HYBRID MASS DAMPER COUPLED TO A WIND TURBINE TOWER: A PARAMETRIC STUDY

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Abstract. In this work, it is studied the application of structural control in the protection of wind turbines subject to external loads, such as wind and earthquake, which can compromise the safety and integrity of the structure due to excessive vibrations. Structural control can be classified as passive, active, hybrid, or semi-active control. The structural control device used is the hybrid mass damper (HMD), which is the combination of a tuned mass damper (TMD) with an active controller. The aim of this research is to analyze, numerically, the behavior and efficiency of the HMD using the Instantaneous Optimal Control (IOC) control algorithm to calculate the control force, and present a parametric study that looks for the most appropriate weighting matrices to ensure the robustness of the control system by minimizing the permanent response of the main system. In this study, the performance of the Instantaneous Optimal Control (IOC) to control the dynamic seismic and wind response in the tower of a wind turbine with a hybrid mass damper will be performed through a parametric study according to the variation of the coefficients of the weighting matrices. The results of the IOC certify the efficiency of the controller through numerical simulations carried out using the computational package MAPLESOFT, MATLAB, and its Simulink control toolbox.

Keywords: Hybrid Mass Damper; Wind Turbine; Seismic; Instantaneous Optimal Control; Weighting Matrices; Parametric Matrices.

1 Introduction

Tall and slender structures become increasingly familiar in everyday life through the sheer volume of building designs and constructions. With wind turbines, the result is no different, the expansive growth of energy generation from renewable sources and the need for ever-larger turbines makes these systems susceptible to excessive vibrations caused by dynamic loads such as wind and sea waves or even by earthquakes.

It is possible to work with prevention to avoid structural collapse, where structural control presents itself as a solution used to reduce excessive vibration levels in these types of systems, seeking stability and safety. Structural control can be classified as passive, active, semi-active or even hybrid. This technology is based on the implementation of external devices, as well as on the actions of external forces, aiming to reduce the levels of vibrations suffered by the structures, promoting variations in the stiffness and damping properties of the main structure [1]. Hybrid control combines the properties of passive and active controls to complement and improve the performance of the passive control system and decrease the need for energy consumption of the active control system, using actuators and power sinks [2].

Hybrid Mass Damper (HMD) performance can be calculated through some control methods. The algorithm, proposed by Yang *et* al (1987) of the Instantaneous Optimal Control (IOC) demonstrates the performance through the performance of the response control and the control force used to stabilize the system in a safe way.

The instantaneous optimal control, as well as the classic optimal one, needs a good determination of the weighting matrices and directly influence the algorithm efficiency. The parametric study of weighting matrices seeks to bring the best performance in terms of efficiency and seeks to compare and analyze the matrices and their respective responses for the control of a large turbine using the wind turbine model reduced to a single degree of freedom [3, 4, 5].

2 Mathematical Formulation

The large wind turbine analyzed in this work is composed of a tall, slender and flexible tower that supports the blades and nacelle at the top. The system has infinite degrees of freedom, however, through a modal reduction technique, it is possible to model it as a discrete system with multiple degrees of freedom (MDOF) [6]. The effects of rotation of the blades and their vibrations in the *flapwise* and *edgewise* directions are not considered in this preliminary model [7]. This structure can be modeled as a system having N degrees of freedom with a TMD device installed and subject to external dynamic excitation. Figure 1 represents the turbine and the reduced modeled system in the form of a cantilever beam with mass at the tip.

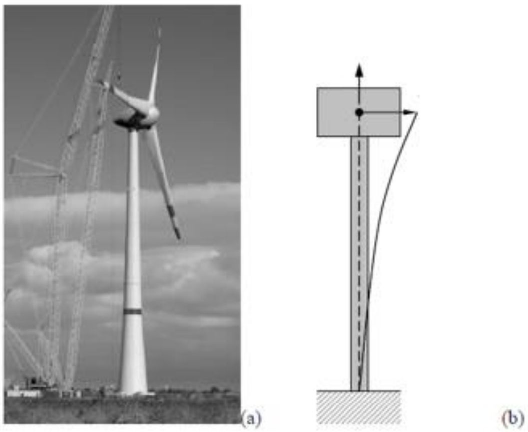

Figure 1. (a) Tubular structures of a wind turbine steel tower; (b) Schematic description of a cantilever beam with end mass [7].

In systems with several degrees of freedom it is possible to reduce the structural response to a single degree of freedom and obtain the structural response for structural systems that vibrate predominantly in a single mode, usually the first [1, 5, 6].

Although the structural model of the analyzed wind turbine has multiple degrees of freedom, the system under study was reduced to a single degree of freedom and a hybrid tuned mass damper was added at the top of the turbine, also as a one degree of freedom model. freedom, which is connected to the main system with the intention of decreasing the vibration amplitude of the structure [1, 7].

The main system represented by a wind turbine reduced to a single degree of freedom associated with a hybrid tuned mass damper with the actuator indicates that the analyzed model, represented by Figure 2, has two degrees of freedom, where $\boldsymbol{M_1}$ is the mass, $\boldsymbol{K_1}$ is the stiffness and $\boldsymbol{C_1}$ is the damping of the main system, while $\boldsymbol{M_2}$ is the upper mass of the HMD, $\boldsymbol{K_2}$ is the stiffness of the HMD and $\boldsymbol{C_2}$ is the damping of the HMD, $\boldsymbol{F}(t)$) is the dynamic loading applied to the structure, $\boldsymbol{u}(t)$ is the control force, while $\boldsymbol{y}(t)$ and $\boldsymbol{z}(t)$ are the horizontal displacements of the main system and the HMD, respectively.

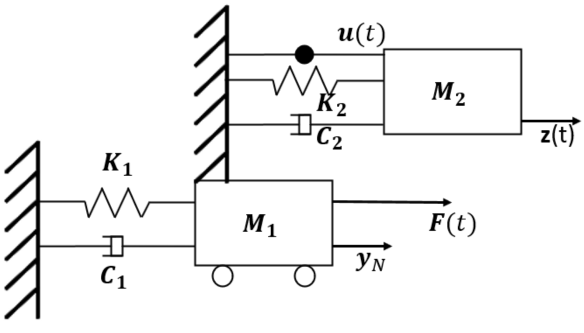

Figure 2. Two degrees of freedom model: main system and tuned mass damper with $\boldsymbol{u}(t)$ actuator (HMD) [8].

The equations of motion of the main system with an HMD connected, including the control force of $\boldsymbol{u}(t)$ have the following form represented by eq. (1) and eq. (2):

$$\boldsymbol{M_1}\ddot{\boldsymbol{y}}(t) + \boldsymbol{C_1}\dot{\boldsymbol{y}}(t) + \boldsymbol{K_1}\boldsymbol{y}(t) = \boldsymbol{F} \tag{1}$$

$$\boldsymbol{M_2}\ddot{\boldsymbol{z}}(t) + \boldsymbol{C_2}\dot{\boldsymbol{z}}(t) + \boldsymbol{K_2}\boldsymbol{z}(t) = -\boldsymbol{M_2}\ddot{\boldsymbol{y}}(t) + g(t) + \boldsymbol{u}(t) \tag{2}$$

For modeling the IOC controller, it is necessary to represent the equations of motion of the system in the form of state space, according to the state equation (3) and the output equation (4) for n degrees of freedom. Where $\boldsymbol{A}$ is the state matrix, $\boldsymbol{B}$ is the input matrix, $\boldsymbol{C}$ is the output matrix and $\boldsymbol{D}$ is the direct transmission matrix. Equations (5) and (6) represent the equations of motion of the system under study in state space.

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{3}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + D\boldsymbol{u}(t) \tag{4}$$

The quadratic performance index, represented by eq. (5), is minimized at each instant of time,

$$\boldsymbol{J}(t) = \boldsymbol{z}^T(t)\boldsymbol{Q}\boldsymbol{z}(t) + \boldsymbol{u}^T(t)\boldsymbol{R}\boldsymbol{u}(t) \tag{5}$$

where $\boldsymbol{T}$ is the modal matrix, whose columns are the eigenvectors of $\boldsymbol{A}$.

$$\boldsymbol{z}(t) = \boldsymbol{T}\boldsymbol{x}(t) \tag{6}$$

From the decoupled state equations, the diagonal matrix with the eigenvalues of $\boldsymbol{A}$ is obtained.

$$\wedge = \boldsymbol{T^{-1}AT} \tag{7}$$

Over a small time interval Δt, the modal state vector $\boldsymbol{x}(t)$ can be expressed in the form

$$d(t-\Delta t) = \exp\,(\wedge\Delta t)\boldsymbol{T}^{-1}\left\{\boldsymbol{z}(t-\Delta t) + \frac{\Delta t}{2}[\boldsymbol{B}\boldsymbol{u}(t-\Delta t) + \boldsymbol{H}\boldsymbol{f}(t-\Delta t)]\right\} \tag{8}$$

The matrix $\boldsymbol{Q}$ is a $2n\ x\ 2n$ real, symmetric and positive semi-definite matrix and $\boldsymbol{R}$ is a real, symmetric and positive definite m x m matrix. The $\boldsymbol{Q}$ and $\boldsymbol{R}$ weighting matrices are also arbitrarily defined for the IOC, they directly influence the control force and the displacement suffered and are represented by equations (9) and (10), respectively.

$$\boldsymbol{u}(t) = -\frac{\Delta t}{2}\boldsymbol{R^{-1}B^T Q z}(t) \tag{9}$$

$$\boldsymbol{z}(t) = \left[\boldsymbol{I} + \frac{\Delta t^2}{4}\boldsymbol{B R^{-1} B^T Q}\right]^{-1}\left[\boldsymbol{T d}(t-\Delta t) + \frac{\Delta t}{2}\boldsymbol{H f}(t)\right] \tag{10}$$

The optimized control force was calculated according to the procedures used by S. Avila and P. Gonçalves [9].

3 Results

In the present work, we consider a wind turbine, previously studied by S. Avila, M. Shzu, M. Morais (2016). The structure was reduced to a simple model with only one degree of freedom and subjected to the following loads: El Centro earthquake [10]; harmonic loading $f(t) = 2500sen(wt)$ applied at the top of the tower. The mass, damping and stiffness properties of the structure are, respectively, $\boldsymbol{M_1} = 34{,}899{,}00$ kg, $\boldsymbol{C_1} = 0{,}00$ Ns/m and $\boldsymbol{K_1} = 463{,}671{,}00$ N/m. The properties of HMD were calculated using the equations of Den Hartog [11]: $\boldsymbol{M_2} = 967{,}98$ kg, $\boldsymbol{C_2} = 427{,}6724$ Ns/m and $\boldsymbol{K_2} = 8{,}9096\text{x}10^3$ N/m The damping ratio, ζ, of the fundamental mode is assumed to be 2% of the critical value.

A detailed parametric optimization study is carried out, in order to determine as $\boldsymbol{Q}$ and $\boldsymbol{R}$ weighting matrices for the instantaneous optimal control algorithm, in the design of hybrid control systems using the HMD. This optimization is applied minimizing the amplitude of the permanent harmonic response of the structure with the lowest possible control force, where the designer has the freedom of choice in the weighting matrices. According to Avila (2002), the first two lines of the $\boldsymbol{Q}$ weighting matrix do not interfere in the calculation of the control force and the values of $\boldsymbol{Q}$ and $\boldsymbol{R}$ are not independent, that is, the relationship between the q_{ij}/R ratio offers greater efficiency to the HMD and that the values should be fixed at $q_{41}/R = 10^3$ for best system stability. This screening is essential to obtain the optimal weighting matrix and to avoid unnecessary analysis. Table 1 shows the variations of the weighting matrices analyzed for later comparison of responses.

Table 1. Values used for the $\boldsymbol{Q}$ and $\boldsymbol{R}$ weighting matrices.

N°	$\boldsymbol{Q}$	$\boldsymbol{R}$
Case 1 ONLY WIND	$\boldsymbol{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 6.654 * 10^3 & 0 & 0 & 0 \\ 1 * 10^{-3} & 0 & 0 & 0 \end{bmatrix}$	$\boldsymbol{R} = [1 * 10^6]$
Case 2 WIND + EARTHQUAKE	$\boldsymbol{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 6.654 * 10^3 & 0 & 0 & 0 \\ 1 * 10^{-3} & 0 & 0 & 0 \end{bmatrix}$	$\boldsymbol{R} = [1 * 10^6]$
Case 3 WIND + EARTHQUAKE	$\boldsymbol{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 6.654 * 10^2 & 0 & 0 & 0 \\ 1 * 10^{-4} & 0 & 0 & 0 \end{bmatrix}$	$\boldsymbol{R} = [1 * 10^5]$
Case 4 WIND + EARTHQUAKE	$\boldsymbol{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 6.654 * 10^1 & 0 & 0 & 0 \\ 1 * 10^{-5} & 0 & 0 & 0 \end{bmatrix}$	$\boldsymbol{R} = [1 * 10^4]$
Case 5 WIND + EARTHQUAKE	$\boldsymbol{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 6.654 * 10^4 & 0 & 0 & 0 \\ 1 * 10^{-2} & 0 & 0 & 0 \end{bmatrix}$	$\boldsymbol{R} = [1 * 10^7]$

The calculations of controlled responses and control forces were performed using MATLAB software, where it was possible to obtain the response in a time interval of 600 seconds. The figures below show the evolution over time of the displacement of the main structure controlled by an HMD through the IOC and also show their respective control forces referring to the weighting matrices used in the Cases in the previous table. In all analyzes the wind turbine blades were not considered.

The control force and displacement responses in Case 1 are represented in Figure 3, where the action of the seismic force was not considered and was calculated only with the excitation of the sinusoidal wind force. The Case 2 is already represented by Figure 4 and presents the earthquake forcing added to the wind. From Case 2, in all analyses, the seismic forcing is considered.

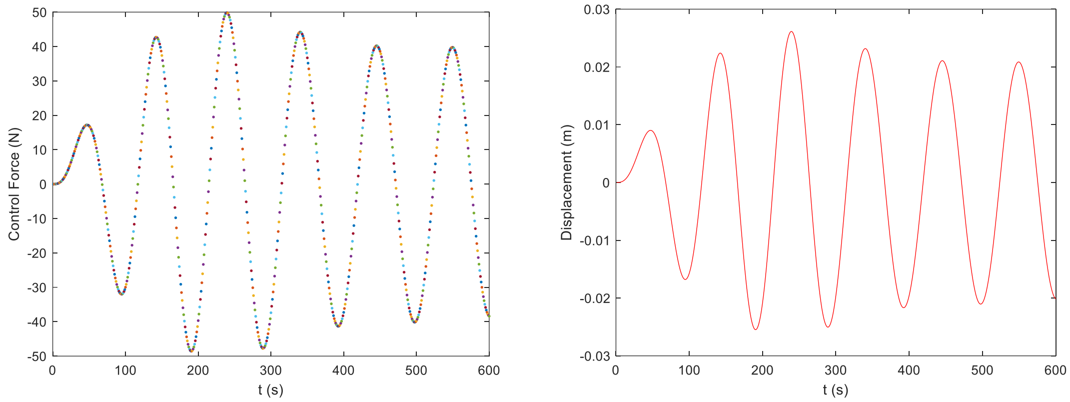

Figure 3. Time evolution of the control force required and time history displacement of the structure controlled controller to wind excitation.

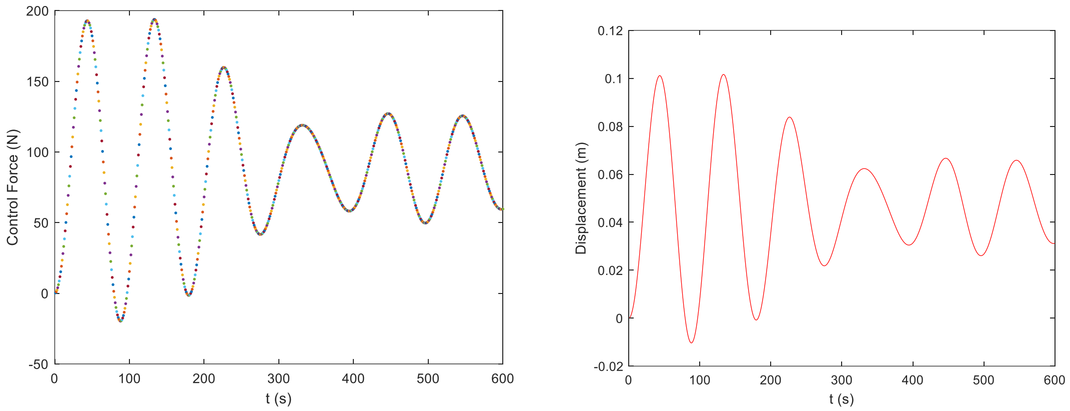

Figure 4. Time evolution of the control force required and time history displacement of the structure controlled by an HMD for wind and seismic excitation.

According to Figures 3 and 4, it is possible to observe that when added to the seismic excitation, the displacement and forcing of the system increase considerably. Table 2 presents the root mean square (*rms*) values for the control forces and displacements and their respective maximum points of the graphs for each weight matrix. From Case 2 to Case 5 the wind and seismic force are considered.

Table 2. Parameterization answers

Cases	**Force rms (N)**	**Displacement rms (m)**	**Maximum Force (N)**	**Maximum Displacement (m)**
Case 1	28.57808959366	0.0149887435950570	49.67458715419	0.026053513745351
Case 2	101.713634974600	0.0533471486874094	193.760928068456	0.101624458137393
Case 3	1.0214415794	0.0535729509885559	1.9717615796	0.103415690717700
Case 4	0.1034156907	0.0535761520110401	0.0197210593	0.103433751859614
Case 5	9513.267926001560	0.0498955443563000	15585.64800859540000	0.081744191121651

According to the results, the $\boldsymbol{Q}$ weighting matrix influences the System state while the $\boldsymbol{R}$ matrix influences the control strength. For $q_{41}/R = 10^3$ fixed, the displacement has negligible variation, while the control force can be optimized, as in Case 4.

It is verified, therefore, that the maximum displacement of the system and the maximum force that presents the best performance, considering the rms, have an optimal effectiveness for the parameterization of Case 4.

4 Conclusions

In this work, the use of a hybrid mass damper is studied for the analysis of the dynamic response and necessary control force of a wind tower model. Analyzes are performed using the wind turbine parameters presented by S. Avila, M. Shzu, M. Morais *et* al. [7]. The results are obtained through numerical analyzes under wind and earthquake excitations by the Matlab software. Using the Intantaneous Optimal Control as a control algorithm to carry out a parametric study of the weighting matrices.

Although the IOC controller has a response dependent on the weighting matrices and, therefore, the response may vary according to the designer's choice, optimal parameterization techniques indicated by Avila (2002) are used, which indicate that only the indices q_{31} and q_{41} matrix $\boldsymbol{Q}$ influence the answer and that by setting the value of $q_{41}/R = 10^3$ it is possible to obtain an optimal answer.

It is verified, therefore, that the system considering the seismic and wind forcing is possible, through the optimal control algorithm parameterizing the weighting matrices, to obtain an optimal response of a displacement of approximately 0.10 meters using only 0.02 Newtons. Although its efficiency is quite sensitive to the choice of suitable weighting matrices, its performance does not require additional computational effort and has the advantage of not relying on the solution of Riccati's problem, which can be computationally costly in the case of one-order problems. bigger.

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