Reliability analysis of reinforced concrete sections for ultimate limit states

Daiane D. L. Leite[1], Herbert M. Gomes[2]

[1]*Dept. of Civil Engineering, Universidade Federal do Rio Grande do Sul*
Av. Osvaldo Aranha, 99, 90035-190, Porto Alegre, RS, Brazil
daianedll@hotmail.com
[2]*Dept. of Mechanical Engineering, Universidade Federal do Rio Grande do Sul*
Av. Osvaldo Aranha, 99, 90050-170, Porto Alegre, RS, Brazil
herbert@mecanica.ufrgs.br

Abstract. The application of probabilistic methods in the design of reinforced concrete structures has paramount importance. Several uncertainties in the structural design, which range from variations in material properties and model hypothesis to the applied loads can affect the final predicted structural behavior. Besides, even if the design is developed with the highest degree of strictness, deviations in the final executed structure may happen. Factors such as, the opening of the formwork when concreting, the lack of spacers in the reinforcement and the inadequate concrete dosage, all can contribute to the uncertainty. These are factors that may result of unsatisfactory structural performance, because they can compromise safety. In this context, the objective of this paper is to evaluate the reliability of reinforced concrete sections under bending loads in ultimate limit states. The uncertainties that can affect the ultimate strength comprise reinforcement cover, concrete strength, external loading, and cross-section dimensions. This study uses First Order Reliability Method (FORM) and the Monte Carlo Simulation (MCS) with Importance Sampling (MCIS) to obtain the reliability indexes. Uncertainty propagation is also performed by MCS to study cross-section strength sensitivity to uncertainties. At the end, the most important parameters are discussed and probability density plots and reliability index curves presented.

Keywords: uncertainty propagation; Monte Carlo simulation; reinforced concrete; FORM; structural reliability.

1 Introduction

Reinforced concrete (RC) structures are very versatile means of construction. They present advantages in terms of robustness, maintenance low cost, slow degradation, and high degree of resistance for ambient aggressiveness. Due to inevitable unpredictability present in every design and loading, and uncertainties that comes from modeling assumptions, geometric deviations, and construction defects, all the advantages may be harmed.

Choi *et al.* [1] classify uncertainties according to the type of sources, as Aleatory and Epistemic. Aleatory (random or objective) uncertainty is also called irreducible or inherent uncertainty. Epistemic (subjective) uncertainty is a reducible uncertainty that stems from lack of knowledge and data. A number of probabilistic analysis tools have been developed to quantify uncertainties, but some complex systems are still designed with simplified rules. Factor of safety have been used to maintain some degree of safety in structural design. When the scatter in the variables is considered, the factor of safety could potentially be less than unity, and the traditional factor of safety based design would fail. Particularly on RC cross-section designs, it is of interest to obtain and analyze the degree of safety involved in such designs based on NBR 6118:2014 [2] which uses safety factors as means of dealing with uncertainty.

A number of papers can be found in literature about reliability of RC members like in Scherer *et al.*[3], Saleh *et al.*[4] , Santos *et al.*[5], Baji and Ronagh [6], Gomes *et al.* [7], but few deals with the sensitivity analysis and the effects of geometric factors in the reliability of the RC sections.

In this context, the objective of this work is to evaluate the structural reliability of RC cross section in bending situations taking into account the uncertainties related to geometric and material property uncertainties. Besides it is evaluated the sensitivity of some important parameters on the resisting bending moment of RC cross-sections and their effect on the final reliability for ultimate limit state function for RC cross section designed based on safety factors semi-probabilistic approach from Brazilian NBR6118:2014 [2]. Monte Carlo Simulation method by Importance Sampling (MCIS) and the FORM (First Order Reliability Method) will be used to the reliability evaluation or the sensitivity investigation.

The Brazilian standard for structural design, NBR 6118:2014 [2], establishes basic requirements for the design and detailing of reinforced and prestressed concrete structures. Allied to other specific action standard, NBR 8681:2004 [8] presents the basic guidelines for building design. The first standard uses a semi-probabilistic theory to ensure the proper safety of structures, where, depending on the predictability and frequency of occurrence, the loads and strengths are treated in different ways and characteristic values are assigned to each variable. Design values are obtained by strength reduction coefficients and action loads increase coefficients [9].

However, as the structure's dimension and material's strength are random in nature, there is always some degree of uncertainty regarding the values they will assume. Dead and live actions are also considered random variables, as they present a considerable scattering. Therefore, the maximum value of the loads to which the structure will be subjected cannot be accurately predicted [10].

There are other uncertainties, which are related to the manufacturing methods, which include the errors due to employee skills and the technology of the used equipment. All the aforementioned uncertainties allows part of the relevant quantities to be assumed as random variables [9].

2 RC Cross-section sizing

2.1 Load actions according to Brazilian Codes

The influence of all actions that cause effects on the safety of the structure must be considered (NBR 6118:2014). These actions can be classified as dead, live or exceptional (rare) actions. dead actions are those that occur with constant values or with negligible variability, throughout the service life. Live actions are those that have variable intensity, duration or direction over the service life of the structure like weight of furniture, vehicles, wind action, effect of temperature variations, etc. Without loss of generality, in this work, only dead (G) and live (Q) actions will be considered as they are main action in RC buildings.

According to NBR 6118:2014 [2] and NBR 8681:2004 [8], live action characteristic values correspond to values that have 25% to 35% of probability of being exceeded, in the unfavorable sense, during a period of 50 years (35% in adopted in this paper). For dead load actions, the probabilistic model adopted is of normal distribution [11] and for the live actions, an extreme Type I Gumbel probabilistic model is used [12]. For geometric variables, a lognormal distribution type mode is adopted since the values cannot assume negative values.

The combination of dead and live actions, in general, is presented in the expression below:

$$F_d = \gamma_g . F_{gk} + \gamma_q . F_{qk} \ , \tag{1}$$

where F_d is the design load value; F_{gk} and F_{qk} are the characteristic grouped dead and live load values and γ_g and γ_q are the weighting coefficients for the combined loads. The weighting coefficients of the grouped dead and live actions. In case of buildings of type 2 (where live loads are less than 5 kN/m^2) the values are $\gamma_g = 1.4$ and $\gamma_q = 1.4$.

For strength values, nominal values corresponding to the lower quantile of 5% are used. The characteristic values $\boldsymbol{f_k}$ for strength are those that, in a given lot, are likely to be exceeded, in the unfavorable sense of safety in 95%. Therefore, for a probability of 45%, analyzing the standardized normal distribution curve, there is a score of 1.645. Thus, the definition of the characteristic strength of concrete and steel is given by $f_k = \mu - 1{,}645\ \sigma$, where μ is the mean value and σ the corresponding standard deviation.

2.2 Ultimate limit state function

Safety conditions must be verified for all limit states, in all load combination, and resistances cannot be less than demands. Being $R_d \geq S_d$, where R_d is the design strength and S_d is the design load action.

The cross-section modelling assumes basic plain section hypotheses [2]. Since material properties, and geometric dimensions are random, a complete verification of the resisting bending moment is performed at the cross-section level. To evaluate the actual resisting moment of the section, subjected to short duration loading, the peak stress of 0.85 f_{cd} was replaced by f_{cm} (mean compressive strength of concrete) in the rectangular diagram.

It is not assumed any defined strains levels at concrete or reinforcing bar fibers, but those defined in the NBR6118:2014 [2] for concrete grade up to C50, as specific deformation of shortening of concrete at the beginning of the plastic limit, ε_{c2} =2°/₀₀, specific deformation of the concrete shortening in the rupture, ε_{cu} =3,5°/₀₀, and as the ultimate compressive strain for concrete and steel and ε_s =10°/₀₀ for tension steel. Although the steel reinforcement sizing is meant and performed in a ductile situation, this may not the assured for random geometric and material properties. The basic calculation hypotheses adopted for this work are presented in Fig. 1. The calculation script for obtaining the resistant moment of the concrete section is indicated in Tab. 1.

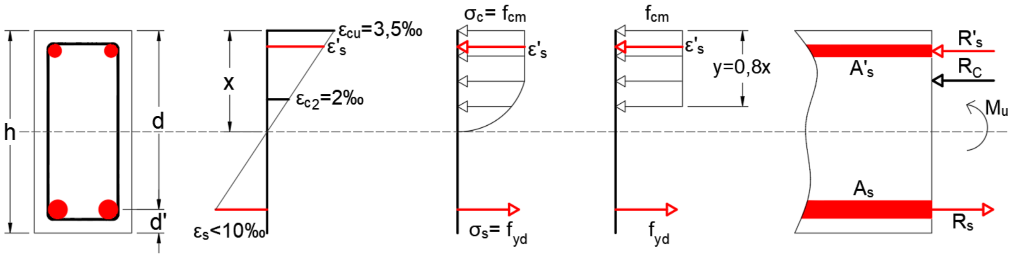


Figure 1 – Rectangular Section with Double Reinforcement - Basic hypotheses

Table 1 - Steps for bending strength evaluation for a rectangular section with double reinforcement (without the Rüsch effect coefficient).

a. Evaluate $\beta_x = (A_s\sigma_s - A_s'\sigma_s')/(b_w d\, 0.8 f_c)$;
b. If $\sigma_s = f_{yd}$ and $\sigma_s' = f_{ycd}$, is verified that $\beta_x < \beta_{lim}$, and $\beta_x \geq \beta_{lim}'$, then β_x value is correct and the resisting bending moment is $M_u = b_w d^2\, 0{,}8\beta_x f_c(1 - 0{,}4\beta_x) + A_s'\sigma_s'(d - d')$;
c. Otherwise:
-If $\beta_x \geq \beta_{lim}$ then evaluate $\varepsilon_S = \varepsilon_{cu}(1 - \beta_x)/\beta_x$ and $\sigma_s = E_s\varepsilon_S$.
- If $\beta_x < \beta_{lim}'$, then evaluate $\varepsilon_S' = \varepsilon_{su}(\beta_x - (d'/d))/(1 - \beta_x)$, and $\sigma_s' = E_s\varepsilon_S'$.
d. Take these values σ_s and/or σ_s' into **a.** and solve for β_x and the resisting bending moment is:
$M_u = b_w d^2\, 0.8\beta_x f_c(1 - 0.4\beta_x) + A_s'\sigma_s'(d - d')$

where f_c is the ultimate concrete stress, f_y is the ultimate yielding rebar stress, b_w is the cross-section width, A_s' is the compressed steel area and A_s is the tensioned steel area, σ_s' and σ_s are the steel stress and d is distance from the outmost compressed fiber to the steel's CG and d' is the concrete cover+ stirrup diameter+ half diameter of bottom reinforcement.

2.3 Uncertainty propagation

The value of a variable is obtained by measuring, which may be subject to uncertainties related to the used equipment, the measurement process, the operator or due to the action of variables that are not being quantified. So, this uncertainty resumes the dispersion of the obtained values. It can be quantified from statistical data and distribution curves. In some cases, it is not possible to make a direct measurement of the quantity and its uncertainty, but indirectly by the measurement all other uncertain variables that are relevant. Thus, the propagation of uncertainties is a useful tool to assess these specific cases [14]. The uncertainty propagation can be performed in a robust way by simulation techniques like Monte Caro Simulation (MCS). In order to reduce the number of samples necessary to precisely define distribution functions and statistical data, Latin Hypercube, a very efficient sampling technique, is used for the uncertainty propagation by MCS.

3 Reliability analysis and methods

Structural reliability can be understood as the probability of a system not violating a limit state, either by failure or by not meeting the expected performance. The main objective of a structural design is to conceive structures that simultaneously meet safety and economy requirements. In some cases, the safety requirements are not adequately quantified, as are the economy requirements. It is not uncommon to find economic structures, however unsafe, or reversely, very safe structures, but not so economic [8].

The failure probability of a structural element is given by the probability of load action (S) be greater than the load resistance (R). Thus, a safety margin can be defined as $M = R - S$ and extreme situation will be when a safety margin is zero. The probability of failure P_f can be evaluated from equation:

$$P_f = \int_0^\infty F_R(s) f_S(S) ds = P[M \leq 0] = \int_{-\infty}^0 f_M(m) dm, \tag{2}$$

where $F_R(s)$ is the cumulative probability function for the resistance, $f_S(S)$ is the probability density function for load action and $f_M(m)$ is the probability density function of the safety margin M.

For this work, the assumed limit state function is defined as $M(\mathbf{X}) = \theta_R M_R(\mathbf{X}) - \theta_S M_S(\mathbf{X})$, where θ_R is a random variable that represents the model uncertainties, θ_S represents the uncertainties of external loads, $\mathbf{X}$ is the vector of correlated random variables M_R is the resisting bending moment of the cross-section (ultimate bending moment) and M_S the combination of dead and live load action, ($M_S = M_G + M_Q$). The reliability index β is related to the probability of failure associated with the limit state function as $\beta = -\Phi^{-1}(P_f)$.

There are several methods to solve the integral in eq. (2), in this work the FORM method (First Order Reliability Method) and the Monte Carlo Simulation Method with Importance Sampling (MCIS) will be used.

FORM uses a first order approximation of the limit state function and transforming the joint probability density function $f_X(\mathbf{X})$ into an uncorrelated multivariate standard normal distribution $f_U(\mathbf{U})$. By finding the shorter distance from the origin of this new transformed variables $(\mathbf{U})$ to the limit state function $M(\mathbf{U})$ (most probable point, $\mathbf{U}^*$) one can approximate the reliability index β by the contribution of failure domain at this point by $\beta = \|\mathbf{U}^*\| = \sqrt{\sum_{i=1}^{n}(U_i^*)^2}$ [14].

In Monte Carlo Simulation (MCS) samples (N) of the random variables $(\mathbf{X})$ are generated by the inverse cumulative functions $(F_X^{-1}(\mathbf{U}))$ and realizations of the limit state function $M(\mathbf{X})$ are performed to check the number of failure cases (N_f) or safety condition. As the number of realizations tends to a large number, the probability of failure $(P_f \cong N_f/N)$ tends to the exact result for the failure probability. When the probability of failure is small, it is necessary a larger number of samples to reach some points in the failure domain [15]. Here, the MC with Importance Sampling (MCIS) technique is used. MCIS is an efficient sampling technique tailored to avoid generating samples far from the failure domain, reducing the number of samples and the variance of Failure probability estimates.

For the development of this work, it was used an equation for the relationship between the dead and live load, expressed as $\chi = Q_K/(G_k + Q_K)$. So, the equation for the characteristic value for the dead (G_k) and live (Q_K) loads is expressed, respectively by:

$$G_k = \frac{M_d}{\gamma_g + \gamma_q \frac{\chi}{(1-\chi)}}, \tag{3}$$

$$Q_k = \frac{M_d}{\frac{\gamma_g(1-\chi)}{\chi} + \gamma_q}, \tag{4}$$

were M_d is the design bending moment of the cross section.

4 Section sizing

It is intended to analyze the section of a beam corresponding to an ordinary classroom of a school (Fig. 2). The aim was to investigate whether, changing the proportion of the dead and live loads that would compromise the beam safety. It was also investigated if the geometric deviations can impact the reliability of the beam.

Besides, it will be considered uncertainties from geometric dimensions, concrete compressive strength and live loading. Other uncertain variables, described in Tab. 3, were also considered due to their inherent randomness. The design of the beam follows NBR 6118:2014 [2] recommendations. The design is presented in Tab. 2. The statistical parameters for the random variables as well as the probability distributions are presented in Tab. 3 and are based on Ellingwood and Galambos [13], JCSS [16], Santos *et al.* [5] and El-Redy [17].

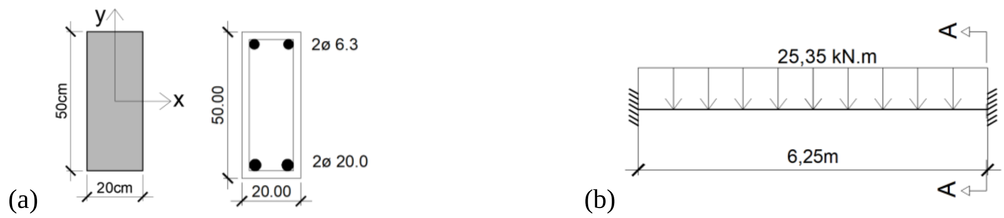


Figure 2 – (a) Cross-section dimensions and (b) Nominal geometric beam's dimensions and loads.

Table 2 – Design parameters for the beam.

Input data								
f_{ck} (MPa)	f_{yk} (MPa)	b (m)	h (m)	d (cm)	d' (cm)	M_d Limit (KNm/m)		
30	500	0.2	0.5	46	4	226.71		
Bending reinforcement results								
Moment (KN.m)	M_d (KN.m)	Type	y (cm)	A_s (cm²)	$A_{s\,min}$ (cm²)	A_s (cm²)	Rebars	Assemblage rebars
82.50	115.50	Simply reinforced	7.50	6.29	1.50	6.29	2ø20.0	2ø6.3

Were f_{ck} is the characteristic compressive strength of concrete and f_{yk} is the characteristic tensile strength of steel.

Table 3 – Probabilistic parameters for the random variables.

Variable	Symbol	Unit.	Distribution	μ_x	σ_x
Dead action	G	kN.m	Normal	G_{mean}	$0.1\mu_x$
Live action	Q	kN.m	Gumbel	Q_{mean}	$0.2\mu_x$
Concrete strength	f_c	MPa	Normal	$1.17\, f_{ck}$	*
Steel strength	f_y	MPa	Normal	$1.08\, f_{yk}$	$0{,}05\mu_x$
Height	h	cm	Normal	50	2.25
Width	b	cm	Normal	20	1.2
Lower reinforcement cover + 0.5Ø	d'_l	cm	Lognormal	d'_{nom}	1.1
Upper reinforcement cover + 0.5Ø	d'_u	cm	Lognormal	3.315	1.1
Model uncert. variable for loads	θ_S	-	Lognormal	1	0.05
Model uncert. variable for resisting moment	θ_R	-	Lognormal	1	0.05

$^*\sigma_{f_c} = 0.15\, exp\,[-0.036(f_{ck} - 20MPa)\,] \times \mu_{f_c}$

An equation for the standard deviation of the compressive strength of concrete was proposed based on El-Redy [17]. For concrete with good quality control, the resistance coefficient of variation (CV) decreases as the compressive strength increases. Multiplying the CV for concrete strength according to the concrete class, one obtains a decreasing standard deviation as the mean concrete strength increases. Most of statistical parameters presented in Tab. 3.

5 Numerical Results

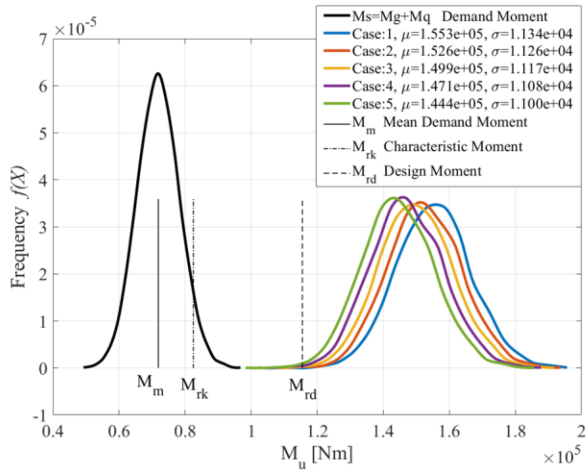
The main uncertainties addressed in this work are related to geometric deviations that can compromise the cross-section safety. They are the lower reinforcement cover + 0.5Ø (d'_l), the characteristic compressive strength of concrete (f_{ck}) and section dimensions (h, b). Fig. 3 shows an uncertainty propagation for these parameters to the final resisting bending moment for the cross-section. It was used 5000 MCS with the Latin Hypercube sampling. It was investigated 5 levels for the nominal values, namely, 0.6, 0.8, 1.0, 1.2, 1.4 which corresponds to values above and below the nominal values to investigate their sensitivity.

As one can notice, varying d'_l, the higher the value assigned, the lower the resisting bending moment of the section. For all cases, the mean acting moment was less than the mean resisting bending moment. Varying f_{ck}, the larger the value assigned, the larger the resisting bending moment the section. For all cases, the mean acting moment was less than the mean resisting bending moment. Varying b, the higher the value, the larger the resisting bending moment of the section. For all cases, the mean acting moment was less than the mean resisting bending moment. For case 1 where M_{u1}=81.8 kNm and for case 2 where M_{u2}=115.9 kNm, as the height varies, the acting moment was larger than the resistant bending moment. This situation is not acceptable, as the section has reached the exhaustion of its resistant capacity. For the other cases the section was stable. The greater the height h of the section, the greater the last moment.

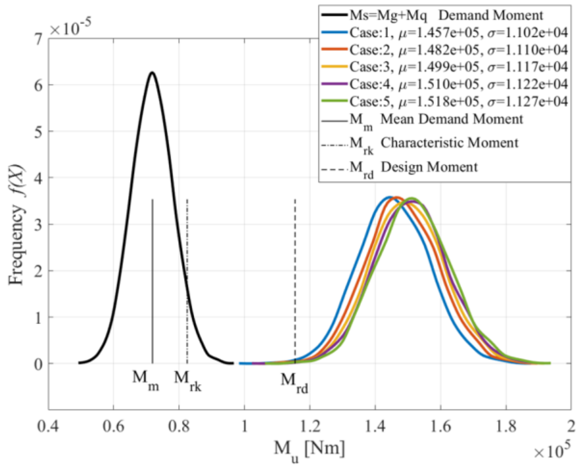
The results obtained for the reliability index of the evaluated section, designed according to NBR 6118:2014 [2], are presented in Fig. 4. The Fig. 4a presents the reliability index with respect to the variation of the beam′s concrete cover. The beam was designed for a d'_l of 4 cm, but to evaluate the influence of the concrete cover on the reliability, the cover was varied according to the environmental aggressiveness class, indicated in NBR 6118:2014 [2] table 7.2. The Class I corresponds to a cover of 2.5 cm (class II = 3.0 cm, class III = 4.0 cm, class IV = 5.0 cm), added to the diameter of the stirrup and half the diameter of the adopted steel, resulting in $d'_l = 0{,}5 + 1 + 2{,}5 = 4\ cm$. For the other classes the same reasoning was followed. The section dimensions, the steel area, the d'_u, the design moment M_d and f_{ck} were not modified, the only change was regarding the position of the bottom steel bar.

Figure 4b shows the reliability index with respect to the variation in the concrete compressive strength. Initially the beam was designed for a f_{ck} of 30MPa, the variations were from C20 to C50 (high strength concretes were not evaluated). The section was not changed, the only variation occurred in the adopted f_{ck}.

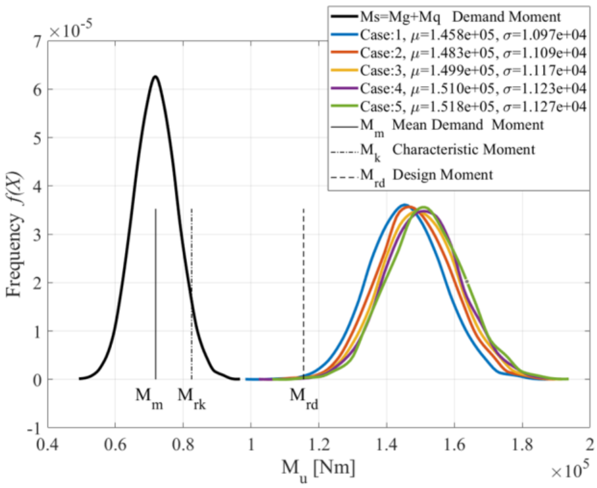
It can be seen in Fig. 4a that as the concrete cover increases there is a decrease in the reliability index. This is an expected behavior, since with the increase in the concrete cover (maintained a fixed height), the lever arm $(d - y/2)$ of the reaction resulting from the concrete compressive stresses, it decreases, making the ultimate moment smaller. The worst-case scenario is when the live action is extremely higher than the dead action, in which case the parameter χ approaches 1.



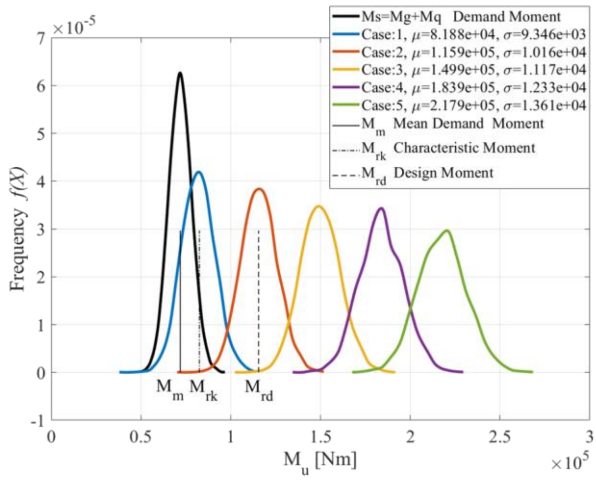
(a) d_l'



(b) f_{ck}

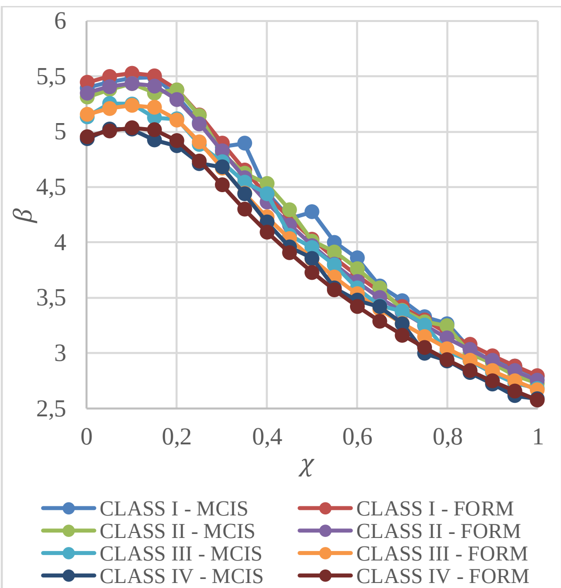


(c) Width b

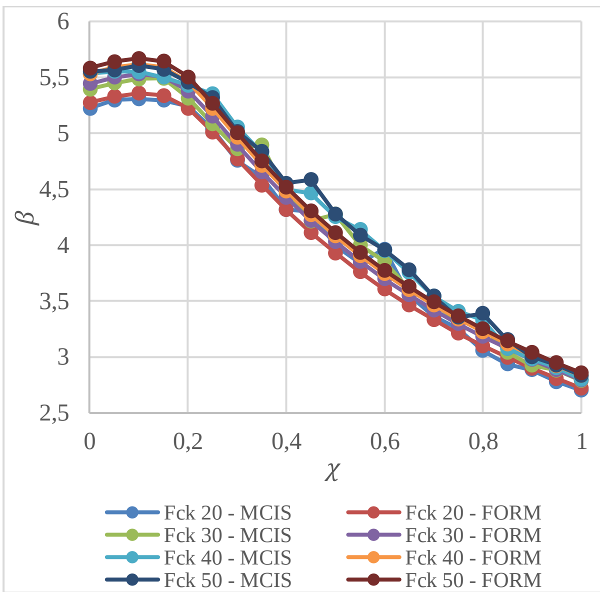


(d) Height h

Figure 3 - Sensitivity analyses for the resisting bending moment and the relevant uncertain parameters.



(a) Ranging d'



(b) Ranging f_{ck}

Figure 4 – Comparison of reliability indices

It can be seen in Fig. 4b that the higher the strength of the concrete, the higher the reliability index. The worst-case scenario is the same as described above, when the live action is extremely higher than the dead action. According to *fib* MC2010 [18], the target reliability index for a building with a moderate failure consequence in ultimate limit state is 3.8 and for ISO 2394:1998 [19] is 3.1.

6 Conclusions

This paper presented a numerical study on sensitivity and reliability of RC section designed by NBR6118:2014 [2] taking into consideration material properties, external loads and geometric imperfections. Several dead to live load ratios were considered in a situation of beam that supports a classroom slab that changes the proportion of dead and live loads. The MCIS and FORM method were used to evaluate the reliability of a reinforced concrete beam section with a considerable variation in dead to live ratio loading.

Depending on the χ parameter, the reliability can be understood as reasonable (close to the limit specified by *fib* MC2010 [18] and ISO 2394:1998 [19]) up to a value of approximately $\chi = 0.6$, or even below the recommended for $\chi > 0.6$. For lower values of χ the reliability values for ultimate limit states are too high, which perhaps prompts a modification of the partial coefficients for this range of loading ratios, as also pointed out in Santos *et al.* (2014). The results showed that the reliability obtained for the section designed according to NBR 6118:2014 [2] is consistent with the values prescribed by ISO 2394:1998 [19] and *fib* MC2010 [18], except when $\chi > 0.8$, in which case the reliability indices are not acceptable.

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