Mixed Dimensional Coupling in GFEM Global-Local

Lorena L. Gomes[1], Felicio B. Barros[1]

[1]*Dept. of Structural Engineering, Federal University of Minas Gerais*
Av. Antônio Carlos 6627, 31270-901, Belo Horizonte/Minas Gerais, Brazil
lorenaleocadio@gmail.com, felicio@dees.ufmg.br

Abstract. The global-local Generalized Finite Element Method (GFEM) is use here to solve mixed dimensional structure problems. Considered as an instance of the Partition of Unity Method (PUM), the GFEM uses enrichment functions that, multiplied with the Partition of Unity (PU) functions, augment the space of problem solving. These enrichment functions are chosen according to the problem analyzed, but they can also be numerically obtained from the results of the analysis of a local problem, the so-called GFEM global-local. The application of this method, however, is limited to models, for which both global and local meshes use the same formulation of finite element. Here, the two scale of the analysis are discretized not only by different meshes, but also with different types of finite elements. Combining mixed-dimensional elements and a multi-scale analysis can be highly effective to capture the local structure features without overburden the global analysis of the problem. An iterative procedure, that balances the forces of the two multi-dimensional models, is combined with the global-local analysis of GFEM. A numerical example is presented, considering the coupling of a large-scale model with Timoshenko beam elements and a small-scale model with quadrilateral plane elements.

Keywords: Generalized Finite Element Method, Global-Local Enrichment, Constraint Equations, Mixed-dimensional Coupling, Multi-scale modeling

1 Introduction

The biggest challenge in large and complex structures is to find an effective numerical method in order to better represent the continuum and to minimize the approximation error without affecting the respective computational cost. The simplest way to reduce these errors is to increase the number of parameters of the discretization. In mesh based methods this is achieved with a large number of small scale elements. Even improving accuracy of the solution, a considerable number of elements can lead to a enormous computational cost and difficulties in processing the analysis.

The use of large elements can capture the global behavior in most structures. However, there are local phenomena, such as stress concentrations, crack initiation and propagation which are normally not represented in the global model. The simulation of multiscale finite elements can provide a better solution for those features. In these types of problems, a simpler analysis is performed, which is able to capture the overall behavior of the structure. At specific regions, a more detailed analysis is required to represent the structural behavior associated with one or more local phenomenon of interest. This can be achieved by a two scale global-local analysis, proposed initially by Noor [1]. In this strategy, the solution of a global problem with a coarse discretization is used to provide the boundary data for a local analysis solved with a fine discretization. Instead of performing two separate analysis, coupling techniques can also be employed, allowing a concurrent multi-scale modelling [2–6].

When the analysis models is not the same for the two scales of the problems, it is necessary to choose an effective coupling method in order to combine such formulations in a single structural model. The challenge when simulating this coupling, according to McCune et al. [7], is to guarantee the continuity in displacement and the stress balance in the region of the interface between the different types of finite elements. Among the several different coupling methods, the multipoint constraint (MPC) method, described in Felippa [8], can be used to define restriction equations that relate the degrees of freedom on the interface between element of different analysis models.

In the present paper, a mixed dimensional global-local analysis is proposed under the Generalized Finite Element Method (GFEM) approach (Strouboulis et al. [9] and Duarte et al. [10]). For this purpose, three problem scales are considered. The first one is the global problem. It is described by a analysis model different from the second problem, that contains the region where one or more local features of interest are present. The mixed dimensional coupling between these two problems is applied by a iterative procedure introduced by Wang et al. [6]. The second problem is just a bridge to the representation of the third local problem. Both of them are represented by the same analysis model. They are related do each other by the strategy proposed by Duarte and Kim [11] in the Global-local GFEM (GFEM^{gl}). The second problem becomes the global scale in this strategy, and provides the boundary data to the third problem. The numerically obtained local solution from the third problem is used as enrichment function to improve the approximation of the second problem.

The main ideias, still under investigation, are summarized on the following sections. Aiming to illustrate this method, a simple problem is analyzed. A beam is globally represented by Timoshenko elements. The local behavior triggered by the presence of an edge crack is reproduced in a two dimensional mesh, considering plane stress state. The results are compared to standard analysis with GFEM.

2 Generalized finite element method (GFEM)

In GFEM [9, 10] the approximate function can be improved by the extrinsic enrichment of standard partition of unity functions (PU), such as the FEM shape functions $N_j(\mathbf{x})$. Local functions $L_{ji}(\mathbf{x})$ multiply the PU resulting in the approximate function $\phi_{ji}(\mathbf{x})$ given by:

$$\{\phi_{ji}\}_{i=1}^{q_j} = N_j(\mathbf{x}) \times \{L_{ji}(\mathbf{x})\}_{i=1}^{q_j}. \tag{1}$$

The local approximation functions $L_{ji}(\mathbf{x})$ can be polynomial functions previously established, or special functions chosen from an a priori knowledge of the behaviour of the problem.

Using the concept of solution decomposition into two scales of analysis, Global Local Generalized Finite Element Method, GFEM^{gl}, provides a framework to enrich the global problem solution space with functions numerically constructed from the solution of a local boundary value problem [12].

The numerical solution obtained in the local problem ($\tilde{\boldsymbol{u}}_L$) is used in eq. 1 as the local approximation function $L_{ji}(\mathbf{x})$ to build the approximate solution of the glocal-local enriched problem.

3 Mixed-dimensional finite element coupling

The objective of this work is to propose a new approach for solving problems with complex geometries and with local behaviors of interest. In this new strategy, a numerical procedure was formulated that combines the GFEM with Global-Local enrichment with mixed-dimensional coupling techniques, enabling the study of the propagation of defects in structures.

In this new technique, the mixed-dimensional finite element coupling proposed by Wang et al. [6] is used both in the first step of the global analysis and in the last step of the MEFG^{gl}, the enriched global model. A specially chosen region of the global model, discretized with plane stress elements, is used to define the local problem, with a finer mesh (Fig. 1). This local problem enables a more accurate analysis of phenomenon of interest, such as cracks and/or damage, in the studied region. Finally, the local numerical solution is used to build the enrichment of the last stage of the MEFG^{gl}.

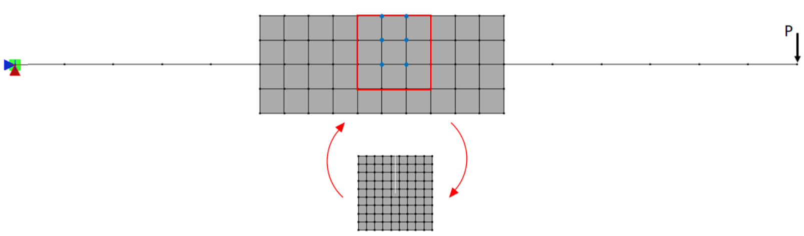

Figure 1. Mixed-dimensional global-local model

The numerical coupling method used for the multidimensional analysis was proposed by Wang et al. [6]. In this method, the principle of virtual works is adopted in order to obtain, at the interface, both the force restraint equations and the displacement restraint equations. For the development of the new iterative method for calculating the coefficients of the constraint equations, Wang et al. [6] use as an example the coupling between a plate and a beam, in which plate is understood as a plane stress model. Once the interface is identified in the mixed-dimensional model, a substructure is extracted directly from the plate elements, Fig. 2.

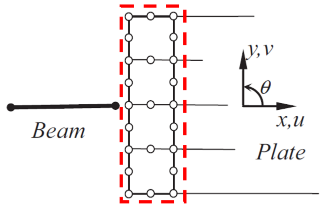

Figure 2. Substructure of the plate at the interface, adapted from Wang et al. [6]

This substructure is used to create a nodal force model (Fig. 3). This model is formed by repeating the substructure using the same mesh and element type. At each end a load node is established, one of the nodes receives unit loads and the other one is totally restrainted. These endpoints represent the beam node on the interface.

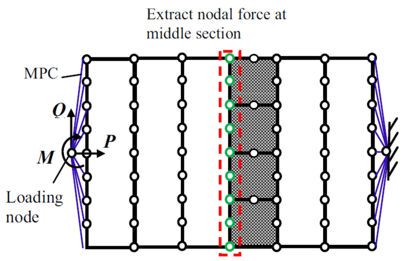

Figure 3. Nodal Force Model, adapted from Wang et al. [6]

To obtain the coefficient matrix of the constraint equation, an iterative process is performed in the nodal forces model. In this iterative process, the first analysis is performed with a rigid connection between the plate nodes and the loading node. From the second analysis, the rigid connection is replaced by the deformable connection and the coefficient matrix found in the previous analysis is used in the deformable multipoint constraint equation. This procedure continues until there is a convergence for the values of these coefficients. The matrix of the displacement constraint equation obtained in the last iteration is then transferred to the main plate-beam model, completing the coupling.

4 Numerical Results

In Fig. 4, the structural problem of a cantilever beam with an edge crack is presented.

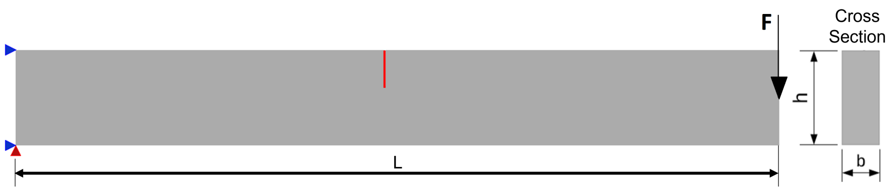

Figure 4. Cantilever beam

The problem has length $L = 320mm$ and cross section with $h = 40mm$ and $b = 8mm$. An isotropic linear elastic material was employed with elastic modulus of $E = 2 \times 10^5 N/mm^2$ and Poisson ratio $\upsilon = 0.3$. The right end of the beam is loaded with a force $F = 12,000N$. Except from the presence of the crack, this problem is similar to the one studied by McCune et al. [7] and Wang et al. [6] for the evaluation of the coupling between beam and plate models.

Based on the model of the cantilever beam, five numerical simulations are presented. The Model A, considered as the reference solution, is depicted in Fig. 5. A mesh of 3200 Q4 elements under plane stress condition was used in a GFEM analysis. The approximate solution was enriched with polynomial functions of the first degree in all nodes and also with the OD singular functions of Duarte et al. [10] in the nodes around the crack.

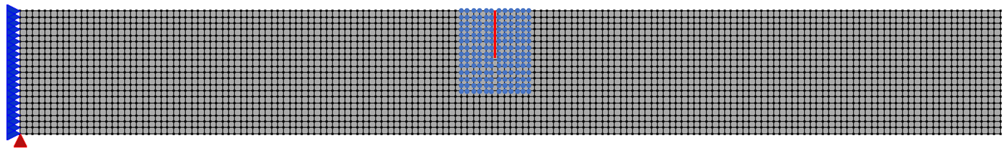

Figure 5. GFEM Reference Model. Crack segment in red color. Circles in blue represent the nodes associated with the OD crack functions.

Two uncracked models were also analyzed in order to be compared their numerical response with the one of the cracked problem. The goal is to show that the strategy proposed here is able to simulate the presence of a crack in the problem that is globally represented by Timoshenko beam elements. The Model B is represented by 16 linear Timoshenko beam elements. The Model C is analyzed under plane stress condition with a mesh of 128 linear Q4 elements and is shown in Fig. 6.

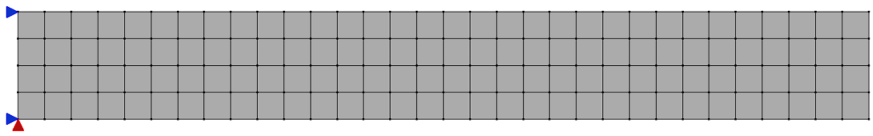

Figure 6. FEM model

For the last analysis, the beam problem is discretized in a multidimensional model subdivided into three sections. The first $100mm$ and the last $120mm$ were discretized with Timoshenko beam elements. The mesh is equivalent to the one used in the Model B. The central region with length of $100mm$ was represented by a mesh of linear Q4 elements, equivalent to the one adopted in Model C. In this problem, the behavior of the model was analyzed with the inclusion of a crack in the center of the section with the plate model (Fig. 7).

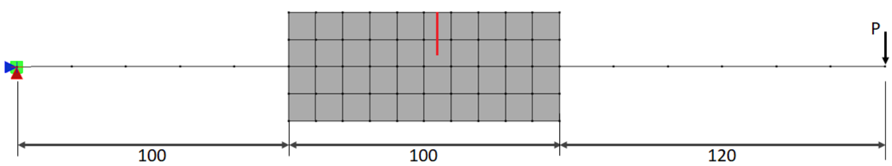

Figure 7. Mixed-dimensional model. The crack is represented in red.

Two models representing this last problem were analyzed. In the Model D, standard GFEM was adopted and the crack is represented only by the OD crack enrichment functions. In the Model E, it was used Global-local GFEM analysis, as indicated in Fig. 1. In this analysis the crack is represented only in the local problem, by the OD crack functions and a finer mesh (each global element is replaced by nine elements in the local mesh).

Table 1 presents the results for displacement in the right end of the cantilever beam, the strain energy and also the stress intensity factor KI. Note that the KI is only calculated in the Models A, D and E, where the crack is simulated.

Table 1. Results of the different models

Model	Displacement	Strain Energy	KI
A	-18.23	109372	0.65
B	-14.16	84929	-
C	-15.02	90517	-
D	-16.49	98926	0.55
E	-16.72	100298	0.61

Models B and C are uncracked problems, and for this reason they are stiffer than the other ones, providing smaller values to the displacements and strain energies. In the multidimensional problems, D and E, it is possible to see the influence of the crack reducing the stiffness of the problem and increasing the values of the displacements and strain energies. Both of them delivers results closer to the reference cracked model A. .

Furthermore, it can be seen that the multidimensional model via GFEM^{gl} represents the cracked problem better than model D, which is granted by the finer mesh used in the local problem. From Fig. 8, it is possible to identify the displacement behavior along the model, and how well it was possible to represent the crack from the global-local enrichment.

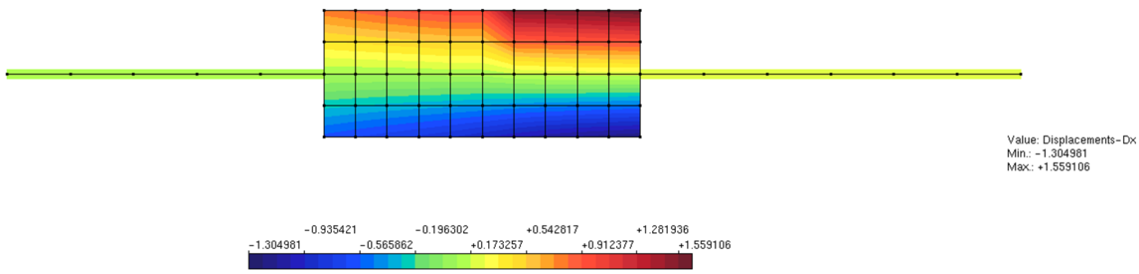

Figure 8. Mixed-dimensional model displacements

5 Conclusions

The Generalized Finite Element Method (GFEM) is consolidated as a useful tool for solving complex structural models that present localized phenomena. Among the various functions that can be used to enrich the solution, there are the functions resulting from the solution process of the Global-Local strategy. In this strategy, a second problem is extracted from the global problem with a finer mesh and whose solution is used to enrich the global model.

Finding an appropriate method for linking different types of elements into a single finite element solution is the foundation of mixed-dimensional modeling. Several coupling methods have been developed to perform these couplings and satisfactory results are being obtained.

Given the consolidation of these two approaches, both multidimensional modeling and GFEM^{gl}, for analysis of complex structures, in this work a new technique was developed to perform analyses via GFEM^{gl}, using the multidimensional modeling.

From the results obtained, it can be concluded that this new technique was able to represent the crack in a model, where the global behavior is simulated by the Timoshenko beam formulation.

Acknowledgements. The authors gratefully acknowledge the important support of the Brazilian research agencies FAPEMIG (in Portuguese “Fundação de Amparo à Pesquisa de Minas Gerais” – Grant APQ-01656 – 18), CNPq (in Portuguese “Conselho Nacional de Desenvolvimento Científico e Tecnológico” – Grants 304211/2019–2, 437639/2018–5, 311663/2017–6).

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] A. K. Noor. Global-local methodologies and their application to nonlinear analysis. *Finite Elements in Analysis and Design*, vol. 2, pp. 333–346, 1986.
[2] Z. X. Li, T. H. T. Chan, Y. Yu, and Z. H. Sun. Concurrent multi-scale modeling of civil infrastructures for analyses on structural deterioration - part i: modeling methodology and strategy. *Finite Elements in Analysis and Design*, vol. 45, n. 11, pp. 782–794, 2009.
[3] T. H. T. Chan, Z. X. Li, Y. Yu, and Z. H. Sun. Concurrent multi-scale modeling of civil infrastructures for analyses on structural deterioration - part ii: model updating and verification. *Finite Elements in Analysis and Design*, vol. 45, n. 11, pp. 795–805, 2009.
[4] Z. X. Li, T. Q. Zhou, T. H. T. Chan, and Y. Yu. Multi-scale numerical analysis on dynamic response and local damage in long-span bridges. *Engineering Structures*, vol. 29, n. 7, pp. 1507–1524, 2007.
[5] P. Mata, A. H. Barbat, and S. Oller. Two-scale approach for the nonlinear dynamic analysis of rc structures with local non-prismatic parts. *Engineering Structures*, vol. 30, n. 12, pp. 3667–3680, 2008.
[6] F. Y. Wang, Y. L. Xu, and W. L. Qu. Mixed-dimensional finite element coupling for structural multi-scale simulation. *Engineering Structures*, vol. 92, pp. 12–25, 2014.
[7] R. W. McCune, C. G. Armstrong, and D. J. Robinson. Mixed-dimensional coupling in finite element models. *International Journal for Numerical Methods in Engineering*, vol. 49, n. 6, pp. 725–750, 2000.
[8] C. A. Felippa. *Introduction to Finite Element Methods*. Sciences Department Of The University Of Colorado, University of Colorado, Boulder, Colorado, 2004.
[9] T. Strouboulis, I. Babuška, and K. Copps. The design and analysis of the Generalized Finite Element Method. *Computer Methods in Applied Mechanics and Engineering*, vol. 181, n. 1-3, pp. 43–69, 2000.
[10] C. A. Duarte, I. Babuška, and J. T. Oden. Generalized finite element methods for three-dimensional structural mechanics problems. *Computer & Structures*, vol. 77, n. 2, pp. 215–232, 2000.
[11] C. A. Duarte and D.-J. Kim. Analysis and applications of a generalized finite element method with global-local enrichment functions. *Computer Methods in Applied Mechanics and Engineering*, vol. 197, n. 6-8, pp. 487–504, 2008.
[12] V. Gupta, D. Kim, and C. A. Duarte. Analysis and improvements of global–local enrichments for the generalized finite element method. *Computer Methods in Applied Mechanics and Engineering*, vol. 245-246, pp. 47–62, 2012.