Analysis of solution by traveling waves for an in situ combustion model

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Abstract. In-situ combustion is a medium and high viscosity oil recovery technique. In this work, a mathematical model was proposed describing the in situ combustion technique. Also, the study of the existence and uniqueness of the solution using traveling waves is presented. Unlike previous works, we use the Ideal Gas Law and the fractional flow theory, resulting in a more realistic model. This physical phenomenon is described by a mathematical model composed of three partial differential balance equations: energy balance, the molar mass balance of gas, and the molar mass balance of oil.

Keywords: In situ combustion, Traveling waves, Fractional flow theory

1 Introduction

Several techniques for the Enhanced Oil Recvery (EOR) are developed to reduce the impact of the production decline. In particular, the in-situ combustion technique consists of injecting the oxygen-containing gas, air, or the enriched air into the reservoir. These gases react with crude oil creating a high-temperature combustion zone, generating combustion gases resulting in a heat front which propagates through the reservoir decreasing the gas viscosity and improving the oil recovery factor. There are several mathematical works addressing this phenomenon. Due to the complexity of the problem, a significant number of them address the filtration combustion (Aldushin et al. [1], Chapiro et al. [2], Zavala and Chapiro [3], Ghazaryan et al. [4]), i.e., instead of mobile oil phase, the authors consider immobile fuel. Others address the mobile fuel phase (Gargar et al. [5], de Assis et al. [6]). Unfortunately, these works do not consider realistic fractional flow functions and neglect gas compressibility.

To model the in-situ combustion, in the present work, we consider a long cylinder of porous rock, thermally insulated throughout the lateral area and containing oil. Diffusion is neglected following some studies, which show that simplified models neglecting diffusion still present qualitatively acceptable solutions (Zavala and Chapiro [3], Chapiro and Marchesin [7]). The model is described by a system of balance equations corresponding to fuel density, oxygen density, and enthalpy.

As appeared in experiment (Gargar et al. [8]) and numerical simulations (da Silva Pereira and Chapiro [9]) there is a stable combustion front in this type of combustion process. This observation motivated several works looking for the solution of the model in the form of traveling waves. There exist two commonly used methods for that. The first method considers that the reaction ceases when the temperature drops to negligible values, see, for instance, (Aldushin et al. [1], Gargar et al. [5]) and references therein. The second method considers that the low-temperature reaction is not negligible; in this case, a long reaction tail may appear, (Chapiro et al. [2]). We follow the the second approach as more accurate from the mathematical point of view.

In summary, the idea of this work is to present the first steps in the mathematical analysys of a model that generalizes (Gargar et al. [5]) and (de Assis et al. [6]) by considering gas compressibility and fractional flow curves

described by Corey-Brooks model (Corey [10]).

This paper is divided into five sections. Section 2 presents a brief introduction to the fractional flow theory, addresing relative velocities of the two-phase fluid and explaining some simplifications used further. In Section 3, the physical model is presented with the simplifications necessary to obtain its dimensionless form. In Section 4, we show the necessary and sufficient conditions for the existence of the solution in the form of a traveling wave for a particular case. Finally, Section 5 presents some conclusions.

2 Fractional flow theory

The standard way to model the one-dimensional two-phase flow displacement in porous media is described by the Buckley-Leverett theory (Buckley and Leverett [11]). In the present model we consider that the saturations (relative volume occupyied by fluid) is never zero, since a portion of residual oil (gas) is always kept in pores. In other words, after the injection, the fluid cannot be completely removed, so that its saturation will never be zero. Thus, the interval in which the oil saturation varies is $[S_{or}, 1 - S_{gr}]$, where S_{gr} is the residual gas saturation and S_{or} is the residual oil saturation. We consider saturated porous medium: $S_g = 1 - S_o$.

The fractional flow of a phase is defined as the flow of this phase in relation to the total flow of fluids being produced. Thus, the functions that describe fractional flows of oil (f_o) and gas (f_g) can be defined as:

$$f_o = \frac{\lambda_o}{\lambda_o + \lambda_g}, \quad f_g = \frac{\lambda_g}{\lambda_o + \lambda_g} \tag{1}$$

where λ_o and λ_g are oil and gas mobilities. Note that mobility depends on the relative permeability, commonly described by the Corey-Brooks model (Brooks and Corey [12]). For determine the flows f_0 and f_g is considered the oil viscosity μ_o and gas viscosity μ_g and following (Gargar et al. [5]) for $T = 273K$ results:

$$\mu_g = 0.016, \qquad \mu_o = 1.297. \tag{2}$$

In (de Assis et al. [6]) is considered constant relative permeabilities and in this work the relative permeabilities of oil k_{ro} and gas k_{rg} are used following the Corey-Brooks model, (Corey [10]):

$$k_{ro}(S_o) = (1 - S_o)^2, \qquad k_{rg}(S_o) = 0.6(S_o)^2. \tag{3}$$

The fractional flows holds on the relationship $f_o + f_g = 1$. Thus, the Darcy velocities of oil and gas phases are given by:

$$u_o = uf_o, \qquad u_g = uf_g \tag{4}$$

therefore $u_o + u_g = u$, where u is the Darcy velocity for mixture composte by oil and gas. The fractional flows (see Fig. 1) of oil and gas are given by:

$$f_o(S_o) = \frac{0.0096S_o^2}{0.0096S_o^2 + 1.297(1 - S_o)^2}, \qquad f_g(S_o) = \frac{1.297(1 - S_o)^2}{0.0096S_o^2 + 1.297(1 - S_o)^2}. \tag{5}$$

3 Mathematical model of the in-situ combustion process

The model proposed in this paper is similar to the models presented in Gargar et al. [5] and in de Assis et al. [6]. In this paper we neglect diffusion (capillary) effects. It is assumed one-dimensional flow, with air (oxygen)

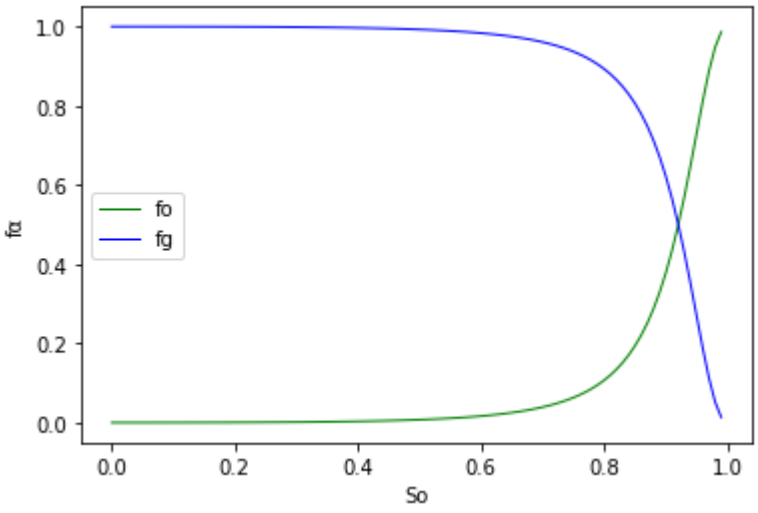

Figure 1. Fractional flow for oil $f_o(S_o)$ and gas $f_g(S_o)$

injected to the left of a porous rock cylinder containing a non-reactive gas and a mobile fuel. The model with time coordinate t and space coordinate x and main variables T (temperature), S_o (molar concentration of oil), Y (molar concentrarion of oxygen) is composed of the equations of heat balance, eq. (6), molar balance for oxygen, eq. (7) and molar balance for oil, eq. (8):

$$C_m \partial_t T + \varphi \partial_x (c_o \rho_o S_o u_o (T - T_{res})) = Q_r Y S_o \rho_o W_r \varphi, \tag{6}$$

$$\varphi \partial_t (Y S_g \rho_g) + \varphi \partial_x (Y S_g \rho_g u_g) = -w \varphi \rho_o Y S_o W_r, \tag{7}$$

$$\varphi \partial_t (\rho_o S_o) + \varphi \partial_x (\rho_o S_o u_o) = -\varphi S_o \rho_o Y W_r, \tag{8}$$

where ω is a stoichiometric coefficient bettwen the mol of gas and the mol of oil. Following de Assis et al. [6] and Gargar et al. [5], the reaction velocity W_r (s^{-1}) is given by Arrhenius law:

$$W_r = k_p \, exp\Big(\frac{-E_r}{RT}\Big), \tag{9}$$

where all parameter name and values are given in Table 1.

3.1 Dimensionless model

For a better analysis of the problem, some dependent and independent dimensionless variables (denoted by bars) were introduced as dimensions and reference quantities (indicated by stars)

$$\bar{t} = \frac{t}{t^*}, \quad \bar{x} = \frac{x}{x^*}, \quad \bar{\theta} = \frac{T - T_{res}}{T^*}, \quad T^* = T_{res} \quad x^* = L_{res}, \quad t^* = \frac{x^*}{u}, \quad \bar{\rho} = \frac{\rho_g}{\rho^*}, \quad \rho^* = \rho_{g_o} \tag{10}$$

Therefore, the system of equations eq. (6), eq. (7) and eq. (8) are written as:

$$\partial_{\bar{t}} \bar{\theta} + a_1 \partial_{\bar{x}} (S_o f_o \bar{\theta}) = b_1 (S_o \frac{S_y}{S_g}) \Phi, \tag{11}$$

$$\partial_{\bar{t}} \Big(\frac{S_y}{\theta + 1}\Big) + a_2 \partial_{\bar{x}} \Big(\frac{S_y f_g}{\theta + 1}\Big) = b_2 (S_o \frac{S_y}{S_g}) \Phi, \tag{12}$$

$$\partial_{\bar{t}} (S_o) + a_3 \partial_{\bar{x}} (S_o f_o) = b_3 (S_o \frac{S_y}{S_g}) \Phi, \tag{13}$$

Table 1. Dimensional parameter values for in situ combustion

Symbol	Physical quantity	Valor	Unit (SI)
T_{res}	Initial reservoir temperature	273	K
C_m	Rock matrix heat capacity	2×10^6	$J/(m^3K)$
Q_r	Enthalpy of immobile fuel at T_{res}	4.4×10^5	J/mol
E_r	Activation energy	58747	J/mol
k_p	Frequency factor for the reaction	4060	$1/s$
R	Gas constant	8.314	$J/(molK)$
φ	Porosity	0.3	
P	Pressure in the reservoir	1013250	Pa
ρ_o	Average molar density of oil	1366.88	mol/m^3
c_o	Molar heat capacity of oil	1097.4	$J/(molK)$
u	Darcy velocity of oil and gas mixture	8.0×10^{-7}	m/s
c_g	Molar heat capacity of gas	27.42	$J/(molK)$
ρ_{g_o}	Densidad molar inicial del gas	401	mol/m^3
L_{res}	Reservoir length	2×10^{-1}	m

where:

$$a_1 = \frac{t^*}{x^*}\frac{\varphi u c_o \rho_o}{C_m}, \quad a_2 = a_3 = \frac{t^*}{x^*}u, \quad b_1 = \frac{t^* k_p}{T^*}\frac{Q_r \rho_o \varphi}{C_m}, \quad b_2 = \frac{-t^* k_p w \rho_o R T^*}{P}, \qquad b_3 = -t^* k_p, \tag{14}$$

$$S_y = Y S_g. \tag{15}$$

4 Analysis of solution by traveling waves

To determine the solution of the proposed model in the form of a traveling wave, we perform a change of variables $(t, x) \to (t, \xi)$, where $\xi = x - ct$, $c > 0$ and assume that the solution in the new (traveling) variables does not depend on t. Defining dimensionless constants:

$$v = \frac{t^*}{x^*}u, \qquad h = \frac{\varphi c_o \rho_o}{C_m}, \qquad \frac{P}{RT^*\rho^*} = \varsigma, \tag{16}$$

omitting the bars in equations eq. (11), eq. (12), eq. (13), and indicating the derivative in ξ as prime ($d_\xi(\cdot) = (\cdot)'$), the system can be written as:

$$-c(\theta)' + vh(S_o f_o \theta)' = b_1(S_o \frac{S_y}{S_g})\Phi, \tag{17}$$

$$-c(\frac{S_y}{\theta+1})' + v(\frac{S_y f_g}{\theta+1})' = b_2(S_o \frac{S_y}{S_g})\Phi, \tag{18}$$

$$-c(S_o)' + v(S_o f_o)' = b_3(S_o \frac{S_y}{S_g})\Phi. \tag{19}$$

In order to determine the equilibrium states of (S_y, S_o), the right side of the equations eq. (17) - eq. (19), must be zero and this implies that:

$$S_o^L = 0, \quad S_y^R = 0. \tag{20}$$

Replacing the eq. (19) into eq. (18), results:

$$\Big[\frac{S_y}{\theta+1}[vf_g - c] + bS_o[vf_o - c]\Big]' = 0, \tag{21}$$

where $b = -\dfrac{b_2}{b_3}$. Applying limit $\xi \to \pm\infty$ to eq. (21) and then isolating c, we obtain the traveling wave velocity:

$$c = \frac{v\Big[Y^L - bS_o^R f_o^R(\theta^L+)\Big]}{Y^L - bS_o^R(\theta^L+1)}. \tag{22}$$

From equation eq. (21) we can obtain:

$$S_y = \frac{\theta+1}{vf_g - c}\Big(\frac{Y^L}{\theta^L+1}[v-c] - bS_o[vf_o - c]\Big). \tag{23}$$

Substituting eq. (19) into eq. (17) implies:

$$[(vhf_oS_o - c)\theta + \bar{b}(vf_o - c)S_o]' = 0, \tag{24}$$

where $\bar{b} = -\dfrac{b_1}{b_3}$. Applying limit $\xi \to \pm\infty$ into eq. (24) we can obtain θ^L and θ:

$$\theta^L = \Big(1 - \frac{vhf_o^R S_o^R}{c}\Big)\theta^R + \bar{b}S_o^R\Big(1 - \frac{vf_o^R}{c}\Big), \tag{25}$$

$$\theta = \frac{\bar{b}cS_o - \bar{b}vf_oS_o - c\theta^L}{vhf_oS_o - c}. \tag{26}$$

From eq. (19) it is possible to find an Ordinary Differential Equation (ODE) for $(S_o)'$:

$$S_o' = \frac{b_3S_y\Phi - vf_o'S_g}{S_g(vf_o - c)}S_o =: F(S_o). \tag{27}$$

The traveling wave solution of the system eq. (11)-eq. (13) must satisfy the ODE eq. (27) where θ is defined in eq. (26) and S_y is defined in eq. (23) with assimptotic boundary conditions given by

$$\lim_{\xi\to-\infty}(\theta, S_y, S_o) = (\theta^L, S_y^L, S_o^L), \quad \lim_{\xi\to\infty}(\theta, S_y, S_o) = (\theta^R, S_y^R, S_o^R). \tag{28}$$

Proving the condition eq. (28) can be really challenging even for much simpler models, (Chapiro et al. [2], Ghazaryan et al. [4]). We proceed by analysing a particular case in the next section.

4.1 Particular case analysis

As the starting analysis we consider the boundary states:

$$Y^L = 1, \qquad \theta^R = 0, \qquad S_o^R = 0.9. \tag{29}$$

As we proved that $S_o^L = 0$ and $S_y^R = 0$, then from eq. (25) we calculate θ^L and the velocity for traveling wave c:

$$\theta^L = 0.0826, \qquad c = 0.523. \tag{30}$$

For this case the phase portrait of eq. (27) is one-dimensional and depends exclusively of the sign of the flux function $F(S_o)$, which is plotted in Figure 2. As one can see it is strictly positive.

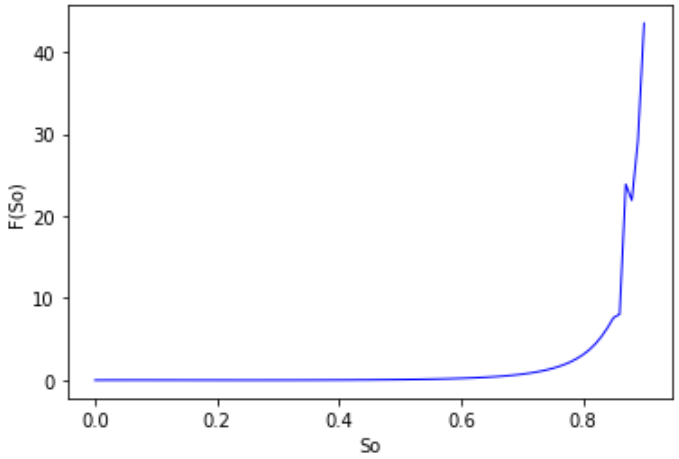

Figure 2. Flow function $F(S_o)$ for the paramenter values given in Table 1.

As we considered that $S_o^L < S_o^R$, the phase portrait of eq. (27) is simple (see Fig. 2) and indicates the existence of the traveling wave connection joining the limit states S_o^L and S_o^R.

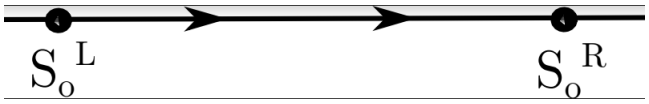

Figure 3. Phase portrait of the system eq. (27), where θ is defined in eq. (26) and S_y is defined in eq. (23).

5 Conclusions

This paper proposes a model of the in-situ combustion which generalizes the ones presented in the literature by considers the molar density of gas dependend of temperature (compressibility) and realistic fractional flow functions. Using the traveling wave hypothesis, the original system was rewritten as a system of ODEs. After some manipulations we managed to ontain the traveling wave velocity and reduce the system to one contaning two algebraic equations and one ODE. We managed to prove the existence and uniqueness of the traveling wave solution for one set of the boundary conditions using semi-analytical approach. The analysis of the general solution is part of the ongoing work.

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