Computational analysis of crack propagation in structures with imposition of a deformation field

Felipe A. Bacelar[1], Ícaro C. A. Almeida[1], Luiz C. Wrobel[1]

[1]*Department of Civil and Environmental Engineering, Pontifical Catholic University of Rio de Janeiro*
R. Marques de São Vicente, 225 - Gávea, Rio de Janeiro - RJ, 22541-041
felipealvesbacelar@gmail.com, icaro_caique@hotmail.com, luiz.wrobel@puc-rio.br

Abstract. Cracks have been present in several types of structures since man started using them. However, currently, accidents due to loss of structural integrity can be of larger proportions causing great social, financial and environmental loss since, due to the modern technological advances, the structures are getting bigger and more complex. Fracture mechanics is the science that studies this type of failure. This work aims to analyze pre-fractured structures composed of fragile materials subjected to the imposition of deformations, better understanding the crack behavior in this case and verifying the fracture propagation path. The modeled structures were presented by Portela *et al* [1] and the objective of this work is to present a comparison between the paths formed by the crack propagation with the extended finite element method (XFEM) and the boundary element method (BEM) presented as a reference.

Keywords: fracture mechanics, crack propagation, extend finite element method.

1 Introduction

Cracks have been present in society since man started to use structures, but currently, accidents are of great proportions causing great social, financial and environmental loss, since, due to technological advances, structures are getting bigger: buildings skyscrapers, planes with extraordinary capabilities, slender dams, large rock tunnels, etc.

Fortunately, evolutions in fracture mechanics have brought more safety to designs, thus offsetting the catastrophic potential of structural failures. In addition to the improvement in predicting possible failures. the research community also understood better how the collapse of the materials occur [2]. Much is still being developed in this field, although it is not always applied.

Fracture Mechanics studies the behavior of structural failure, which are unavoidable in structures [3]. Although quality control in the manufacturing process is as efficient as possible, defects may arise in many ways, in addition to those inherent in the material itself. In this situation, the concepts of strength of materials based on properties such as yield strength or breaking strength cannot be applied directly, as they do not take into account the fracture toughness of the material, which is defined by fracture mechanics as the property that measures the propagation resistance of the material to a crack.

The evolution of computational mechanics directly influenced the amount of research generated in the field of fracture mechanics, considering that problems in this area have a very onerous calculation behind them, making manual calculation unfeasible today. This problem can be analyzed analytically as well as numerically.

There are several ways to apply fracture mechanics to analyze the propagation of a crack, these formulations are based on finite element methods (FEM) or boundary element methods (BEM). In the case of boundary elements, the technique fits very well to this type of analysis, for the finite element method there is a need to adapt the formulation so that it is possible to analyze the structure as a non-continuous medium.

Thus, the main objective of this work is to perform a comparison between the crack propagation path by BEM presented in [4] and by XFEM. All physical and geometric properties will be presented in the text.

2 Extended Finite Element Method

Developed by Belytschko & Black [5] based on the Partition of Unit theory proposed by Melenk & Babuska [6], the method aims to model the singularity and the generated discontinuities, in a way that they do not depend on the model's meshing. The core of the methodology is the insertion of enriched functions for a better approximation of the discontinuous displacements field.

According to Silva [7], the function of the MEF displacement field in Equation is added by two functions that will be responsible for describing the fracture behavior. The first will represent the asymptotic fields close to the fracture tip region, while the second is a Heaviside function, defined along the crack, which represents the jumps in the displacement field at positions distant from the tip, thus generating the XFEM characteristic equation, represented by Equation 2.

$$u(x) = \sum_{i=1}^{n} u_i N_i(x) \tag{1}$$

N_i being the form function associated with the node, u_i being the degree of freedom in the node and n being the number of nodes in the model.

$$u(x) = \sum_{i=1}^{n} u_i N_i(x) + \underbrace{\sum_{i=1}^{m} c_i N_i(x) H(x)} + \underbrace{\sum_{i=1}^{k} N_i(x) \left(\sum_{l=1}^{4} c_i^a F_a(x) \right)} \tag{2}$$

m is the number of nodes in the elements cut, k is the number of elements at the tip of the fracture, c_i is the degree of freedom added to the nodes m, c_i^a is the degree of freedom added to the nodes k and F_a is the set of asymptotic functions represented by Equation 3.

$$[F_a(r,\theta)]_{a=1}^{4} = \left[\sqrt{r} \cdot sin\left(\frac{\theta}{2}\right), \sqrt{r} \cdot sin\left(\frac{\theta}{2}\right) \cdot sin(\theta), \sqrt{r} \cdot cos\left(\frac{\theta}{2}\right), \sqrt{r} \cdot cos\left(\frac{\theta}{2}\right) \cdot sin(\theta)\right] \tag{3}$$

r e θ are the polar coordinates at the tip of the fracture exemplified in Fig 1.

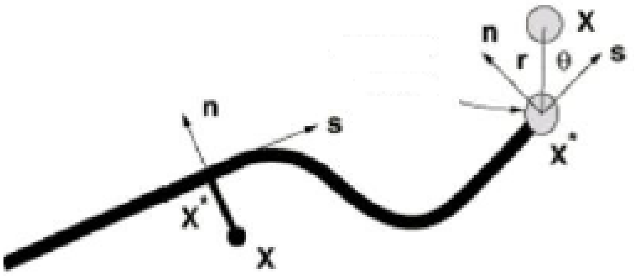

Figure 1. Scheme of polar coordinates for enriching nodes with asymptotic functions [8]

The term that describes the behavior of the formulation is the first term in the set of Equation 3, that is, this term is extremely important because it describes the discontinuities on the fracture surface. This term is defined according to Equation 4.

$$\sqrt{r} \cdot sin\left(\frac{\theta}{2}\right) = \begin{cases} \sqrt{r}, \text{se } \theta = \pi \\ -\sqrt{r}, \text{se } \theta = -\pi \end{cases} \tag{4}$$

to demonstrate the lower surface θ=-π, for the fracture direction θ=0 rad and for the upper surface θ=π. The other terms are responsible for improving the approximation of convergence criteria (MARTÍNEZ, 2015)[9].The Heaviside function is defined as: López Bendezú (2015)[10] in Equation 5.

$$H(x) = \begin{cases} +1, \text{se } (x - x^*) \cdot n \geq 0 \\ -1, \text{se } (x - x^*) \cdot n < 0 \end{cases} \tag{5}$$

x is any point or point of integration, x^* is the projection of x onto the fracture surface and n is the unit vector normal to the fracture at x*. Figure 2 exemplifies a mesh with node enrichment by the Heaviside function.

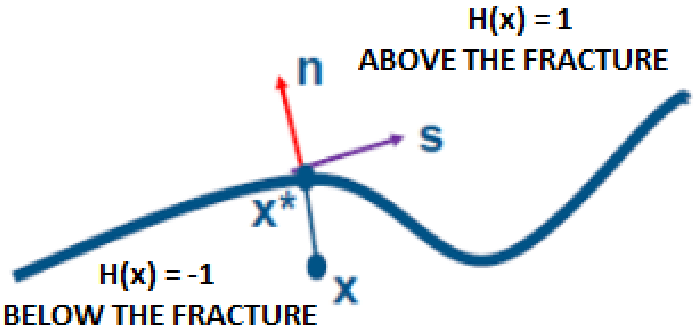

Figure 2. Scheme for enriching nodes with the Heaviside function H(x) [8]

The Extended Finite Element Method (XFEM) has a great advantage over the traditional FEM, as the cracks are inserted in the model independently of the mesh, allowing the fracture to travel inside the element without the need for refined meshing. Silva [7], further adds that, with this method, it is possible to obtain good results in the calculation of stress intensity factors when it comes to crack propagation.

3 Problem under Study

One of the problems studied in [1] is the case of a plate shaped as a crucifix (Fig. 3).

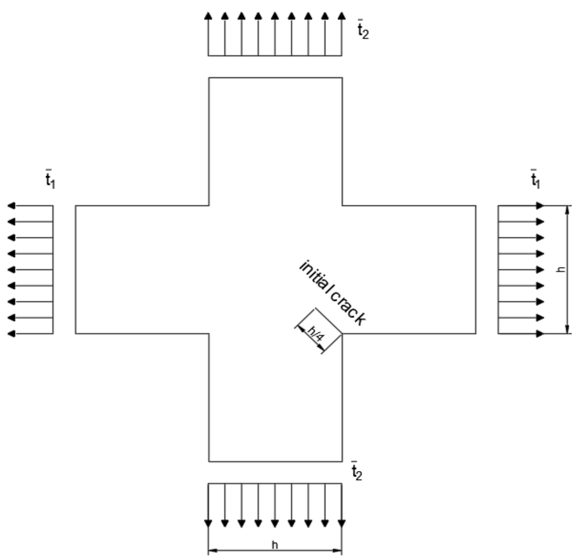

Figure 3. Fracture model of a plate shaped as a crucifix

For this same structure, four analyzes are performed, modifying only the applied load t_1 and t_2, where: $t_1 = t_2$, $2t_1 = t_2$, $t_1 = 0$ and $t_2 = 0$. This analysis was made considering a fragile rupture of the material, but in their work Portela *et al* [1] do not specify the modulus of elasticity nor the maximum resisted stress. Based on the geometry presented, the physical properties of the lean concrete material of fck 30 MPa were adopted. Table 1 presents the properties adopted for the models.

Table 1. Properties of the adopted modeling

Properties	Magnitude	Value	Unit
Young's modulus	material	30.672,46 x 10^6	Pa
Max principal Stress	material	2.896,46 x 10^3	Pa
Poisson's Ratio	material	0,2	
Displacement at Failure	geometric	0,1	m
h	geometric	0,3	m

The material properties were calculated from NBR 6118:2014 [11], which characterizes concrete as a function of its characteristic strength.

The structures were modeled in Abaqus 2020® software, using a simplified mesh divided into 0.01m formed by CP4SR-type elements in a two-dimensional plane stress model. Fig. 4 shows the mesh used for the models. The criterion adopted for the crack propagation was the maximum allowable stress, thus, whenever the material's rupture stress is reached, the crack will be propagated and the time increment will be redone. In the model, the load was applied as an imposition of a deformation field.

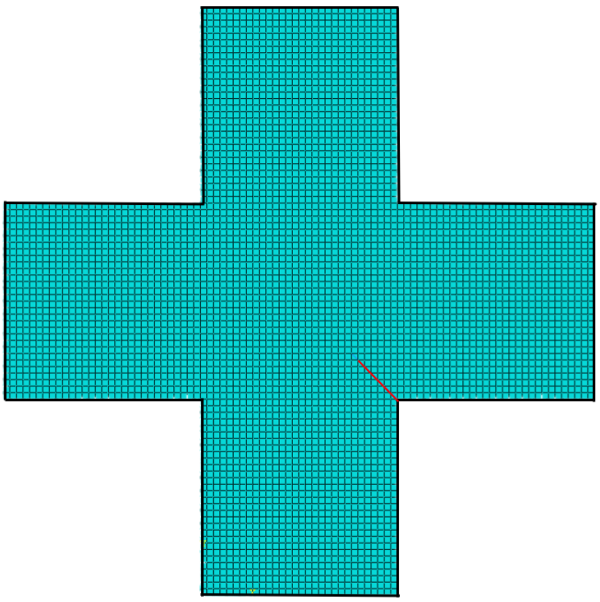

Figure 4. Extended finite element mesh

4 Results e Discussion

Fig. 5 shows the results obtained by Portela *et al* [1].

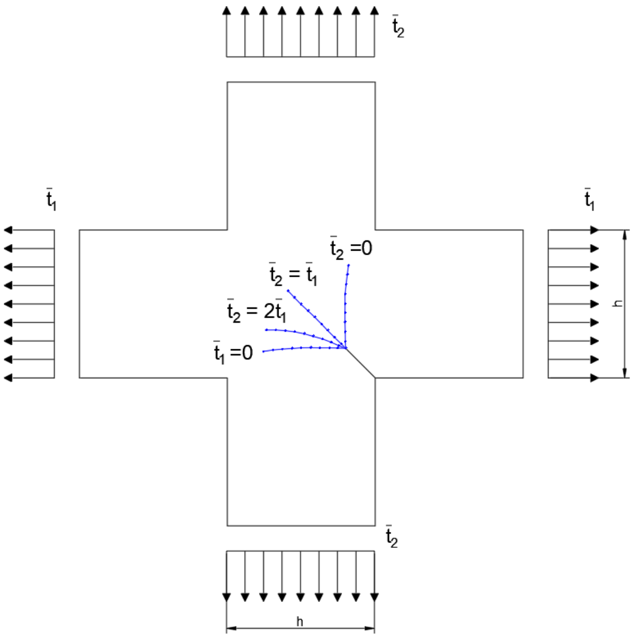

Figure 5. Results presented by Portela *et al* [1]

The results obtained through the modeling were very close to those obtained by Portela *et al* [1], the small difference can be justified because the calculation methods are different, while the present numerical solution was solved based on the finite element method, Portela *et al* [1] used the boundary element method. Another factor that justifies this difference is that *Portela et al* [1] did not make it clear which Poisson's coefficient was used to obtain the results. Thus, in Fig. 6, there is a comparative graph between both solutions.

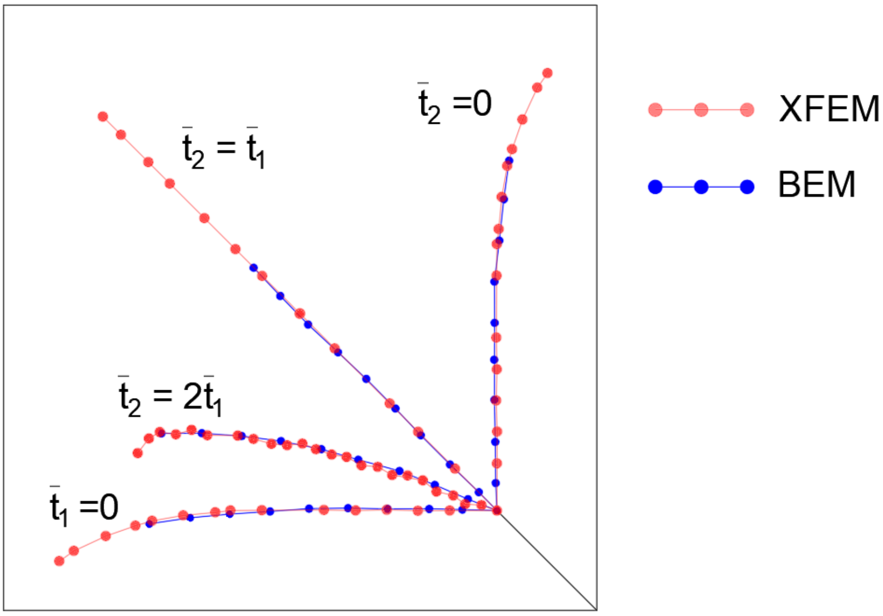

Figure 6. Comparison between XFEM and BEM results

5 Conclusions

This work aims to show that although the BEM and XFEM methods have different formulations, they can present a similar result. The references adopted treat the BEM as the most accurate for the study of fracture mechanics, but for this analysis there is a strong similarity of the results.
The authors understand that comparing only the propagation path does not mean that the results are close, but an important detail is that this work is still in progress and the next step is to numerically compare the two methods.

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