Comparison between the global flow stability of isothermal and adiabatic gaps in a boundary layer

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Abstract. The global stability of a flow provides great insight to its behavior in the real world. Small imperfections in flat surfaces may accelerate the transition to turbulence, which has a great impact on the flow behavior downstream as well as the overall lift and drag caused by such flow. The literature contains several simulations of the flow over open cavities – of which gaps are a subset – however, there is no standard treatment for wall temperature, which is usually either isothermal or adiabatic, whereas the real-world condition is somewhere between both scenarios. We wish to compare the global stability of both cases to measure the impact the wall temperature modeling has over the overall flow. To achieve this, we will use and in-house developed open-source Direct Numerical Simulation (DNS) code, coupled with its global stability routine. The simulation is subsonic yet compressible and the parameters are chosen so that the flow is close to a critical stability condition, so that any differences between both temperature treatments are maximized. Different temperature boundary conditions to cause the base flow to settle at different temperatures, which in turn affects the flow density as well as the local Mach number. Our goal is to better understand the role of surface temperature on flow stability so that we can better compare our simulations, as well as those found in the literature, to experimental results as well as to real-world scenarios.

Keywords: Global Stability, Direct Numerical Simulation, Wall Temperature

1 Introduction

When designing and analyzing an aerodynamic model, both computational and experimental data may be used, each with their own intrinsic challenges and advantages. In computational models, it is often possible to predetermine every variable in the system. On the other hand, experiments in wind tunnels are subject to external variables, such as environmental conditions, which may influence the outcome. Therefore, it's important to understand how each variable may affect the flow's stability.

In our previous works, we have analyzed the impact of various parameters on the global stability of a compressible flow, namely incoming boundary layer thickness, Mach number and cavity geometry [1, 2]. When the flow is globally unstable at a small cavity, it may accelerate the transition to turbulence and, hence, drag [3]. In this paper, we wish to add a new parameter to our search and understand its effect on the flow stability: wall temperature.

2 Methods

2.1 Governing equations

In this work, the flow is modeled by the compressible Navier-Stokes equations. Five variables are needed to fully define the flow: density (ρ), internal energy (e) and the three velocity components (u,v,w). All other flow variables, such as pressure and temperature, can be computed as a function of those five.

Two types of temperature boundary conditions were defined in this study. An isothermal condition, where the wall temperature is fixed at a certain predetermined value; and an adiabatic condition, where wall temperature depends on the flow, such that no thermal energy transfer happens between the wall and the flow, this is done by forcing the wall-normal derivative of temperature to be null.

2.2 Flow solver

We used an in-house Direct Numerical Solver (DNS), which features structured meshes that are refined in regions of interest. A fourth-order Runge-Kutta scheme is used for time marching and fourth-order compact spectral-like finite differences are used for the spatial derivatives [4]. A pencil-slab domain decomposition is used for code parallelization [5]. A tenth-order spatial high-frequency filter is also employed [6] to prevent very short wavelength spurious oscillations. Buffer zones are placed around the useful domain to attenuate undesirable open boundary condition effects such as reflections. They employ a combination of grid stretching, lower order spatial derivatives and Selective Frequency Damping (SFD) [7]. The SFD acts as a low pass temporal filter and may also be turned on in the whole domain to allow base flows to be generated faster or at unstable conditions. Further details of these methods and their implementation in our codes are given by [8–11].

2.3 Global stability analysis

To access the global stability, we use a global analysis routine, that uses a time-stepping approach, in which the Jacobian matrix of the governing equations is not explicitly needed [12, 13]. The method uses the Arnoldi algorithm [14] which is based on Krylov subspaces. It just requires the ability to compute vector multiplications which, due to the way in which the algorithm is built, corresponds to a call to the flow numerical solver, in our case, the code described in the previous section.

The time-stepping global instability analysis can be regarded as an established procedure and the current implementation closely followed that of Chiba [15] and Tezuka and Suzuki [16]. In summary, the method iteratively disturbs the base flow and uses the DNS to capture its response. The successive iteration involves disturbances that are orthogonal to all previous ones. The flow response is used to form a corresponding Hessemberg matrix, which is several orders of magnitude smaller than the flow's Jacobian matrix. If the number of iterations is sufficiently large, the leading eigenvalues and eigenvectors computed from this matrix are good representations of the flow modes and provide good estimates of their respective amplification rates and frequency. In our convention, the real part of the eigenvalue represents the growth rate in time, while the imaginary part represents its angular frequency. Further details on the implementation are given by Mathias and Medeiros [11].

3 Results

3.1 Flow parameters and base flow

The reference flow for this study is a flat plate with a small rectangular gap. All variables are non-dimensional; the reference length is the boundary layer displacement thickness at the gap's leading edge, δ_0^*, the reference velocity, density and temperature are those of the free flow. Reynolds number is $Re_{\delta_0^*} = 1000$, Mach number is $Ma = 0.5$ and the gap's length and depth are $L = 10\delta_0^*$ and $D = 5\delta_0^*$. Five values for the wall temperature (T_w) were chosen, from 80% to 120% of the reference temperature (T_0), one case with adiabatic walls, instead of isothermal, was also used. Using $Re_{\delta_0^*}$ as the reference Reynolds number allows us to easily access the flow's spatial stability near the cavity; in this particular case, the flow is already spatially unstable.

The base flow for this scenario is shown in figure 1. This figure is for the $T_w/T_0 = 1$ case, however there were no visible changes in the velocity field of the base flows for other wall temperatures.

The distinction between the cases becomes clearer when we plot contours of temperature for each case, as shown in figure 2. The flow temperature inside the gap becomes almost uniform and close to the wall temperature. In the adiabatic case, the wall temperature has settled at $1.034T_0$ in the gap. Figure 3 shows the temperature profiles for all cases, before, at, and after the cavity.

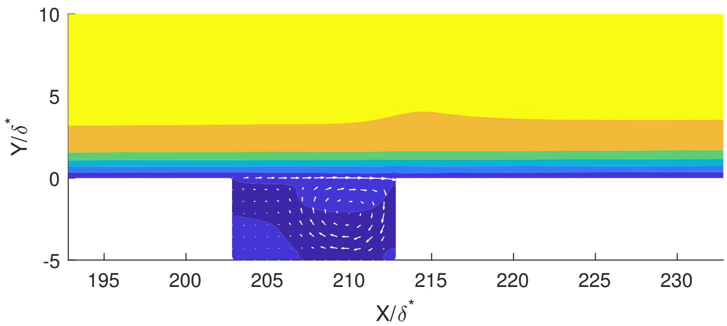

Figure 1. Base flow with the wall at the reference temperature. Contours of stream-wise velocity every 20% of the free-flow velocity.

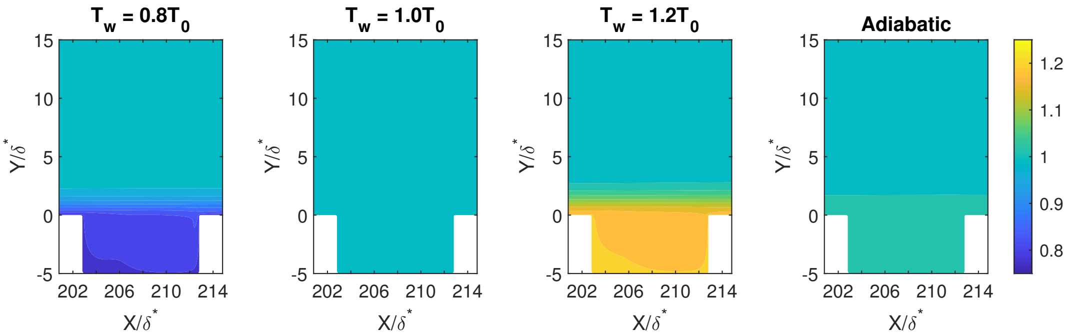

Figure 2. Contours of flow temperature with the wall at 80%, 100% and 120% of the reference temperature.

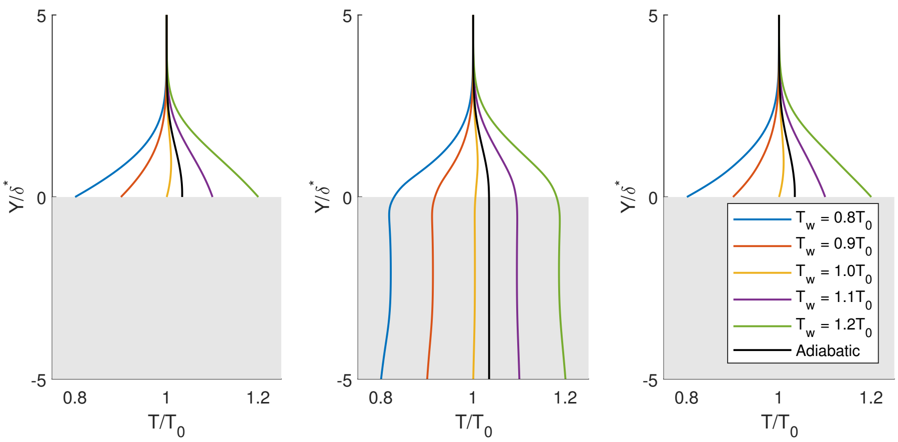

Figure 3. Profiles of flow temperature for various wall temperatures. (Left) $10\delta_0^*$ upstream from the leading edge of the cavity. (Center) Middle of the cavity. (Right) $10\delta_0^*$ downstream from the trailing edge of the cavity.

3.2 Global stability analysis

By using the global stability analysis, we can evaluate how the wall temperatures influences the flow stability and its leading modes. Figure 4(Left) shows the eigenvalues of the most unstable (or least stable) modes for each scenario. A positive real part in the eigenvalue indicates an unstable mode. The flow is only globally stable if all its modes are stable, as any unstable mode would grow exponentially until reaching a limit cycle due to non-linear effects.

Two different modes were identified as being the most sensitive to changes in the wall temperature, those are highlighted in Figure 4(Left) and have their real and imaginary parts shown in Figure 4(Right). Those modes were identified as Rossiter modes 1 and 2 [17]. For $T_w/T_0 \leq 0.9$ mode 1 becomes unstable and for $T_w/T_0 \leq 0.8$ mode 2 is also unstable. Once more, the adiabatic case is positioned very similarly to an isothermal case with $T_w/T_0 \approx 1.03$.

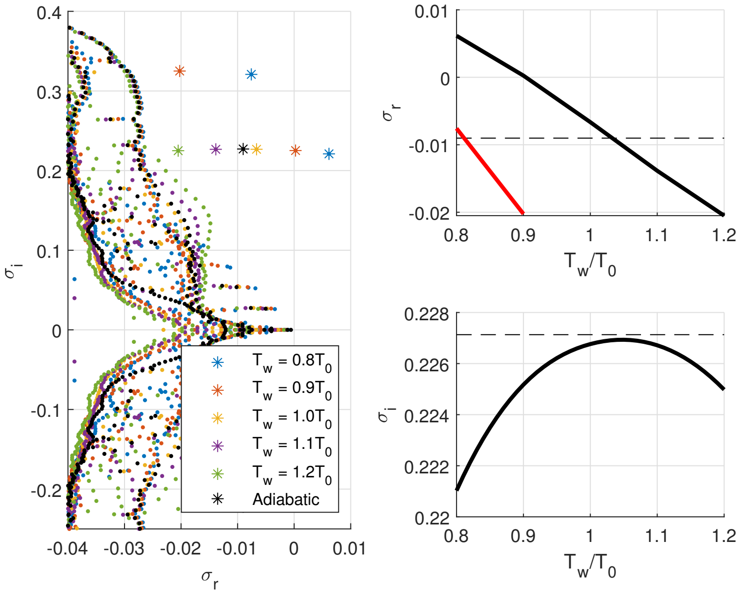

Figure 4. (Left) Global stability eigenvalues for various wall temperatures. (Right) Real part (top) and imaginary part (bottom) of the eigenvalue corresponding to Rossiter mode 1 (black) and mode 2 (red). The dashed line corresponds to the adiabatic case.

One possible explanation for this destabilizing effect of reducing the wall temperature is what happens to the Mach number. As the flow becomes colder in and close to the gap, the speed of sound is also reduced in this region. Considering that the flow velocity is not changed, a reduction in wall temperature ends up causing an increase in the Mach number in the gap, which is known to have a strong destabilizing effect on Rossiter modes [1].

However, the change in Mach number is not enough to explain this destabilizing effect. Mathias and Medeiros [2] bring a Mach number sweep for a flow with these parameters and $Ma = 0.7$ results in the least stable flow; nonetheless, no subsonic Mach number was found that has caused this flow to become unstable with $T_w/T_0 = 1$.

Comparing all cases shown here, we have found that the flow pressure in the gap is almost independent on the wall temperature. Therefore, as the flow becomes colder, its density must increase to maintain the pressure. Figure 5 shows density profiles at the same positions as the temperature profiles of figure 3.

The increased flow density caused by the colder walls consequently increases the local Reynolds number, which also has a strong destabilizing effect on the flow. Along with the increased Mach number, as discussed earlier, this has caused this flow to become unstable as the wall temperature dropped below $0.9T_0$.

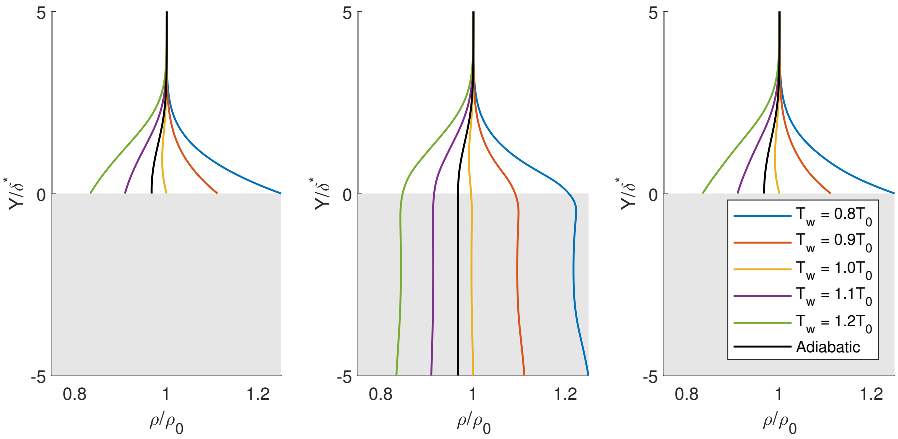

Figure 5. Profiles of flow density for various wall temperatures. (Left) $10\delta_0^*$ upstream from the leading edge of the cavity. (Center) Middle of the cavity. (Right) $10\delta_0^*$ downstream from the trailing edge of the cavity.

4 Conclusions

In this work, we have observed there is a destabilizing effect when the walls of a flat plate with gap are colder than the incoming flow. Two mechanisms were found that justify this observation. First, the cooler flow has a lower speed of sound, which increases the Mach number and, therefore destabilizes Rossiter modes. However, this effect alone is not able to explain the large differences in global stability as the wall temperatures changed. The second phenomenon we have found was that flow density increased as the walls became colder, as to maintain the pressure; this causes an increase in the inertial forces and, therefore, the Reynolds number, further destabilizing the flow. The adiabatic case was found to behave very similarly to an isothermal case with slightly heated walls.

These observations can be used by future works to allow better comparisons between computational simulations, in which wall temperature is often controlled, and simulation data, in which wall temperature is often uncontrolled and depends on environmental and wind tunnel characteristics.

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