# OPTIMAL DESIGN OF VISCOELASTIC LINKS CONSIDERING TEMPERATURE INFLUENCE IN VIBRATION CONTROL

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***Abstract.*** *Viscoelastic links can be characterized by the connection of a structure to the ground by a device with a layer of viscoelastic material. When the structure vibrates, it provokes a relative displacement between the ground and itself, leading to a deformation on the viscoelastic material. Due to high damping, viscoelastic materials are used in devices for vibration control, working through dissipation of vibratory energy, and introducing stiffness. Mechanical properties of viscoelastic materials vary, mainly, according with temperature and vibration frequency. Changes in environmental conditions, like temperature, can lead to a non-optimal behavior of devices designed with viscoelastic materials. The GVIBS group, which the authors are part of, has been developing over decades a methodology for optimal design of passive vibration control devices, including some that use viscoelastic materials. The goal of this paper is to expand the group methodology to design viscoelastic links, as well as model, into a graphical interface, the effects of temperature changes on the behavior of viscoelastic links. A Fortran code is developed to optimize the physical parameters and location of viscoelastic links, while considering the effects of temperature changes. Experiments are done on a steel plate, with and without viscoelastic link, to validate the developed methodology.*

**Keywords:** viscoelastic links, passive vibration control, viscoelastic materials.

## 1 Introduction

Being dynamic in nature, vibration may be a common occurrence for mechanical systems. In some cases, vibration can lead to reduced lifespan, compromised safety, and discomfort upon use of mechanical components. When caused by mechanical instability in systems with low damping factor, vibration can be controlled by the addition of links, a type of damper defined by the connection of a structure to the ground by a device with a layer of viscoelastic material.

Viscoelastic materials have both viscous and elastic properties, and can be an interesting option to introduce damping on a system, as they are cheap and can be cut into a variety of shapes and sizes. In comparison to hydraulic dampers, a more commonly used type of damper, viscoelastic links can use less space and be of easier placement. However, properties of viscoelastic materials vary, between other factors, with temperature, and shifts in room temperature can lead to non-optimal behavior. Thus, temperature effects have to be taken into consideration during design.

The GVIBS group, which the authors are part of, has been developing over the past decades a methodology for optimal design of dynamical neutralizers, some of which employ viscoelastic materials. This methodology has been implemented in a proprietary software called LAVIBS_ND.

This work expands the GVIBS group methodology to viscoelastic links, while also updating models to allow temperature influence to be considered during design. The software LAVIBS_ND is updated and new graphical visualization tools are introduced to manage temperature changes.

# 2 Mathematical model

## 2.1 Viscoelastic material

The properties viscoelastic materials vary, mainly, in respect to frequency and temperature. Classical mathematical models for viscoelastic materials use an association of purely elastic springs and purely viscous dampers in parallel (Kelvin-Voigt) or in series (Maxwell). Some models, as the generalized Maxwell model, utilize as many springs and dampers as necessary to properly model the viscoelastic material behavior.

By introducing the concept of fractional derivative, it is possible to achieve a precise model without the need of considering as many springs and dampers (Ciniello, Bavastri and Pereira [1]). Using the four parameters fractional derivative Zener model, the complex shear modulus is given by

$$G_c(\Omega_{red}) = \frac{G_0 + G_\infty b_1 (i\Omega_{red})^\beta}{1 + b_1 (i\Omega_{red})^\beta}, \tag{1}$$

where $G_0$ is the asymptotic value for very low frequencies, and $G_\infty$ the asymptotic value for very high frequencies. $b_1$ is a complementary parameter, and $\beta$ represents the fractional derivative value, being lower than 1. $\Omega_{red}$ is called reduced frequency, that can be defined as:

$$\Omega_{red} = \alpha_T(T)\Omega. \tag{2}$$

In the equation, $\Omega$ represents frequency, and $\alpha_T(T)$ is the shift factor, which will be set by the William-Landel-Ferry equation:

$$log_{10}\, \alpha_T(T) = \frac{-\theta_1 (T - T_0)}{\theta_2 + T - T_0}, \tag{3}$$

with $T_0$ being a reference temperature and T the temperature at which the viscoelastic material operates in, both in Kelvin. $\theta_1$ and $\theta_2$ are additional parameters that vary according to the material.

The viscoelastic material dimensions also influence its behavior, as described by Nashif, Jones, and Henderson [2], in the equation

$$K_c(\Omega_{red}) = \vartheta G_c(\Omega_{red}) \ or\ K_c(\Omega_{red}) = \vartheta E_c(\Omega_{red}), \tag{4}$$

where $\vartheta$ is a geometric factor.

For pure shear, $\vartheta$ is set as the ratio of the sheared area and the length of the material. For pure compression, $\vartheta$ is three times the ratio between the compressed and the material height.

## 2.2 System with multiple degrees of freedom

The vibrating system, or primary system, to be controlled in this paper is modelled as a multiple (n) degree of freedom system, with time invariant parameters, written as

$$M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = f(t). \tag{5}$$

M, C, and K represent the mass, damping, and stiffness matrix, respectively, with size n x n. f(t) is a force exciting the primary system, and q(t) the generalized coordinates of the primary system.

In order to simplify the resolution of eq. (5) a coordinate transformation to modal space can be used. This transformation can be done through use of the eigenvalues and eigenvectors, derived from

$$K\phi_j = \lambda_j M \phi_j. \tag{6}$$

In eq. (6), j varies from 1 to n degrees of freedom, with $\phi_j$ representing the jth eigenvector, and $\lambda_j$ the jth eigenvalue associated with the jth eigenvector. The proportional damping model, which assumes the damping matrix as a linear combination of the stiffness and mass matrixes, is considered. For this model,

$$\lambda_j = \Omega_j^2, \tag{7}$$

with $\Omega_j$ being the jth natural frequency.

Not all eigenvectors are needed to correctly represent the system. Therefore, only $\hat{n}$ eigenvectors will be considered, with $\hat{n} \ll n$. The eigenvectors are normalized by the modal mass matrix. The orthonormalized eigenvectors are defined by $\psi_j$. The orthonormalized modal matrix ($\Psi$) and the spectral matrix ($\Lambda$) are set as

$$\Psi = [\psi_1, \psi_2, \dots, \psi_{\hat{n}}] \; and \; \Lambda = diag(\Omega_j^2), \tag{8}$$

and are n x n in size.

A Fourier transform is applied on eq. (5), changing the equation from time domain to frequency domain, therefore resulting in:

$$[-\Omega^2 M + i\Omega C + K]Q(\Omega) = F(\Omega). \tag{9}$$

Equation (9) is then transformed to the modal space by pre-multiplying both sides by $\Psi^T$ and establishing

$$Q(\Omega) = \Psi P(\Omega) \; and \; N(\Omega) = \Psi^T F(\Omega). \tag{10}$$

Thus, eq. 9 becomes

$$\left[-\Omega^2 I + i\Omega diag(2\xi_r \Omega_r) + diag\left({\Omega_r}^2\right)\right] P(\Omega) = N(\Omega), \tag{11}$$

where r goes from 1 to n̂, and $\xi_r$ the damping factor related to the rth mode, which can be determined experimentally.

Receptance ($\alpha(\Omega)$) is a frequency response function (FRF) that establishes a relationship between the system response displacement and the excitation force. Mathematically, it can be defined as

$$\alpha(\Omega) = \Psi\left[-\Omega^2 I + i\Omega diag(2\xi_r \Omega_r) + diag\left({\Omega_r}^2\right)\right]^{-1} \Psi^T. \tag{12}$$

For an excitation force applied in the position k and being measure on position s, eq. (12) becomes

$$\alpha_{ks}(\Omega) = \sum_{r=1}^{\hat{n}} \frac{\Psi_{kr}\Psi_{sr}}{-\Omega^2 + i2\xi_r \Omega_r \Omega + {\Omega_r}^2}. \tag{13}$$

Similarly, other FRFs can be defined, as inertance, that relates excitation force to system response acceleration.

### 2.3 Adding dampers to the system

Similar to what has been done by Bavastri [3] with neutralizers, a viscoelastic link can be modeled by a stiffness element that varies with frequency, as shown in eq. (4). Hence, if p links are added into the system, eq. (9) becomes

$$[-\Omega^2 M + i\Omega C + K + K_d(\Omega)]Q(\Omega) = F(\Omega). \tag{14}$$

with

$$K_d(\Omega) = \begin{bmatrix} 0 & & & & 0 \\ & k_{c1}(\Omega) & & & \\ & & \ddots & & \\ & & & k_{cp}(\Omega) & \\ 0 & & & & 0 \end{bmatrix}. \tag{15}$$

The links stiffness matrix $K_d(\Omega)$ can be cast to the modal space and, with this model, eq. (12) turns into

$$\alpha(\Omega) = \Psi\left[-\Omega^2 I + i\Omega diag(2\xi_r \Omega_r) + diag\left({\Omega_r}^2\right) - K_a(\Omega)\right]^{-1} \Psi^T, where \; K_a = \Psi^T K_d(\Omega) \Psi \tag{16}$$

### 2.4 Link optimization

The goal of the optimization is to find the ideal link dimensions and placement position within the structure to reduce vibration within a certain bandwidth. For that purpose, the objective function is defined as

$$f_{obj}(x) = \left\| \max_{\Omega_{inf} < \Omega < \Omega_{sup}} \hat{P}(\Omega, x) \right\|_2, \tag{17}$$

respecting

$$\Omega_{inf} < \Omega < \Omega_{sup} \; and \; x_{inf} < x < x_{sup}. \tag{18}$$

$\Omega_{inf}$, $\Omega_{sup}$, $x_{inf}$, $x_{sup}$ are the lowers and upper boundaries for the frequency and design vector. The design vector is composed of the modal positions where the links will be inserted as well as the geometric factor for each link. Genetic algorithm is used as the optimization method.

The optimization is done by the proprietary software LAVIBS_ND. The software implements a Fortran code, to run the optimization method, and a Java application, which displays the results in a graphical way.

# 3 Methodology and Results

In this paper, vibration control is done on a steel plate with dimensions 543mm x 351mm 10.5mm, as shown in Fig. Figure 1

Figure 1. Steel plate used on experiments

The goal of the experiments is to validate if the FRFs match the ones predicted by the software LAVIBS_ND after optimization. The measurement was done by exciting the steel plate with an impact hammer, model 086C04, and the response was measure with a piezoelectric accelerometer 352C68, both manufactured by PCB PIEZOTRONICS. The data is gathered by a Photon 2 analyzer, made by LDS DRACTON, and processed by the software RT Pro Photon. The measurements were performed with the plate hanged vertically by a thin nylon line, to minimize altering the system behavior. The range of frequencies measured was from 0 to 657Hz with a discretization of 1024 points.

## 3.1 Structure model

In order to acquire modal parameters, a simulation is performed with software ANSYS, version 17.0, using 504 elements (20 node hexahedral). The steel Young modulus is assumed 200GPa, the Poisson coefficient 0.3 and density 7900 Kg/m³.

The nylon line holding the plate vertically is assumed to have little impact in the system behavior. Thus, for the boundary conditions, all 4 edges of the plate were considered free.

As the modal damping factor can be difficult to model, it was calibrated to match approximately the peaks of the experimental tests. The excitation and measurement points were chosen to avoid vibration nodes within the measured frequency range and are displayed on Fig. Figure 2.

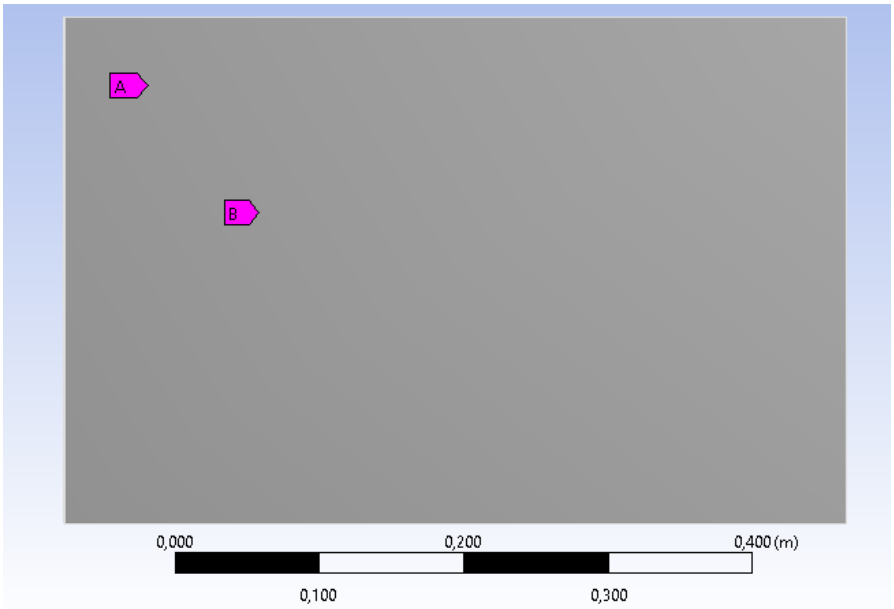


Figure 2. Point of response measurement (A) and point of excitation (B)

The frequency range for vibration control was set between 300Hz and 600Hz, due to the high number of modes within that range. Inertance was simulated within this frequency range and the results compared to the measured values for the steel plate, shown on Fig. Figure 3.

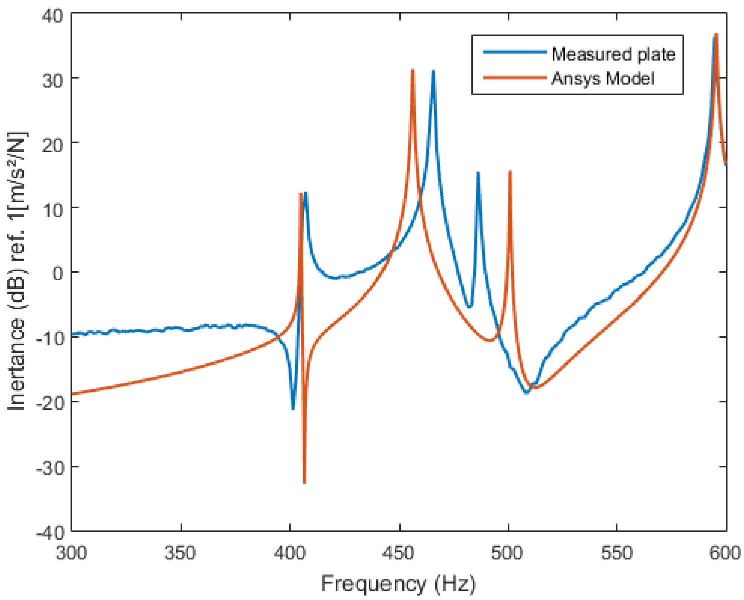


Figure 3. Measured inertance(blue) vs model inertance(orange)

The differences between the model and the measurement are attributed to poor manufacture quality of the plate and uncertainties over the material property. Table 1 displays the natural frequencies from the model and measurement, as well as the relative error.

Table 1. Natural frequencies measured vs modelled

| Mode | Measured natural frequency (Hz) | Model natural frequency (Hz) | Relative error (%) |
|---|---|---|---|
| 3rd | 407.2 | 404.9 | 0.56 |
| 4th | 465.8 | 456.2 | 2.06 |
| 5th | 486.3 | 500.8 | 2.98 |
| 6th | 594.7 | 595.5 | 0.14 |

The relative error is relatively low, below 3% for all modes. For this reason, the modal parameters calibrated will be used on the links optimal design.

### 3.2 Links design

The links geometric factor and position of placement in the primary structure are optimized using the LAVIBS_ND software. For the optimization technique, a genetic algorithm is utilized. The algorithm considers 100 generations, with 100 individuals each, a crossover rate of 50% and 7% mutation rate. The range to be controlled is between 300Hz and 600Hz, with a discretization of 800 points.

The viscoelastic material used for the links was the elastomer BT 806/55, with properties described by Silva [4]. The optimization considered 2 links. The boundaries for the links geometric value are set as 0.1 at minimum and 2 at maximum. Those boundaries are set based on geometric limits for the construction of links. For the position optimization no boundaries were set; the links can be placed on any modal node of the mesh. The optimization was performed for a temperature of 288K.

The optimization results lead to the first link(A) to have a geometric factor of 0.58 and the second link(B) to have a geometric factor of 2. The optimal placement can be seen below, on Fig 4.

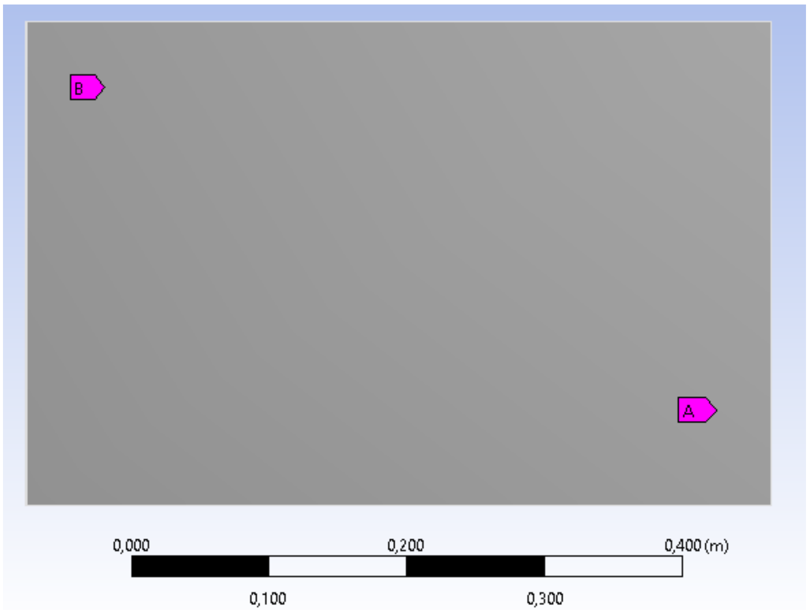


Figure 4. optimized position of the links

Figure 5. displays the optimization results on the inertance curves. The graph also displays two additional curves showing the links behavior under different temperatures, 273K and 303K. These values are chosen to reflect normal seasonal temperature changes.

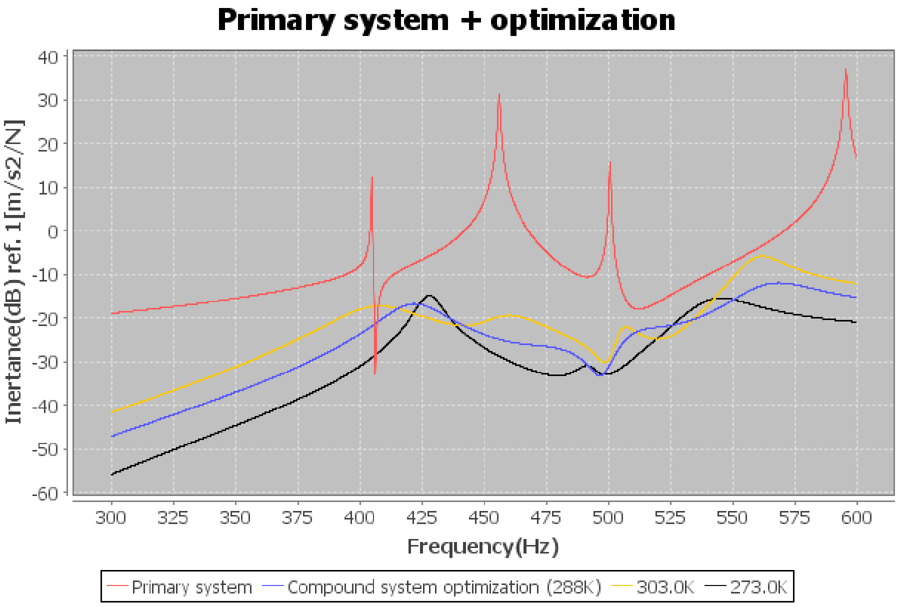


Figure 5. Inertance of the primary system (red) vs composite system on temperatures 288K (blue), 303K (yellow), and 273K (black)

As displayed, there was a significant reduction throughout the entire frequency range. The reduction on the inertance is of at least 20db on all natural frequencies and an overall reduction in the vibration levels is perceived. Temperature changes can cause a slight underperformance or be beneficial depending on the frequency range observed.

# 4 Conclusions

The optimal design of links proposed has shown great theoretical results. Experiments are still going to be conducted to validate the methodology developed, but significant reduction on vibration levels are expected. Temperature variation can theoretically lead to small changes on the effectiveness of control, at around 10db for the case in study.

Further works can apply the methodology developed on this paper to more complex structures and to different types of control devices.

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