

# Dynamic Performance of Controlled Composite Structure of Footbridge

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**Abstract.** Footbridges are structures subject to dynamic loads caused by a single or a varied number of pedestrians. The dynamic interaction between pedestrians and the structure may bring discomfort to the users. Based on this, the design of this type of footbridges require the evaluation of the behavior and the verification of the structure strength under the combined action of static and dynamic loadings. This article presents a footbridge to be constructed in composite steel-concrete structure considering only static loadings. Based on this, a dynamic analysis of the footbridge was carried out to investigate the performance of the structure subjected to the pedestrians traffic, considering two loading conditions, established by the Sètra guide: (i) a single pedestrian; (ii) a crowd of pedestrians. Based on the maximum vertical accelerations, it was verified that the footbridge does not attend the user comfort criteria to vertical vibration established by Sètra guide and ISO 10137 standard. Thus, it has been made a parametric study to choose the features and design details of the passive attenuator of the TMD type (Tuned Mass Damper) that is more appropriate. The results for the controlled footbridge show that the structure meet the required comfort criteria.

**Keywords:** footbridge, composite steel-concrete structure, vibration control, user comfort.

## 1 Introduction

In addition to loads with static characteristics, structures can be subject to dynamic loads with varying intensity and different application points along the structure. In the case of footbridges, these loadings are caused by the passage of one or more pedestrians through the structure. Thus, the dynamic analysis of footbridges is necessary for a better understanding of their structural behavior, in order to avoid performance problems, damage and collapse.

If the footbridge presents unwanted vibrations or outside the stipulated limits, it is essential to implement vibration control systems that guarantee the stability and good performance of the vibratory system. Modernly, the design of footbridges built of slight and slender structures to overcome large spans has been considering, already in the preliminary phase of development, the use of dynamic control systems in order to mitigate the effects of the pedestrian-structure dynamic interaction and, consequently, give an economic structure, with a safety margin and fatigue life required by design standards and that, also, meets the user comfort criteria recommended by international standards.

An efficient way to promote the reduction of footbridge vibration is to use passive attenuators, known as tuned mass dampers (TMDs), as the dynamic response of a footbridge is usually dominated by only a few vibration modes, which makes the TMD a very good device suitable for the problem of excessive structural vibration (REZENDE *et al* [1]). Control devices of the TMD type have the advantage of having low cost and low maintenance, and can be installed without interrupting the operational activities of the structure, in addition to

being considered versatile devices that can be designed in different shapes and sizes, according to architectural aspects of the structure (VARELA and BATTISTA [2]).

As for the tolerable comfort limits, these vary from author to author, considering that several authors have carried out studies on the dynamic behavior of footbridges when subjected to pedestrian loading (SARAMAGO *et. al* [3]). However, in this study, the criteria of the Sètra guide [4] and ISO 10137 [5] will be used.

## 2 Characteristics of the Footbridge Analyzed

The footbridge to be studied is shown in Figure 1, having three spans of different lengths and totaling a length of 58.7 m. The first span is 11.30 m long and have 3 transverses girders, the second span is 33.00 m long and have 10 transverses girders, while the third span is 13.50 m long and have 4 transverses girders. The footbridge is supported by 4 gantries measuring 6.85 m in height. It has a concrete slab and pre-slab which, added together, have a thickness equal to 12 cm, with a width of 2.4 m and the distance between the longitudinal girders equal to 2.04 m. This footbridge will be built on a busy avenue in the city of Rio de Janeiro, connecting points close to public transport stations.

The structure will be analyzed for the dynamic loading of a pedestrian crossing the footbridge and a crowd of pedestrians, as indicated by Sètra [4], and the comfort evaluation will be made considering the criteria established by Sètra [4] and ISO 10137 [5].

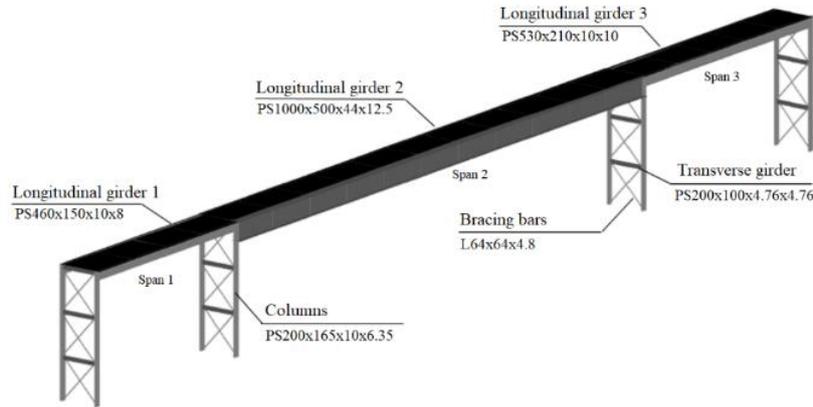


Figure 1. Structure of the analyzed footbridge.

## 3 Pedestrians Walking Forces and Comfort Evaluation of Footbridges

### 3.1 Human walking dynamic load

For the consideration of the force variation in space, the force that varies in time is multiplied by the modal shape function ( $\phi$ ). This function was obtained through the software SAP2000 for the vertical bending and torsional modes of vibration of the footbridge. The generalized exciting force for a single predominant mode is:

$$F_j(t) = \phi F(t) \quad (1)$$

It is noteworthy that, in the case of torsion analysis, the force was applied to only half of the footbridge, for the case of a crowd of pedestrians, and in one edge, for the case of a single pedestrian, so that it represents the worst case for this mode of vibration. In the case of bending analysis, the load was applied to the entire passageway area, for the case of a crowd of pedestrians, and in the longitudinal axis, for the case of a single pedestrian.

- Mathematical Modeling of Pedestrian Walking

According to Sètra [4], experimentally obtained signals of force over time indicate that the vertical reaction of a person walking on footbridge can be described using the Fourier series (BACHMANN *et al* [6]):

$$F(t) = G + \sum_{i=1}^{nh} G_i \alpha_i \text{sen}(2\pi i f_p t - \varphi_i) \quad (2)$$

Where  $G$  is the weight of a person, taking as an average value of 700N;  $t$  the instant of time;  $i$  the harmonic of the series;  $nh$  the number of harmonics in the Fourier series ( $nh = 3$ );  $\alpha_i$  the dynamic coefficient of the  $i$ -th harmonic of the Fourier series ( $\alpha_1 = 0,4$ ;  $\alpha_2 = 0,1$ ;  $\alpha_3 = 0,1$ ; according to Sètra [4]);  $f_p$  the fundamental frequency of the human walking step; and  $\varphi_i$  the phase angle between the  $i$ -th and the first harmonic ( $\varphi_1 = 0$ ;  $\varphi_2 = \pi/2$ ;  $\varphi_3 = \pi/2$ ; according to Sètra [4]).

- Mathematical Modeling of the Walking of a Pedestrians Crowd

In practice, the footbridges are subjected to the simultaneous action of several people. The force per unit area to be applied on the footbridge considering the second harmonic of the stresses caused by pedestrian walking, suggested by Sètra [4], is:

$$F(t) = d \times (70N) \times \cos(2\pi f_v t) \times 1.85 \times \sqrt{\frac{1}{n}} \times \psi \quad (3)$$

Where  $d$  is the pedestrian density (equal to 1.0 for Class I footbridge);  $f_v$  is the frequency of the structure considering the crowd of pedestrians mass;  $\zeta$  is the structure's critical damping ratio;  $n$  is the number of pedestrians on the footbridge ( $d \times$  crossing area);  $\psi$  is the factor that considers the resonance risk, taken equal to 0.4375 according to the Sètra abacus [4].

### 3.2 User comfort criteria to vibrations

To evaluate the comfort level of the footbridge for the vibration modes by vertical bending and torsional, the criteria presented in the Sètra guide [4] and ISO 10137 [5] will be used, shown in Figure 2, which consider as a parameter the maximum acceleration value, in the case of the Sètra [4], and the maximum R.M.S. value acceleration in the case of ISO 10137 [5], calculated considering an averaging time of 1 second. In the case of ISO 10137 [5], the level of vibrations in the vertical direction for footbridges over roads or waterways must not exceed those obtained in the graph in Figure 2.b multiplied by a factor equal to 60.

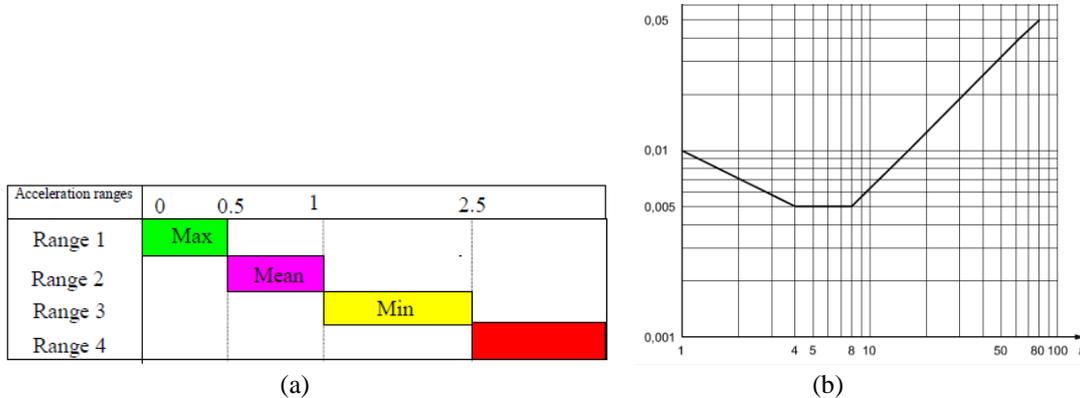


Figure 2. (a) User comfort rating on footbridges for vertical vibrations. Acceleration values in  $m/s^2$  (SÈTRA, 2006). (b) Vertical acceleration limits RMS ( $m/s^2$ ) as a function of natural frequency (Hz) (ISO 10137, 2007).

## 4 Tuned Mass Damper for Vibration Control

Tuned Mass Damper (TMD) is a type of passive vibration control system, characterized by dissipating the vibratory energy of a structure through the movement of a mass-spring-damper system coupled to the structural system. This type of control system works by attenuating the vibrations of only one mode of vibration of the structure, to which the TMD frequency is tuned, through the transfer of energy from the structure to the attenuation system, whose mass goes into oscillatory movement, reducing the movement of the structure.

In the dynamic analysis of the structure with an TMD system, it is common to use a simplified system of two degrees of freedom, corresponding to the generalized degree of freedom of the structure (associated with the vibration mode for which you want to reduce the vibrations) and the degree of freedom associated with the

movement of the TMD mass. The footbridge without the attenuator can be represented by a system with a generalized coordinate  $x$ , and, consequently, with the equation of motion (4).

$$m_s \ddot{x} + c_s \dot{x} + k_s x = F_j(t) \tag{4}$$

Where  $m_s$  is the generalized mass of the structure;  $k_s$  the generalized stiffness of the structure;  $c_s$  the damping of the structure; and  $F_j(t)$  the exciting force of the structure.

Considering now that coupled to the structure is a vibration control system of the Tuned Mass Damper type, the set will then have two generalized degrees of freedom and, consequently, two equations of motion (CHOPRA [7]):

$$m_s \ddot{x} + (c_s + c_a) \dot{x} - c_a \dot{y} + (k_s + k_a)x - k_a y = F_j(t) \tag{5}$$

$$m_a \ddot{y} - c_a \dot{x} + c_a \dot{y} - k_a x + k_a y = 0 \tag{6}$$

Where  $m_a$  is the mass of the attenuator;  $k_a$  the stiffness of the attenuator;  $c_a$  the damping of the attenuator.

For the design of control systems of this type, it is common to carry out parametric studies of the properties of the mass-spring-damper system, in which the dynamic response of the structure without the control system and provided with the system is evaluated, in which they are varied the mass, frequency and damping rate of the TMD, to select those that result in a greater reduction in acceleration and displacement values in the structure.

## 5 Numerical Modeling and Footbridge Dynamic Analysis

To accurately determine the modal properties of the footbridge: frequency, modal mass and mode shape, a three-dimensional numerical model of the structure was developed in the SAP 2000 software. The footbridge model, shown in Figure 3, was made considering its meso and superstructure, where the pillars were considered only fixed in the ground. At the top of the end pillars, displacement restrictions were imposed in the longitudinal direction of the footbridge. The model considered straight frame type elements for beams, bracing bars and columns, and shell type elements for the slab.

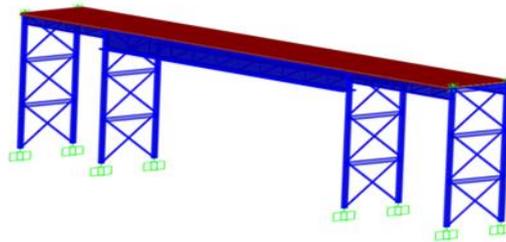


Figure 3. Numerical model of the footbridge on SAP2000.

It was observed that the first two vibration modes of footbridge vibration are torsion and vertical bending. The following vibration modes have high frequencies and will hardly be excited by human walking. It was possible to extract the mode shape of the footbridge for each analyzed mode, and the respective natural frequencies. The damping coefficient was obtained from the damping rate of 0.6%, indicated in the Sètra guide [4] for composite steel-concrete structures. To consider the presence of a crowd of pedestrians, the Sètra guide [4] recommends that the mass of pedestrians be included in the numerical model of the footbridge, so that the natural frequencies of the first two modes of vibration of the footbridge are presented in Table 1, considering the two occupation conditions.

Table 1. Natural frequencies of the footbridge with and without crowd mass.

Empty Footbridge		Busy Footbridge	
$f_1$ (Hz) torsion	$f_2$ (Hz) bending	$f_1$ (Hz) torsion	$f_2$ (Hz) bending
2.551	3.074	2.442	2.920

### 5.1 Torsional vibration (first vibration mode)

Table 2 presents the results in terms of maximum accelerations, from the time domain analysis for the uncontrolled footbridge. The load of a single pedestrian walking (along one of the longitudinal edges of the footbridge) and the load of a crowd of pedestrians (applied to only half, longitudinal, of the central span of the footbridge) were considered, in a way that represents the most critical situation. In both cases the resonance situation is considered.

Table 2. Displacements and accelerations values considering excitation force for torsional mode.

Uncontrolled footbridge considering the load of a pedestrian		Uncontrolled footbridge considering a crowd of pedestrians	
y (mm)	a (m/s <sup>2</sup> )	y (mm)	a (m/s <sup>2</sup> )
0.225	0.058	0.216	0.051

From the results presented, it can be observed that for the load cases considered to evaluate the torsional mode, the walking situation of only one pedestrian generated the most unfavorable results. However, the answers obtained for this case can be considered small, as shown in Table 2 and Figure 4. Considering the comfort criteria established in the Sètra guide (Figure 2.a) and ISO 10137 (Figure 2.b), it can be concluded that the comfort level of the footbridge for this vibration mode is maximum according to the criteria of Sètra [4] and is below the limit according to the criteria of ISO 10137 [5] for the analyzed loads. Therefore, this mode of vibration will not need to be attenuated.

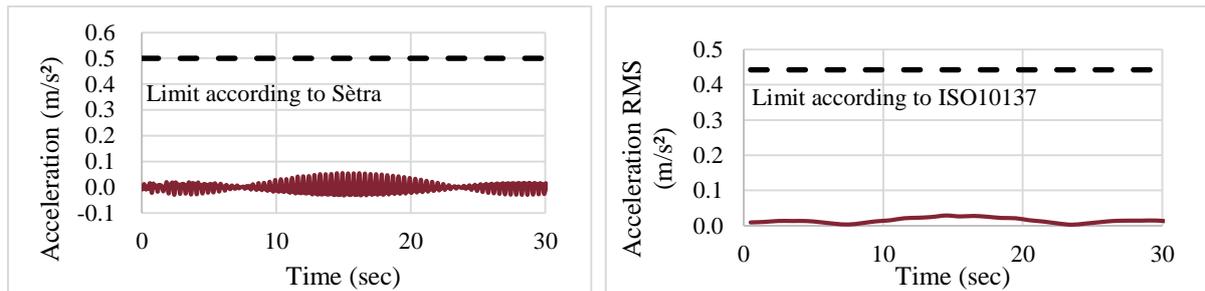


Figure 4. Vertical acceleration (on the left) and RMS (on the right) on the middle point of the central span of the footbridge for the torsional mode considering one person walking and the limits for user comfort (dashed lines) for Sètra and ISO.

### 5.2 Vibration in vertical bending (second vibration mode)

Table 3 shows the maximum accelerations, according to the time domain analysis, for the uncontrolled footbridge considering the walking load of a pedestrian along the axis of the footbridge, with walking frequency equal to half the natural frequency of the structure for vertical bending mode (1.534 Hz). The results for the walking of a crowd of pedestrians obtained by applying the load along the entire footbridge are also presented, considering the resonance situation.

Table 3. Displacement and acceleration values obtained for excitation forces in the vertical bending mode.

Uncontrolled footbridge considering the load of a pedestrian		Uncontrolled footbridge considering a crowd of pedestrians	
y (mm)	a (m/s <sup>2</sup> )	y (mm)	a (m/s <sup>2</sup> )
0.520	0.167	2.120	0.713

From the results presented in Table 3, it is possible to conclude that the crowd load is the most critical case. Therefore, to measure the comfort level of the footbridge and designs the attenuator, the results referring to the

dynamic load of a crowd of pedestrians will be used. The results of the time domain analysis for the uncontrolled footbridge, in terms of maximum accelerations, and the acceleration limits values by the comfort criteria considered in this work are shown in Figure 5.

Based on the above, taking into account the comfort criteria presented in this paper, it is concluded that the footbridge presents an average level of comfort according to the Sètra guide [4] and is above the limit recommended by ISO 10137 [5]. Therefore, the objective of the attenuator design will be to reduce the vibration of the structure so that the footbridge presents the adequate level of comfort according to both criteria (maximum comfort for Sètra [4] and below the limit established by ISO 10137 [5]).

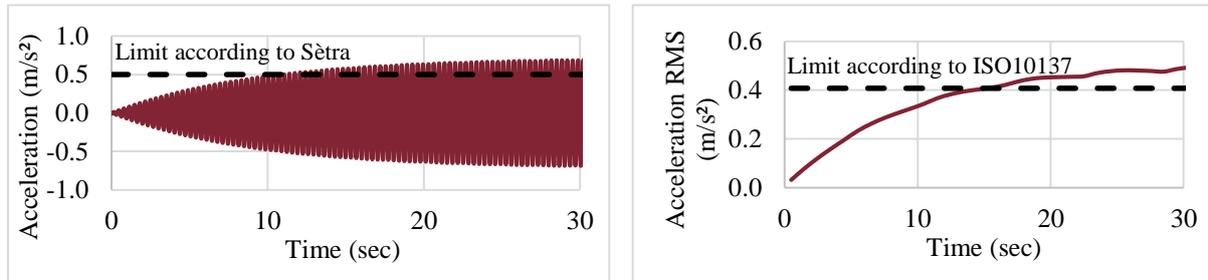


Figure 5. Vertical acceleration (on the left) and RMS (on the right) on the middle point of the central span of the footbridge for the vertical bending mode considering a crowd and the limits for user comfort (dashed lines) for Sètra and ISO.

## 6 Design and Analysis of the Tuned Mass Damper

### 6.1 Parametric study for determination of attenuator properties

To determine the attenuator parameters, a parametric study was carried out to investigate the performance of the control system regarding the reduction of footbridge responses (Figure 6). Then, different damping rates ( $\xi$ ), different mass percentages related to the structure mass ( $m_a/m_s$ ) and different frequency percentages related to the excitation frequency ( $\omega_a/\omega_s$ ) were adopted, and the vibration reduction of the footbridge was verified.

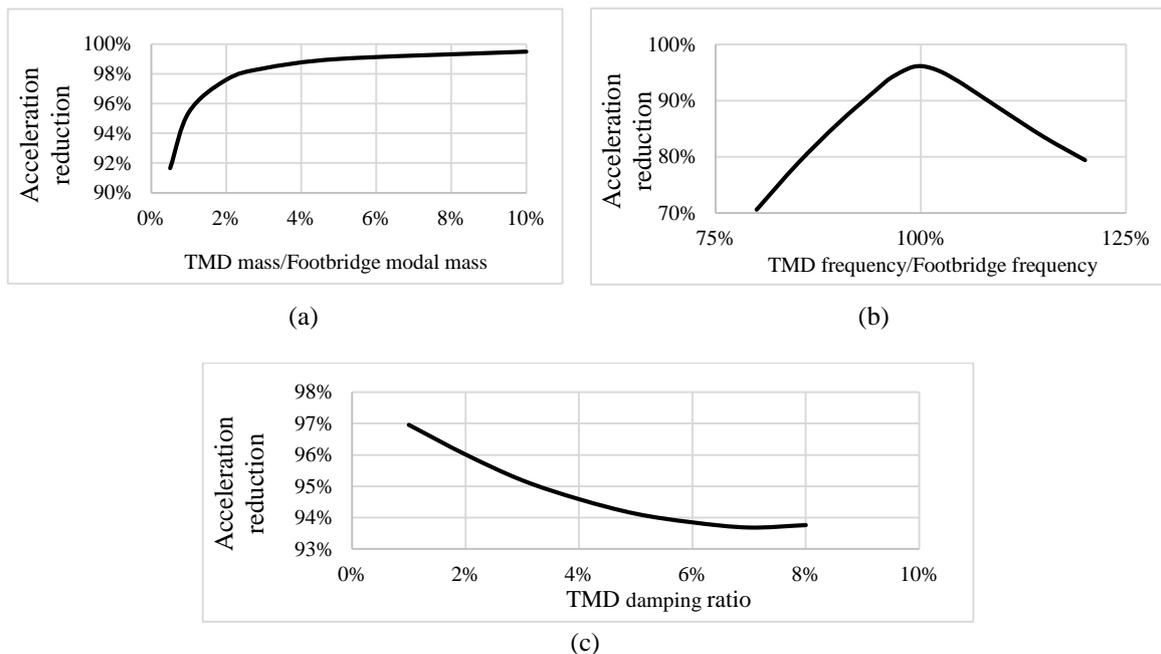


Figure 6. Reduction of the structure's vertical acceleration according to the variation of the parameters associated with the TMD properties: (a) mass ratio; (b) frequency ratio; (c) attenuator damping ratio.

Then mass ratio equal to 0.5% was chosen, resulting in a mass of 133 kg for the attenuator; the frequency ratio chosen is equal to 99%, so the attenuator frequency will be 2.89 Hz; and attenuator damping rate is equal to 1%. With these properties of the attenuator, it was possible to obtain a 97% vibration reduction regarding the acceleration of the structure without control (Table 3), as shown in Figure 7, so that the two limits for maximum user comfort to vertical vibrations were met.

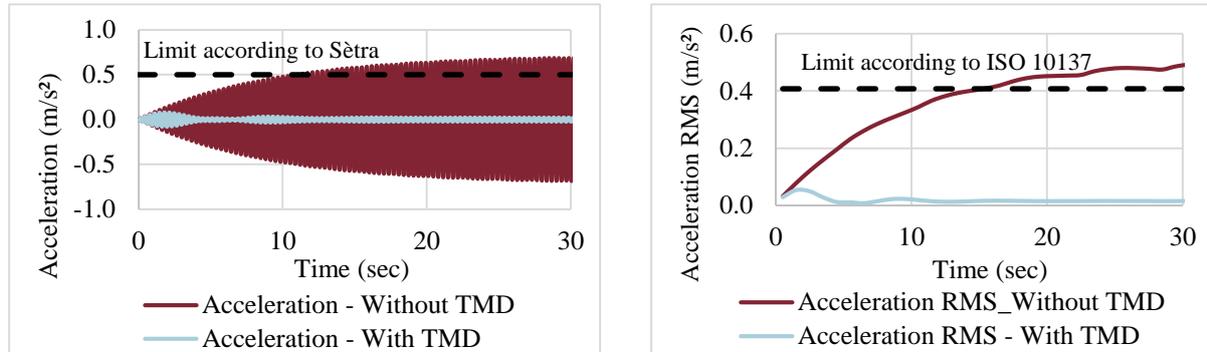


Figure 7. Vertical acceleration (on the left) and RMS (on the right) on the middle point of the central span of the footbridge with and without the control system for the vertical bending mode considering a crowd and the limits for user comfort (dashed lines) for Sètra and ISO.

## 7 Conclusion

Based on the results obtained from the dynamic analysis of the footbridge subjected to pedestrian walking loading, it was concluded that the torsional vibration mode did not generate comfort problems in the structure, unlike the vertical bending vibration mode. In this case, it was found that the maximum accelerations resulting from walking of a crowd of pedestrians lead the footbridge to a medium level of comfort to vertical vibrations, according to the Sètra guide [4] and is above the acceleration limit stipulated by the ISO 10137 [5].

It was concluded that to reach the level of comfort adequate to vertical vibrations, without changing the original structural design, it would be necessary to design a tuned mass damper (TMD) to the structure's vertical bending mode. To choose the properties of the attenuator, a parametric study was carried out in which the TMD mass equal to 133 kg, TMD frequency equal to 2.89 Hz and TMD damping rate equal to 1% were obtained. Thus, it was possible to obtain a 97% reduction in the maximum vertical acceleration and the footbridge presented a maximum level of comfort to vibrations according to Sètra [4] and is within the limit stipulated by ISO 10137 [5].

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