Nonlinear Solution of Tuned Liquid Column Dampers Compared to Linearized Analytical Solution

Marcos Wilson Rodrigues de Lima[1], Juliano Ferreira Martins[2], Suzana Moreira Avila[1], Marcus Vinicius Girão de Morais[1]

[1]*Graduate Program in Engineering Materials Integrity, University of Brasília*
Special Industry Area Projection A, East Sector, 72.444-240, Brasília, Distrito Federal, Brazil
engmarcoswilson@gmail.com, mvmorais@unb.br, avilas@unb.br
[2]*Dipart. di Ingegneria Strutturale e Geotecnica, Sapienza Universitá di Roma,*
Piazzale Aldo Moro, 5, 00185, Roma, Lazio, Italy
juliano.martins@hotmail.com

Abstract. This article compares the nonlinear and equivalent linearized solutions of an isolated Tuned Liquid Column Dampers (TLCD), a kind vibration control device. A numerical simulation of the nonlinear behavior of the structure under harmonic excitation was performed to determine the frequency response function (FRF). The stepped sweep-sine procedure was performed to numerical estimation of nonlinear transmissibility FRF. With TLCD's transmissibility FRF due to base acceleration, the equivalent damping coefficient was determined using the linearized solution proposed by Gao et al. [1]. Both non-linear and linearized FRF was compared to verify the precision of Gao's equivalent damping technique. The present implementation were done in Python programming language and its native scientific libraries Numpy, Scypy and Matplotlib. The solution of nonlinear TLCD ODE was calculated with the Odeint tool, a general-purpose integrator developed under the LSODA procedure (Livermore Solver for Ordinary Differential Equations with the automatic switching method for rigid and non-rigid problems).

Keywords: TLCD, numerical simulation, vibration control.

1 Introduction

Relevant current structural systems (high buildings with multiple floors, bridges, wind turbines, among others) are subject to high levels of vibration due to actions, denoted as examples of the capacity and intensity of influence of mechanical vibrations on the integrity of arrangements structural (Karimi *et al.* [2]).

The scenarios described, qualified by thin structures - that is, with high modulus proportional relationships between their dimensions -, according to Martins *et al.* [3], show low natural vibration frequencies and small damping coefficients. These physical characteristics manifest a situation of arguing incapable of receiving external oscillations, under the risk of subsequent structural damage.

Disturbances of external origin to the systems arise from different sources, highlighting mechanical impacts related to human activity and the installation of machinery in the structure's domain. However, the main matrices of oscillations concern actions of natural causes and random frequency, exemplified by seismic movements and wind currents.

Therefore, as a result of the clinical analysis of the scenarios imposed on structural complexes with a low slenderness index, an expressive stream of academic and applicational research related to the control of structure vibrations has been developed, under a scope of guarantee and maintenance of properties durability, safety, economy and visual comfort related to the life of a building (Martins et al. [3]). Thus, according to Soong and Dargush [4], systemic equipment was developed whose primary functionality denotes the prevention of external vibratory requests: vibration absorbers.

As elucidated by Savi and Paula [5], the methodologies applied to the manipulation of a dynamic system can be divided into active control interventions – related to the inclusion of a new type of energy to the original arrangement and passive control, with an inverse definition to first. In this context, the solution formulations and execution of the Tuned Liquid Column Damper (TLCD) stood out, a passive aspect device and the target of several theoretical and experimental studies, mainly related to its use in wind turbines, as evidenced by scientific production by Al-Saif *et al*. [6]; Jaksic *et al*. [7] and Zhang *et al.* [8].

This device has a non-linear operation, a characteristic explained by the loss of head in turbulent conditions of its liquid when disturbed in oscillatory movement Martins *et al*. [2]. The history of literature points to the work of Sakai and Takaeda [9] as the first proposition for the use of TLCD in tall buildings. Later, Yalla and Kareem [10] propose an approximate solution concerning the optimized damping rate equivalent to the TLCD. Furthermore, Alkmin *et al*. [11] optimize linearized parameters of an TLCD subject to an arbitrary wind spectrum. To solve the equivalent damping of the vibration-absorbing system in question, researchers resorted to the application of the statistical linearization method proposed by Roberts and Spanos [12].

Shum [13] presents the approximate resolution for an optimized nonlinear TLCD. The scientific collection of the area points to the research of Kizilay and Cigeroglu [14], whose result was the non-linear resolution of a differential system denoted by the main structure coupled to the TLCD, converting it into a system of non-linear algebraic equations under the possibility of computation by applying Newton's method. Munoz et al. [15] presents a non-linear application of structural analysis submitted to seismic actions by the Harmonic Equilibrium Method. The last two references are important nonlinear solutions in the frequency domain with possibilities for analyzing actions of random color spectra (wind, waves and seismic actions).

This article compares the nonlinear and equivalent linearized solutions of an isolated Tuned Liquid Column Dampers (TLCD), a kind vibration control device. A numerical simulation of the nonlinear behavior of the structure under harmonic excitation was performed to determine the frequency response function (FRF). The stepped sweep-sine procedure was performed to numerical estimation of nonlinear transmissibility FRF. With TLCD's transmissibility FRF due to base acceleration, the equivalent damping coefficient was determined using the linearized solution proposed by Gao et al. [1]. Both non-linear and linearized FRF was compared to verify the precision of Gao's equivalent damping technique. The present implementation was done in Python programming language and its native scientific libraries Numpy, Scypy and Matplotlib. The solution of nonlinear TLCD ODE was calculated with `odeint` package, a general-purpose integrator developed under the LSODA procedure (Livermore Solver for Ordinary Differential Equations with the automatic switching method for rigid and non-rigid problems).

2 Tuned Liquid Colum Damper Coupled to Main Structure

Figure 1 shows the graphical representation of the TLCD structure, with its dimensional characteristics. Derived of Lagrangian approach (Di Matteo *et al.* [16]), the mathematical description of liquid column relative elevation $w(t)$ was done by:

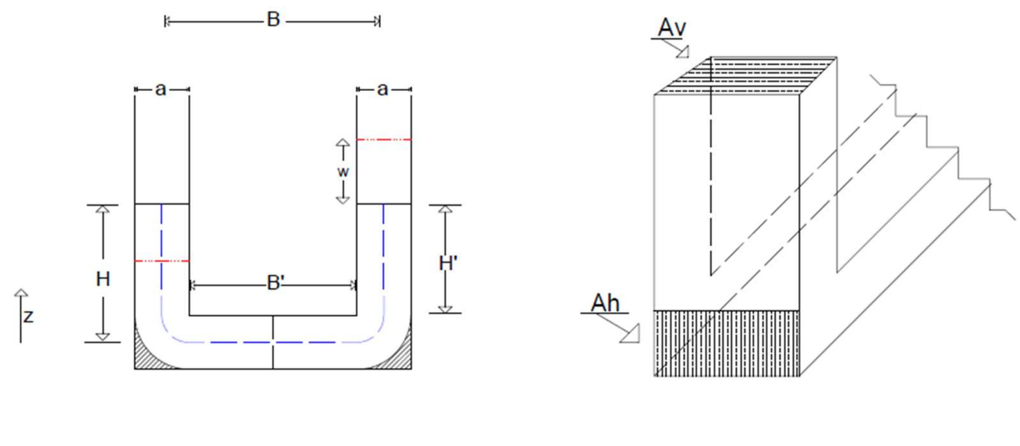


Figure 1. Schematic representation of TLCD geometry.

$$\rho AL\,\ddot{w}(t) + \frac{1}{2}\rho A\varepsilon\ |\dot{w}(t)|\ \dot{w}(t) + 2\,\rho Ag\ w(t) = -\rho AB\,\ddot{u}(t) \tag{1}$$

where ρ is the fluid density, A the fluid cross-sectional area, L the effective length of the fluid column, ε the head loss coefficient, g the gravitational acceleration and $\ddot{u}(t)$ represents the acceleration of base motion. Also, the lateral B and height H of water column, respectively, are indicated in Fig. 1.

As noted by several authors (Di Matteo *et al*. [16], Martins et al. [3], and Shum *et al*. [13]), the TLCD damping is described by a nonlinear damping force represented by $0{,}5\ \rho A\varepsilon\,|\dot{w}(t)|\,\dot{w}(t)$. Thus, Gao *et al* [1] propose a linearization of the nonlinear damping term by harmonic excitation technique . The equivalent damping coefficient is done by:

$$c_{eq} = \frac{\rho A\varepsilon\,\Omega_{exc}}{2}\,\frac{\frac{1}{T}\int_0^T |W_0\cos(\Omega_{\mathrm{exc}}\mathrm{t}+\Phi_0)|\,[W_0\cos(\Omega_{\mathrm{exc}}\mathrm{t}+\Phi_0)]^2dt}{\frac{1}{T}\int_0^T [W_0\cos(\Omega_{\mathrm{exc}}\mathrm{t}+\Phi_0)]^2dt} = \frac{4}{3\pi}\rho A\varepsilon\,\Omega_{\mathrm{exc}}\,W_0 \tag{2}$$

where the liquid column elevation is assumed as a harmonic function, i.e., $w(t) = W_0\cos(\Omega_{exc}t + \Phi_o)$. Then, the terms Ω_{exc} and Φ_0 are, respectively, the excitation frequency and phase difference by report to acceleration of base motion $\ddot{u}(t) = -U_o\Omega_{exc}^2\cos(\Omega_{exc}t)$.

3 Nonlinear FRF Determination by Stepped Sweep-sine Procedure

To describe the spectral behavior of nonlinear TLCD, the transmissibility $W(\jmath\Omega)/U(\jmath\Omega)$ FRF was analyzed using the stepped sweep-sine procedure. This methodology consists of simulating the TLCD fluid column movement $w(t)$ by imposing a same base movement U_o, due to the acceleration base movement $\ddot{u}(t) = -U_o\Omega_{exc}^2\cos(\Omega_{exc}t)$ for a range of excitation frequency. Thus, the water column elevation $w(t)$ harmonic was observed as for a range of excitation frequencies Ω_{exc} of 0 to 3 Hz. Figure 2 present this harmonic behavior of TLCD column elevation subjected to an excitation frequency $\Omega_{exc} = 2{,}705Hz$ with $U_o = 1{,}0m$ of base motion amplitude. Figure 3 compare the temporal evolution nonlinear fluid column elevation with the assumed harmonic motion $w(t) = W_0\cos(\Omega_{exc}t + \Phi_o)$, where W_o is estimated by root mean square, i.e., $\sigma_w^2 = (1/T)\int_0^T [w(t)]^2dt = W_o/\sqrt{2}$.

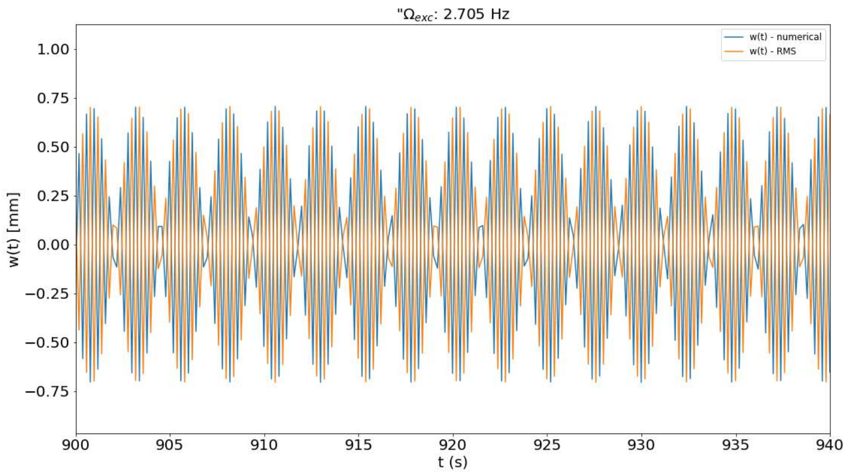


Figure 2. Time evolution of water column elevation $w(t)$ for $\Omega_{exc} = 2{,}705Hz$ and $U_o = 1{,}0m$.

4 Results

Figure 3 show the transmissibility $W(\jmath\Omega)/U(\jmath\Omega)$ FRF of nonlinear water TLCD through Stepped Sweep-Sine Procedure, with a fluid height $H = 50{,}0mm$, base $B = 7{,}75mm$ and an amplitude base displacement $U_o = 1{,}0m$. Figure 4 show the water column elevationW_o as a function of excitation frequency $\Omega_{exc}[Hz]$ to a range between 0,0 to 3,0 Hz.

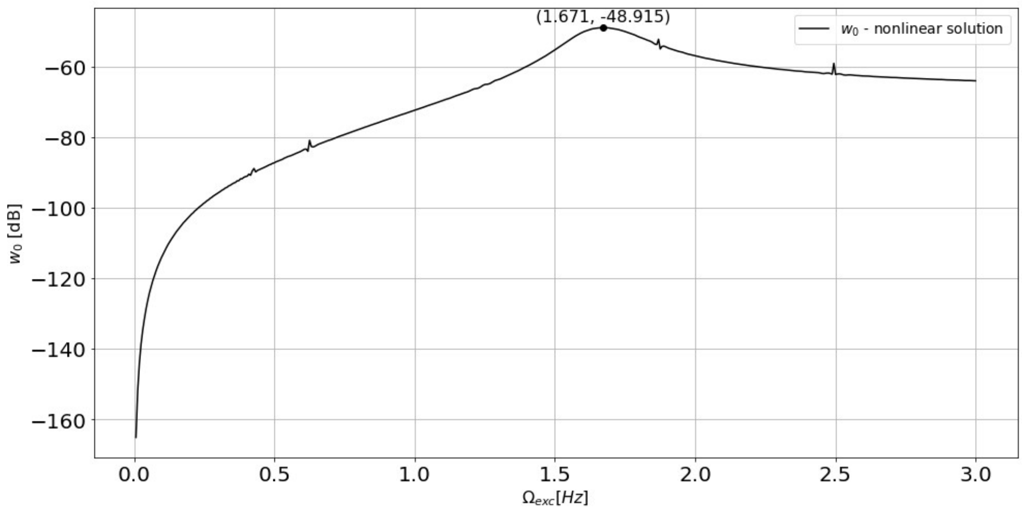


Figure 3. Transmissibility $W(\jmath\Omega)/U(\jmath\Omega)$ FRF of nonlinear TLCD
($H = 50mm$, $B = 7{,}75mm$, $U_o = 1000mm$) by Stepped Sweep-sine

Figure 4 demonstrates the experimental results (Martins et al. [3]), which point to a constant relationship between the pressure loss of the fluid in the TLCD and the height of its liquid column.

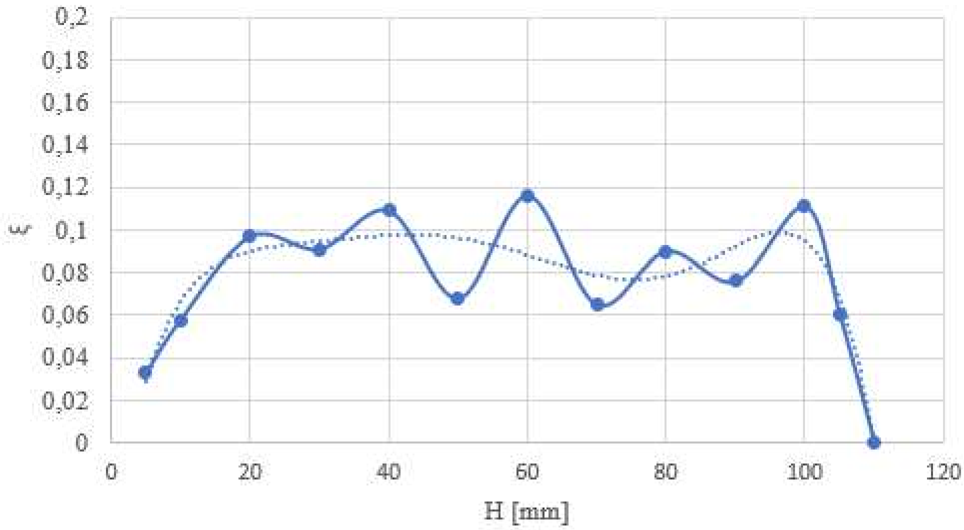
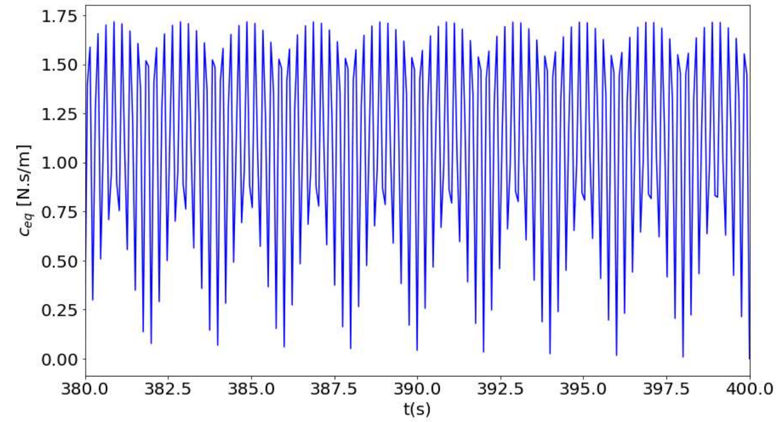
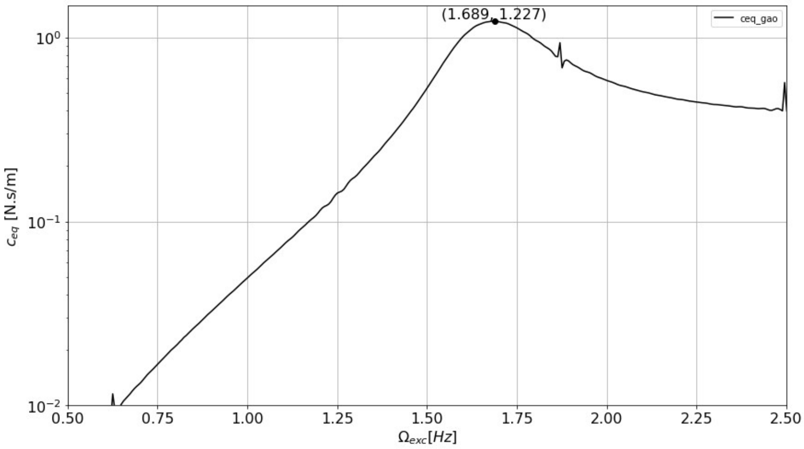


Figure 4. Damping ratio in function of water column elevation

Aware of the data resulting from the non-linear solution of the TLCD, the calculation of the equivalent damping was performed according to the linearized solution proposed by Gao et al. [1]. Figure 5 shows the processing of damping parameters as a function of device, respectively, excitation time and frequency.



(a)



(b)

Figure 5. Equivalent damping as function of (a) time $t(s)$ and (b) excitation frequency $\Omega_{exc}[Hz]$.

Through a joint analysis of the variation of the equivalent damping coefficient as a function of a plane composed of two simultaneous variables - excitation time and frequency -, a damping surface was constructed, shown in fig. 6.

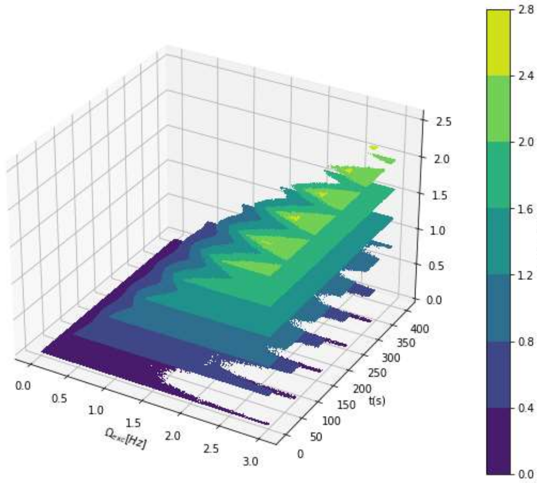


Figure 6. Equivalent damping as function of time $t(s)$ and excitation frequency $\Omega_{exc}[Hz]$.

Figure 7 delimits a comparison between the dynamic behavior of the TLCD under a nonlinear solution and the linearized solution proposed by Gao *et al*. [1], under the conditions of an initial unit displacement and a fluid column height $L = 50mm$.

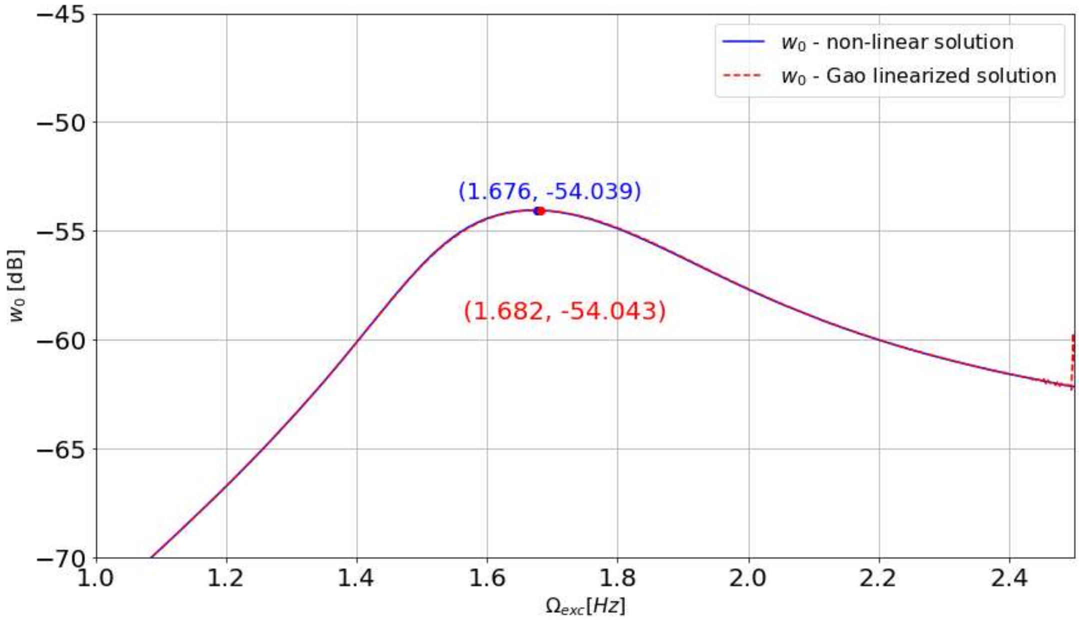


Figure 7. TLCD nonlinear and linearized solution comparation

5 Conclusions & Perspectives

This work compared the nonlinear numerical solution and Gao's equivalent linearized analytical solution of an isolated TLCD. The numerical simulation of nonlinear TLCD was developed by the Stepped Sweep-sine Procedure implemented in Python programming language using Scipy/Odeint.

As pointed out by fig. 7, the Gao's equivalent linearization of TLCD damping show an efficiency of approximately 0,007%. These results must be improved to verify, for several base amplitudes, the concordance of numerical results with experimental results.

Acknowledgements. The authors would like to thank CAPES, the University of Brasília, and the Graduate Program in Engineering Materials Integrity.

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References

[1] H. Gao, K. Kwok e B. Samali, "Optimization of tuned liquid column dampers," *Eng Struct,* pp. 476-486, 1997.

[2] H. R. Karimi, H. Gao, J. Lam e H. Du, "Vibration control systems," *Mechatronics,* pp. 287-288, 2014.

[3] J. F. Martins, M. V. G. d. Morais e S. M. Avila, "EXPERIMENTAL STUDY OF EQUIVALENT 1dof MAIN STRUCTURE COUPLED TO TLCD: VALIDATION OF OPTIMUM PARAMETER OBTAINED BY PARAMETRIC OPTIMIZATION," *ABCM,* pp. 1-4, 2019.

[4] T. T. Soong e G. F. Dargush, Passive Energy Dissipation Systems in Structural Engineering, New York: Wiley, 1997.

[5] M. A. Savi e A. S. d. Paula, Vibrações Mecânicas, Rio de Janeiro: LTC, 2017.

[6] K. A. Al-Saif, K. A. Aldakkan e M. A. Foda, "Modified liquid column damper for vibration control of structures," *International Journal of Mechanical Sciences,* pp. 505-512, 2011.

[7] V. Jaksic, C. S. Wright, J. Murphy, C. Afeef, S. F. Ali, D. P. Mandic e V. Pakrashi, "Dynamic response mitigation of floating wind turbine platforms using tuned liquid column dampers," *Philos Trans R Soc A Math Phys Eng Sci,* 2015.

[8] Z. Zhang, B. Basu e S. Nielsen, "Tuned liquid column dampers for mitigation of edgewise vibrations in rotating wind turbine blades," *Struct Control Heal Monit,* pp. 500-517, 2015.

[9] F. Sakai e S. Takaeda, "Tuned liquid column damper-new type device for suppression of building vibrations," *Proceedings of international conference on highrise buildings,* pp. 926-931, 1989.

[10] S. Yalla e A. Kareem, "Optimum absorber parameters for tuned liquid column dampers," *Struct Eng New York,* pp. 906-915, 2000.

[11] M. Alkmim, M. Morais e A. Fabro, "Vibration Reduction of Wind Turbines Using Tuned Liquid Column Damper Using Stochastic Analysis," *J Phys Conf Ser,* 2016.

[12] J. Roberts e P. Spanos, "Stochastic averaging: An approximate method of solving random vibration problems," *Int J Non Linear Mech,* pp. 111-134, 1986.

[13] K. Shum, "Closed form optimal solution of a tuned liquid column damper for suppressing harmonic vibration of structures," *Eng Struct,* pp. 84-92, 2009.

[14] H. Kizilay e E. Cigeroglu, "Frequency Domain Nonlinear Modeling and Analysis of Liquid-Filled Column Dampers," *Springer International,* pp. 43-57, 2020.

[15] L. Muñoz, P. Gonçalves e A. Silva, "Análise Dinâmica Não Linear De Estruturas Reticulares Submetidas Frequência," *XXXV Iberian Latin-American Congress on Computational Methods in Engineering,* 2014.

[16] A. Di Matteo, M. Di Paola e A. Pirrotta, "Innovative modeling of tuned liquid column damper controlled structures," *Smart Structures and Systems,* pp. 117-138, 2016.