the discretization happened in the first place in the definition of three regions that are coupled, each one being discretized by continuous and discontinuous linear elements, duly observing the nodes of the interfaces. Figure 1 shows the beam definitions as listed in the models.The models are simply supported subjected to uniformly distributed load from top to bottom, varying from zero to failure value (from tests performed in Simão [8]).



(a)

(b)

(c)

(d)

Figure 2. Beam for analysis following Simão [8]: a) definition of the beam and the three subdomains for BEM analysis - model without opening; b) model with 15cm x 10cm opening at 15cm from the face; c) model with 60cm x 10cm opening at 15cm from the face; d) model with 15cm x 10cm opening at 60cm from the face

After processing the cases in the "MEC 2D" program, it was found that the vertical displacement values at points in the center of the beam were very close to those obtained in the reference. This demonstrates the suitability of the Boundary Element Method and the implemented algorithm. Table 1 shows the values found in Simão [8] compared to those obtained using the BEM. The models follow the four shown in Figure 1, with three subdomains, adding the VGP_1D model of the beam without opening with only 1 domain, to verify the accuracy of the discretization.

Table 1. Comparative values of the vertical displacements at the center of span, following reference Simon [8]; the maximum load was defined as the beam failure load in the experimental tests

Model	q_{unit}	Max. Displac. (m)		$q_{máx}$	Max. Displac. (m)	
	(tf/m)	Ref.	BEM	(tf/m)	Ref.	BEM
VGP_1D	1	-0.00146	-0.001344	8.833	-0.0129	-0.011874
VGP_3D	1	-0.00146	-0.00146	8.833	-0.0129	-0.012896
VG1_3D	1	-0.001778	-0.001479	9	-0.016	-0.013311
VG2_3D	1	-0.001762	-0.001777	9.25	-0.0163	-0.016437
VG3_3D	1	-0.001853	-0.001475	9.5	-0.0176	0.014013

When generating the input data, internal points were defined, forming a consistent mesh in whose nodes the displacements and stresses were calculated, including the principal ones. From the stress analysis, graphical maps were created to demonstrate the distribution of principal stresses along the beam. Significant changes can be seen on the side where the openings are located, whose disturbances can be better studied and accurate. Figure 3 shows the principal stress distributions for the models defined in Figure 2.

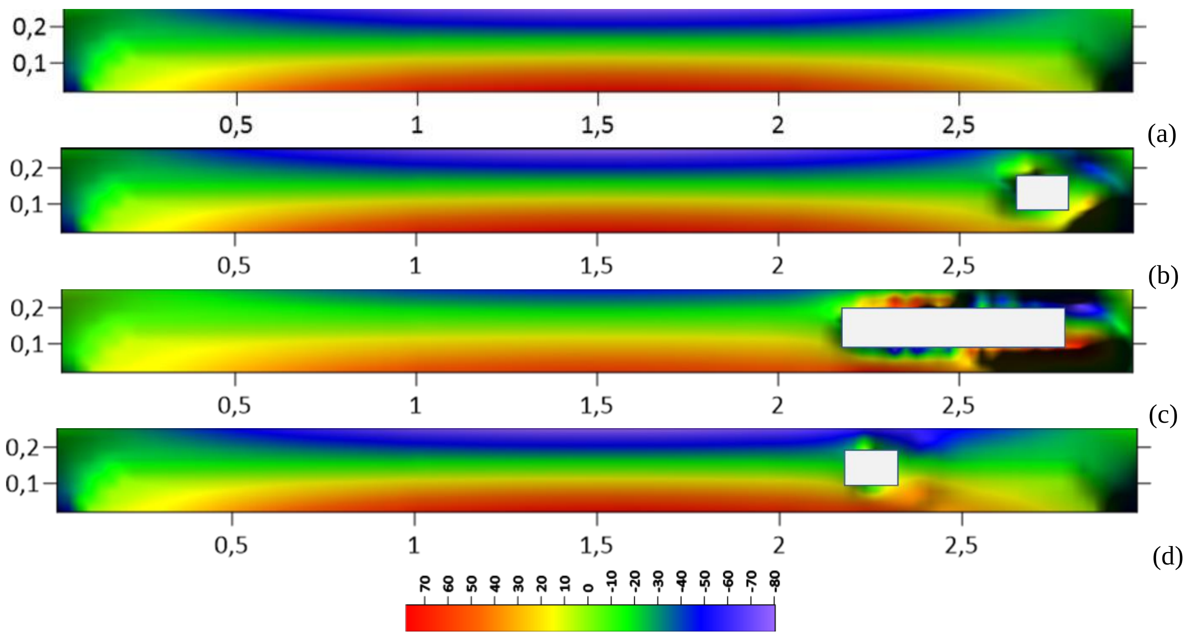


Figure 3. Analysis of the principal stresses for the three models: a) no opening; b) opening 15cm x 10cm at 15cm from the face; c) opening 60cm x 10cm at 15cm from the face; d) opening 15cm x 10cm at 60cm from the face

4 Conclusions

The implementation of the MEC 2D Boundary Element Method, written in PYTHON, proved to be adequate for the analysis of beams with web openings, providing data on displacements and stresses. The comparative tests to those presented in the literature prove the suitability, whose values were satisfactorily close. The areas of stress disturbance in the vicinity of the openings were highlighted, which leads to the warning that the openings should be defined before the due calculation and detailing of the structural parts, since the procedures for stiffening the regions around the openings can be seen before the structures are built. This work has provided knowledge about the BEM numerical method as a tool that can assist in structural analysis, as well as in a better understanding of the behavior of structures as a complement to undergraduate engineering classes.

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