



Three-dimensional geological mapping based on cross sections restoration

Vinicius Almada¹, Luiz Fernando Martha¹, André Luís Müller¹, Márcio R. de Santi¹

¹*Tecgraf Institute of Technical-Scientific Software Development of PUC-Rio (Tecgraf/PUC-Rio)
R. Marquês de São Vicente, 225 - Gávea, Rio de Janeiro - 22451-900 - RJ, Brazil
viniciusalmada@tecgraf.puc-rio.br, lfm@tecgraf.puc-rio.br, muller@tecgraf.puc-rio.br,
marcio@tecgraf.puc-rio.br*

Abstract. The geological restoration of cross sections is one of the main oil and gas industry resources to assist in structural interpretation. In general, geological processes occur three-dimensionally. However, 3D restoration is complex and costly. The present work proposes a methodology and the tools development to map the three-dimensional movement based on the geological cross sections restoration. This methodology addresses the problem in 2 steps. The first maps the movement of the cross sections to the surfaces of the model. Subsequently, the movement of the sections together with the movement of the surfaces maps the movement of the volume. The numerical approach for the first step performs the movement of the surfaces considering control points, restrictions from movement of the cross sections, together with the minimization of a functional tri-harmonic in order to guarantee surfaces of minimum variation. The second step performs the movement of the volume based on control points given by the movement of the cross sections added to the movements of the surfaces, obtained in step 1. The basis for the development of these studies is the RECON-MS System, a software developed by PETROBRAS in partnership with the Tecgraf PUC-Rio Institute, which allows the restoration of geological sections.

Keywords: Geological restoration, Geological mapping, Surfaces and volumes deformations

1 Introduction

Geological restoration search to reverse geological processes, taking a region, from its current geometry to its original configuration, previous to deformation [1]. One of the resources present in geological restoration is the restoration of geological sections. It is the use of geometric assumptions and geological transformations in a cross section of a geological model in order to reverse the tectonic effects that occurred and thus reach the original configuration prior to the deformation. Due to the simplicity of representing the geological model in two dimensions, the restoration of geological sections has been used as a cheap and efficient solution to aid structural interpretation.

Restoring geological sections can give tectonic movement information at a two-dimensional level and a set of geological sections close together can provide even better data under a global view of the geological model.

Generally geological processes occur in three dimensions and the use of geological sections are not able to provide detailed information about what happens to the three-dimensional structures of the geological model as surfaces and volume. However, a 3D restoration is a more complex and expensive process that involves many more parameters to be performed.

Based on this, a methodology and computational tools are presented in this work to perform the three-dimensional mapping of surfaces and volumes from the restoration data of geological sections. The mapping in this work can be defined as search and applied use of data to characterize a geological surface and volume at a given stage of geological restoration.

The computational implementation was done in the Recon MS System, software that allows the restoration of geological sections among other types of analysis for structural interpretation of geological models and used in the oil and gas industry by PETROBRAS.

2 Restoration of geological sections in the Recon MS System

The Recon System [2] has a set of tools to manipulate a geological section, from digitizing the information that defines it geometrically, from the characterization of materials and properties, from the creation of control and monitoring devices for the restoration, to the transformation kit that will deform the section.

To create a geological section in the Recon System, you can use the graphic editor to draw lines and manually assign properties according to their type (if horizon, fault, section limits, etc.). Alternatively, in models with three-dimensional surfaces, sections can be created by intersecting a vertical plane according to a direction given by the user. This action is called *slicing* of the model.

The geological section is represented as a planar subdivision, second Berg et al. [3], a subdivision of the plane through the use of *edges*, *vertices* and *faces*. In Figure 1 one can observe a geological section and some elements, such as the lines where their color represents the type attribute and the faces that are, in simple terms, regions closed by lines. In this example, all faces belong to the same geological layer.

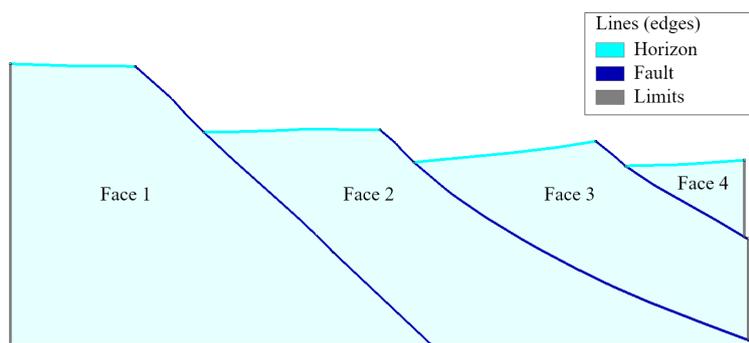


Figure 1. Geological section highlighting the elements of lines and faces.

Faces have a very important attribute for the restoration work, which are triangle meshes. Each face has a mesh independent of the others. The one-face mesh has several uses, one of them being to allow the evaluation of the strain field that occurred in the face due to geological transformations. Since the transformations act directly on the mesh, deforming it.

Restoring sections is a linear process in the sense that each new step depends on the state in the previous step. Any decision about a geological transformation in such a step leads to a different outcome in the end. Furthermore, section balancing is not a one-way activity. The aim is to obtain a plausible geological interpretation for the current geological section, that is, an interpretation that allows its structural components to be restored [4].

Thus, the Recon System makes available in its section manipulation interface a component capable of recording the history of steps in the restoration process: the scene tree. A scene is the representation of a section's restored state. For example, if a transformation occurs from one step to another of the restoration, the previous state can be registered in one scene and the later in another. From each scene you can create several others as if they were different timelines, or different interpretations of that step of restoration.

For models with multiple sections where nearby sections may have a similar restoration process, it is important to organize the scene of sections that correspond to the same geological event into groups. This is the definition of *StepMS*, a set of scenarios from different sections that represent a geological milestone in the restoration process. This restoration organization is an important part of surface and volume mapping.

3 Lines of mapping

The mapping described in this work is based exclusively on the restoration of geological sections. Therefore, there is a need for an information layer from the sections that contains the data to be used in the three-dimensional mapping. This information can be extracted with the aid of an auxiliary geometric object present in the geological sections: *lines of mapping*.

Mapping lines are lines formed by *dots of mapping*. These points are objects mapped in the section's meshes and keep information regarding their location within this mesh. The mapping points in this work are of type *node* and keep the identifier of that node.

For the surface mapping, a derivation of the line of mapping was developed, the *line of model mapping* or *LMMModel*, whose objective is to serve for the mapping of geological lines from sections to surfaces of geological horizons. The *LMMModels* represent a horizon line along the model restoration. Thus, it is possible to monitor what

happens with the geological entities in the section, in addition to being able to verify how the movement of each point of horizon occurred during the restoration. Figure 2 shows *LMModels* from two different horizons. Their representation within the Recon System is made with a line that is thicker than the horizon lines. However, the points are the same as the mesh outline.

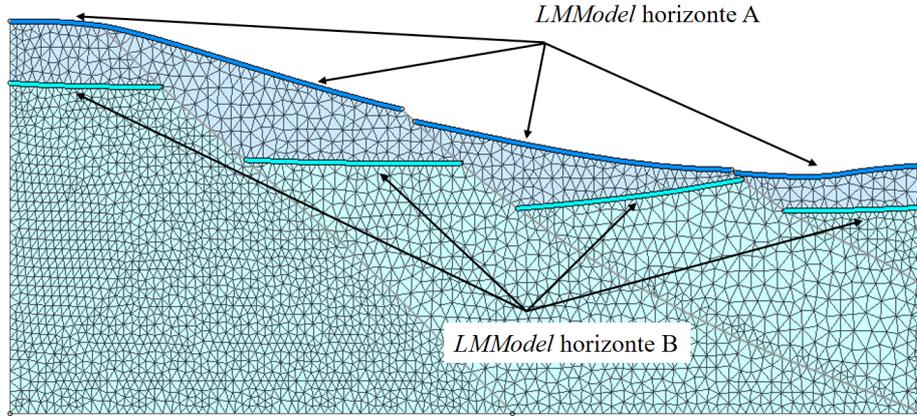


Figure 2. *LMModels* in a geological section in the Recon System

4 Surfaces mapping

By extracting displacement data from the *LMModels* of the sections between restoration steps, tectonic movement information of the entire model is obtained. This information can be used to perform a deformation on geological surfaces and thus perform surface mapping based on section restoration.

4.1 Methodology

The deformation of a surface can be modeled as a variational problem with Lagrangian functions described by the Laplace operator with different orders and that can describe different energies. The membrane surface energy is a second order PDE (partial differential equation), described by the Laplace operator Δq and the equation $\Delta q = 0$ minimizes the surface area. The thin-plate energy that minimizes surface curvature is described by the fourth order PDE $\Delta^2 q = 0$. For the minimization of surface curvature variation, a sixth-order or tri-harmonic PDE can be used, $\Delta^3 q = 0$, and the surface resulting from this minimization is called the minimum variation surface [5, 6].

Consider m known control points $\mathbf{s} = \{s_1, s_2, \dots, s_m\}$ and other n points with unknown final position $\mathbf{v} = \{v_1, v_2, \dots, v_n\}$ of a surface \mathcal{S} , being known the deformed position of the m points $\mathbf{s}' = \{s'_1, s'_2, \dots, s'_m\}$ in \mathcal{S}' , we need to define a function $\mathbf{d} \rightarrow R^3$ that interpolates exactly the m points $\mathbf{d}(s_i) = (s'_i - s_i)$ and that interpolates, guaranteeing the stationary solution of the Lagrangian function, the n unknown points [5].

This problem can be solved through an RBF (radial basis function) represented by the linear combination of radially symmetric kernels $\varphi_j(\mathbf{x}) = \|\mathbf{x}_j - \mathbf{x}\|$ located at centers $\mathbf{b}m.x_j \subset R^3$ weighted by functions weight $\mathbf{w}_j \subset R^3$ added to a low order polynomial (whose base is $\pi(\mathbf{x}) = \{x, y, z, 1\}$ and weighted by $\lambda_k \subset R^3$) to ensure polynomial precision:

$$\mathbf{d}(x) = \sum_{j=1}^m \mathbf{w}_j \varphi_j(\mathbf{x}) + \sum_{k=1}^4 \lambda_k \pi_k(\mathbf{x}) \quad (1)$$

Back to the beginning of the problem, the interest is to solve the tri-harmonic EDP $\Delta^3 \mathbf{d}(x) = 0$. For that, it is necessary to choose a kernel function that is a fundamental solution of this equation. It was shown by Botsch [6] that $\varphi(r) = r^3$ is a fundamental solution that leads to a surface of minimal variation. For surface-based solution, this minimization must be obtained explicitly. The coefficients \mathbf{w}_j and λ_k are obtained by solving a linear system (Eq. 2) of equations $(m+4) \times (m+4)$ as long as the interpolation of the m control points with the restrictions imposed on the positions $x_j = s_j$ [5] is satisfied.

$$\begin{bmatrix} \varphi_1(s_1) & \cdots & \varphi_m(s_1) & \pi_1(s_1) & \cdots & \pi_4(s_1) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \varphi_1(s_m) & \cdots & \varphi_m(s_m) & \pi_1(s_m) & \cdots & \pi_4(s_m) \\ \pi_1(s_1) & \cdots & \pi_1(s_m) & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \pi_4(s_1) & \cdots & \pi_4(s_m) & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} w_1^T \\ \vdots \\ w_m^T \\ \lambda_1^T \\ \vdots \\ \lambda_4^T \end{bmatrix} = \begin{bmatrix} (s'_1 - s_1)^T \\ \vdots \\ (s'_m - s_m)^T \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (2)$$

After solving the system of equations and getting the weighting coefficients, the n unknown points can be obtained with $v'_i = v_i + \mathbf{d}(v_i)$. The computational procedures for the types of surface energy minimization described in this section were implemented by Müller and are available in the Recon System's *MGeo Deformer* [5] library.

4.2 Control points: *LMModels*

This methodology is capable of performing the surface deformation through knowledge of its initial position and also information on the movement of control points, which must be part of the surface domain. Thus, in the context of restoring sections in the Recon System, the surface mapping is obtained with the deformation of this surface according to parameters from the sections: the *LMModels*. In other words, surface mapping uses the points of *LMModels* as control points to achieve equivalent behavior in the three-dimensional environment. Each point of the *LMModels* at the end of the restoration of the sections, has a history of positions in each *StepMS* of the process.

4.3 Surface mesh regeneration

As one of the requirements for surface deformation is that the control points (*LMModels*) are in its domain, a surface remesh must occur. At the end of the restoration of the sections, the surface mesh is remeshed to include the points of the *LMModels* in its domain. The *remesh* is performed with the aid of the surface mesh regeneration algorithm considering curvatures proposed by Miranda et al. [7].

4.4 Definition of source and destination borders

Section restoration has several techniques to arrive at a restored geological layer. When there is a fault reject in an extensional tectonics section, for example, the restoration is with the hanging wall block moving over the fault line until it meets the top of footwall block.

On surfaces, during deformation, it is necessary to define movement limits according to the fault reject being restored in that particular *StepMS*. These boundaries are stretches of surface edge around the fault region. One of the edges is marked as *source* and is the one that will move (analogous to the hanging wall block) until it meets the edge of *destination* which must remain immobile (similarly to the footwall block).

4.5 Movable and non-movable points

It is necessary to define a parameter regarding the freedom of movement for each of the surface points for each *stepMS*. The null displacement of the points of the *LMModels* between two steps indicates that such points are non-movable, otherwise they are marked as movable. Points on the source edge are marked as move and those on the destination edge as non-move as shown in the previous subsection.

All these points are part of the surface mesh. The source and destination edge points and the *LMModels* points that were added after the *remesh*. These are the ones that receive the move and non-move tag as explained above. For the remaining points, the marking is done based on an algorithm of *breadth-first search* (BFS) algorithm. Where a search is performed from all nodes of the first level, then it goes to the second level which are all neighboring nodes of the first level and so on. Always visiting all nodes of the current level before incrementing the position in the data depth. With this, the surface points receive the indication whether or not they should move to that particular stage of the mapping.

To synthesize the surface mapping process as proposed in this work, the following flowchart (Figure 3) presents the order of steps for this activity:

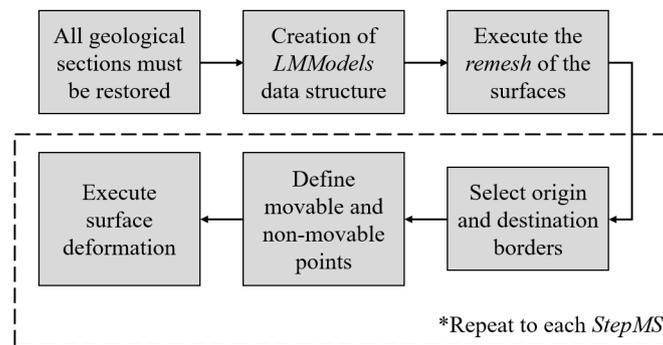


Figure 3. Surface mapping workflow.

4.6 Example and results

This example uses an adapted geological model provided by the Leopoldo Américo Miguez de Mello Research, Development and Innovation Center (Cenpes/PETROBRAS) [8], with two horizons and seven faults with their respective surfaces. In the model, 16 sections were created and restored and a total of 14 *StepsMS*. Figure 4 shows the geological model in a three-dimensional top view (Figure 4-a) with the surface of the most recent horizon, the surface of the oldest horizon in addition to the identification of faults (Figure 4-b) and a view of the geological cross-sections (Figure 4-c).

Following the flow shown in Figure 3, after restoring all sections of the model, the data structure containing the *LMModels* is created and then the surfaces are remeshed. The mapping of surfaces to the steps related to the top layer can be followed in Figure 5.

5 Volume mapping

The volume mapping is intended to provide a way to follow up points arranged in the three-dimensional domain of the geological model during the restoration process. This monitoring is based on the movement of the section points (and also the surfaces in case the surface mapping has been performed). With this, it is possible to obtain more information about what happens outside the geological sections and surfaces domain.

The geological volume in this mapping is represented by a cloud of points arranged in the three-dimensional domain of the geological model. In order to move these points, it is necessary to formulate a solution that can be based on the conservation of mass and momentum conservation equations. The solution to these equations is developed for a medium of discrete points and these points will have a connectivity based on a volumetric grid [5]. The sections and surfaces will also be represented by points, including the displacement of these points that will guide the movement of the points of the volumetric grid.

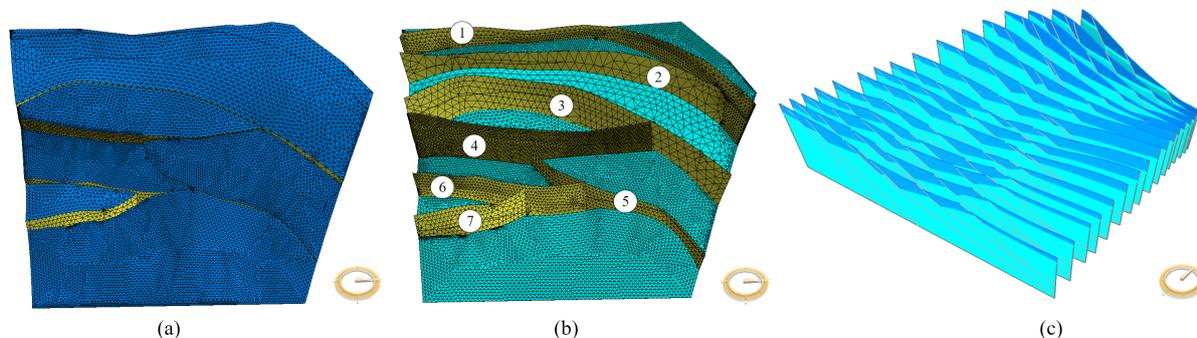


Figure 4. Geological model provided by Cenpes/PETROBRAS. (a) View of horizon top surface, (b) view of horizon bottom surface and faults surfaces with its identification numbers and (c) geological cross-sections of the model.

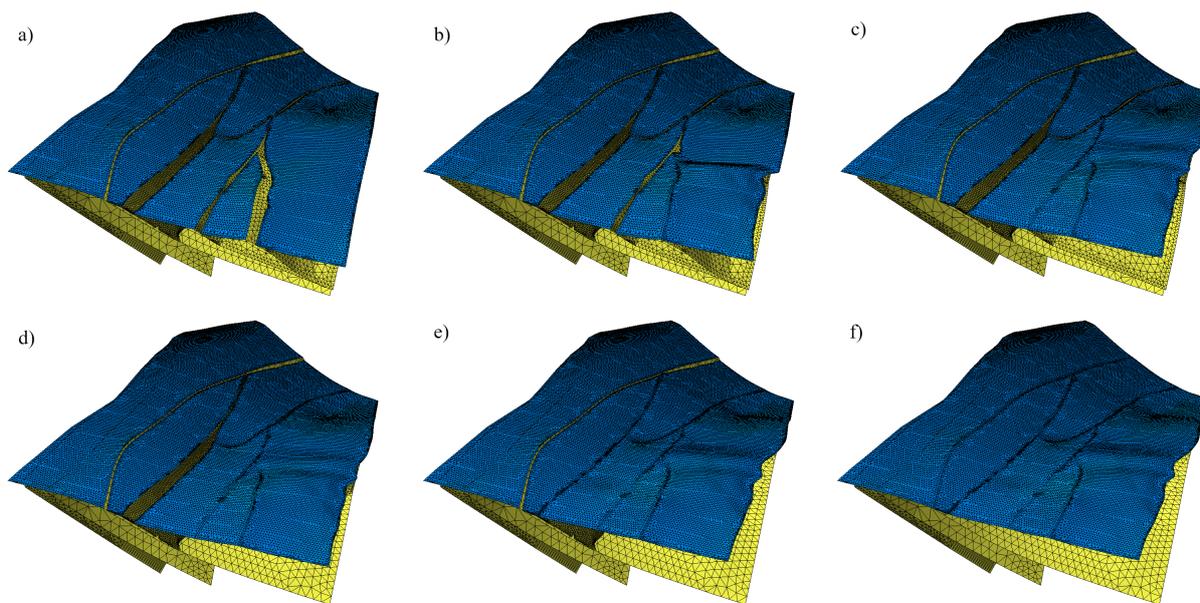


Figure 5. Top surface mapping in the model provided by Cenpes/PETROBRAS.(a) original model, (b) fault 7, (c) fault 6, (d) fault 5, (e) fault 4 and (f) fault 3.

The point cloud creation step takes only the geological surfaces in the initial stage of the model. Points are given a special attribute to indicate which layer they belong to. This information is important for the decompaction *StepMS* where the top layer needs to be removed, in this case the cloud points that correspond to this layer are removed to maintain consistency with the behavior of the sections. The workflow for performing the volume mapping can be seen in Figure 6.

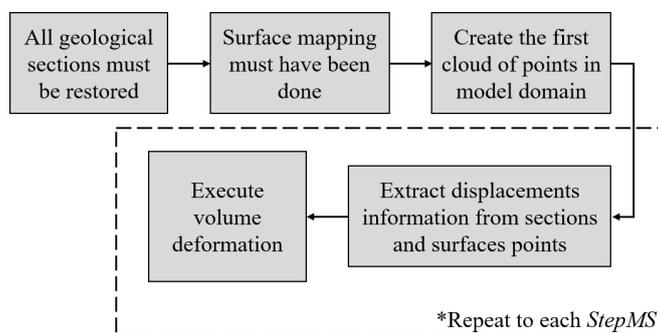


Figure 6. Workflow of volume mapping.

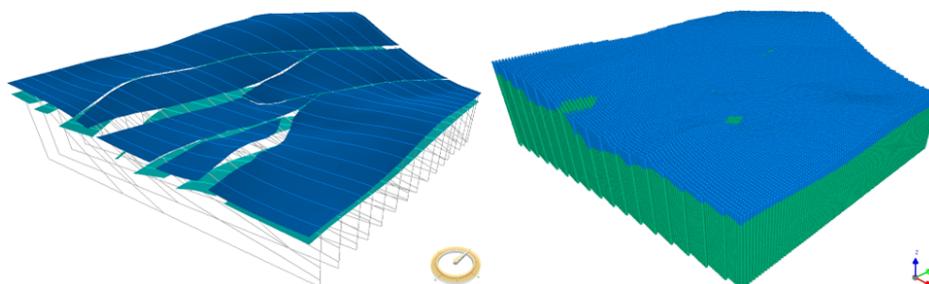


Figure 7. Cloud of points for model provided by Cenpes/PETROBRAS.

5.1 Example and results

The volume mapping as proposed in this work was applied to the same model used in surface mapping. From the workflow of Figure 6, with the sections restored and the surfaces mapped, we start to create the point cloud.

This is shown in Figure 7. In Figure 8 the movement in the region of fault 7 is shown in detail. The restoration of fault 7 is the first step MS of this model.

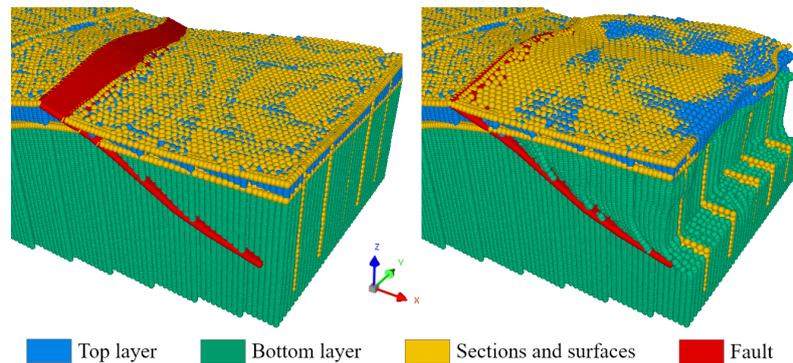


Figure 8. Volume mapping related to restoration of fault 7 on model provided by Cenpes/PETROBRAS

6 Conclusions

This work carried out a search and treatment of information from the restoration of sections (currently more understood) and applied them to deform surfaces and move a cloud of points. This process was called mapping because it is a survey and data processing with the objective of characterizing a geological entity, as is done in geological mapping. In this case, the objective was to obtain a pattern of the behavior of geological surfaces and volume using the results of section restoration.

The surface mapping is able to simulate surface restoration, with a result as plausible as restoring geological sections at a relatively low cost. This cost is even clearer compared to a surface restoration itself. Volume mapping, in turn, brings together the results of restoring sections and mapping surfaces. This makes it possible to move a point cloud in a manner consistent with the tectonic transport presented by those methodologies. Another advantage is that it is not necessary to collect information as in surface mapping. The process is practically automatic.

What was presented in this work was an alternative proposal whose purpose was based on 2D restoration to obtain a three-dimensional information mapping at relatively low computational cost.

Acknowledgements. The authors wish to express their gratitude to the Tecgraf Institute for Technical-Scientific Software Development at PUC-Rio, in particular to team of the Digital Modeling in Geosciences and PETROBRAS.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

- [1] P. Durand-Riard, G. Caumon, and P. Muron. Balanced restoration of geological volumes with relaxed meshing constraints. *Computers & Geosciences*, vol. 36, n. 4, pp. 441–452, 2010.
- [2] Instituto Tecgraf. Manual do usuário - Sistema Recon MS. <https://web.tecgraf.puc-rio.br/~recon/>. Acessado em 25/06/2021, 2020.
- [3] M. Berg, O. Cheong, M. Kreveld, and M. Overmars. *Computational Geometry: Algorithms and Applications*. Springer, 3rd ed. edition, 2008.
- [4] H.Fossen. *Geologia Estrutural*. Oficina de Textos, São Paulo, 2a ed. edition, 2017.
- [5] A. Muller. MGeo Deformer. *Internal documentation TECGRAF*, vol. 1, 2020.
- [6] M. Botsch, L. Kobbelt, M. Pauly, and B. Lévy. *Polygon mesh processing*. A K Peters, Ltd, Natick, 1st edition, 2010.
- [7] A. C. O. Miranda, L. F. Martha, P. A. Wawrzynek, and A. R. Ingrassia. Surface mesh regeneration considering curvatures. *Engineering with Computers*, vol. 25, pp. 207, 2008.
- [8] PETROBRAS. Tecnologia e Inovação. <https://comunidadecientifica.petrobras.com.br/>. Acessado em 02/07/2021, 2020.