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Investigating Satellite Attitude and Orbit Control System Performance of the SDRE Technique regarding Parametric Uncertainty

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Abstract. The satellite attitude and orbit control subsystem (AOCS) can be designed with success by linear control theory if the satellite has slow angular motions. However, for fast maneuvers, the linearized models are not able to represent all the perturbations due to the effects of the nonlinear terms present in the dynamics which compromises the system's performance. Therefore, in such cases, it is expected that nonlinear control techniques yield better performance than the linear control techniques, improving the AOCS pointing accuracy without requiring a new set of sensors and actuators. Nonetheless, these nonlinear control techniques can be more sensitive to parametric uncertainties. One candidate technique for the design of AOCS control law under a fast maneuver is the State-Dependent Riccati Equation (SDRE). SDRE provides an effective algorithm for synthesizing nonlinear feedback control by allowing nonlinearities in the system states while offering great design flexibility through state-dependent weighting matrices. The Brazilian National Institute for Space Research (INPE, in Portuguese) was demanded by the Brazilian government to build remote-sensing satellites, such as the Amazonia-1 mission. In such missions, the AOCS must stabilize the satellite in three-axes so that the optical payload can point to the desired target. Currently, the control laws of AOCS are designed and analyzed using linear control techniques in commercial software. Although elsewhere the application of the SDRE technique with opensource software has shown to yield better performance for the missions developed by INPE, a subsequent important question is whether such better performance is robust to parametric uncertainties. In this paper, we investigate whether the application of the SDRE technique in the AOCS is robust to parametric uncertainties in the missions developed by INPE. The initial results showed that SDRE controller is robust to $\pm 20\%$, at least, variations in the inertia tensor of the satellite.

Keywords: SDRE, Nonlinear, Parametric, Structured, Uncertainty

1 Introduction

The design of a satellite attitude and orbit control subsystem (AOCS) that involves plant uncertainties, large angle maneuvers and fast attitude control following a stringent pointing, requires nonlinear control methods in order to satisfy performance and robustness requirements. An example is a typical mission of the Brazilian National Institute for Space Research (INPE), in which the AOCS must stabilize a satellite in three-axes so that the optical payload can point to the desired target with few arcsecs of pointing accuracy.

One candidate method for a nonlinear AOCS control law is the State-Dependent Riccati Equation (SDRE) method, originally proposed by [1] and then explored in detail by [2–4]. SDRE provides an effective algorithm for synthesizing nonlinear feedback control by allowing nonlinearities in the system states while offering great design flexibility through state-dependent weighting matrices. The SDRE can be considered as the nonlinear counterpart of linear-quadratic regulator (LQR) control method [2–4]. SDRE is based on the arrangement of the system model in a form known as state-dependent coefficient (SDC) matrices.

Accordingly, a suboptimal control law is carried out by a real-time solution of an algebraic Riccati equation (ARE) using the SDC matrices by means of a numerical algorithm. Therefore, SDRE linearizes the plant about the instantaneous point of operation and produces a constant state-space model of the system. The process is repeated in the next sampling steps, producing and controlling several state dependent linear models out of a nonlinear one.

Elsewhere, we showed State-Dependent Riccati Equation (SDRE) is a feasible non-linear control technique that can be applied in such CubeSats using Java [5]. Moreover, we showed, through simulation using a Monte Carlo perturbation model, SDRE provides better performance than the PID controller, a linear control technique.

In this paper, we tackle the next fundamental problem: stability. We evaluate stability from two perspectives: (1) parametric uncertainty of the inertia tensor and (2) a Monte Carlo perturbation model based on a uniform attitude probability distribution. Through the combination of these two perspectives, we grasp the stability properties of SDRE in a broader sense. In order to handle the uncertainty appropriately, we combine SDRE with H_∞. The initial results showed that SDRE controller is robust to $\pm 20\%$, at least, variations in the inertia tensor of the satellite.

It is beyond the scope of the present paper the following related topics: orbital dynamics simulation [6–8], attitude estimation based on noisy sensor measurements [6, 7] and real-time implementation concerns of a SDRE controller based on the Java software [9, 10] or other software languages as C [11].

SDRE was originally proposed by [1] and then explored in detail by [4]. A good survey of the SDRE method can be found in [2] and its systematic application to deal with a nonlinear plant in [3]. The SDRE method was applied by [5, 12–17] for controlling a nonlinear system similar to the six-degree of freedom satellite model considered in this paper. [12] defined a simulator using Euler angles based on commercial software, whereas, [13] applied quaternions on commercial software. [16] applied the SDRE as a filter technique together with a PID controller. [5, 14, 15] showed, through simulation applying opensource software, using a Monte Carlo perturbation model, SDRE based on quaternions provides better performance than the PID controller.

The application of SDRE method, and, consequently, the ARE problem that arises, have already been studied in the available literature, e.g., [11] investigated the approaches for the ARE solving as well as the resource requirements for such online solving. Recently, [13] proposed the usage of differential algebra to reduce the resource requirements for the real-time implementation of SDRE controllers. In fact, the intensive resource requirements for the online ARE solving is the major drawback of SDRE. Nonetheless, the SDRE method has three major advantages: (a) simplicity, (b) numerical tractability and (c) flexibility for the designer, being comparable to the flexibility in the LQR [13]. Taking into account linear techniques such as LQR to control nonlinear spacecraft systems, [18] argued that a linearized spacecraft model that involves three components of the quaternion globally stabilizes the nonlinear system, whereas it locally optimizes the spacecraft performance.

SDRE method can be readily extended to nonlinear H_∞ [4]. The interest in H_∞ optimization for robust control of linear plants is mostly attributed to the influential work of [19], in which the problem of sensitivity reduction by feedback is formulated as an optimization problem. Later, [20] addressed the problem of robustly stabilizing a family of linear systems in the case where such family was characterized by H_∞ bounded perturbations of a normalized left coprime factorization of a nominal system.

This paper is organized as follows. In Section 2, the problem description is presented. In Section 3, the satellite physical modeling in the simulator is reviewed. In Section 4, we explore the state-space models and the controllers. In Section 5, we share simulation results for the satellite and upside-down maneuver. Finally, the conclusions are shared in Section 6.

2 Problem Description

The SDRE technique entails factorization (that is, parametrization) of the nonlinear dynamics into the state vector and the product of a matrix-valued function that depends on the state itself. In doing so, SDRE brings the nonlinear system to a (nonunique) linear structure having SDC matrices given by eq. (1).

$$\begin{aligned} \dot{x} &= A(x)x + B(x)u \\ y &= Cx \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector and $u \in \mathbb{R}^m$ is the control vector. Notice that the SDC form has the same structure as a linear system, but with the system matrices, A and B, being functions of the state vector.

The nonuniqueness of the SDC matrices creates extra degrees of freedom, which can be used to enhance controller performance, however, it poses challenges since not all SDC matrices fulfill the SDRE requirements, e.g., the pair (A,B) must be pointwise stabilizable.

The system model in eq. (1) is subject of the cost functional described in eq. (2).

$$J(x_0, u) = \frac{1}{2}\int_0^\infty (x^T Q(x)x + u^T R(x)u)dt \tag{2}$$

where $Q(x) \in \mathbb{R}^{n \times n}$ and $R(x) \in \mathbb{R}^{m \times m}$ are the state-dependent weighting matrices. In order to ensure local stability, $Q(x)$ is required to be positive semi-definite for all x and $R(x)$ is required to be positive for all x [11].

The SDRE controller linearizes the plant about the current operating point and creates constant state space matrices so that the LQR method can be used. This process is repeated in all samplings steps, resulting in a pointwise linear model from a non-linear model, so that an ARE is solved and a control law is computed also in each step. Therefore, according to LQR theory and eq. (1) and (2), the state-feedback control law in each sampling step is $u = -K(x)x$ and the state-dependent gain $K(x)$ is obtained by eq. (3) [3].

$$K(x) = R^{-1}(x)B^T(x)P(x) \tag{3}$$

where $P(x)$ is the unique, symmetric, positive-definite solution of the algebraic state-dependent Riccati equation (SDRE) given by eq. (4) [3].

$$P(x)A(x) + A^T(x)P(x) - P(x)B(x)R^{-1}(x)B^T(x)P(x) + Q(x) = 0 \tag{4}$$

Considering that eq. (4) is solved in each sampling step, it is reduced to an ARE. Finally, the conditions for the application of the SDRE technique in a given system model are [3]:

1. $A(x) \in C^1(\mathbb{R}^w)$
2. $B(x), C(x), Q(x), R(x) \in C^0(\mathbb{R}^w)$
3. $Q(x)$ is positive semi-definite and $R(x)$ is positive definite
4. $A(x)x \implies A(0)0 = 0$, i.e., the origin is an equilibrium point
5. $pair(A, B)$ is pointwise stabilizable (a sufficient test for stabilizability is to check the rank of controllability matrix)
6. $pair(A, Q^{\frac{1}{2}})$ is pointwise detectable (a sufficient test for detectability is to check the rank of observability matrix)

2.1 SDRE with H_∞

SDRE method can be readily extended to nonlinear H_∞ [4]. Consider the general nonlinear dynamics using SDC as:

$$\begin{aligned} \dot{x} &= A(x)x + B_1(x)w + B_2(x)u \\ z &= C_1(x)x + D_{12}(x)u \\ y &= C_2(x)x + D_{21}(x)u \end{aligned} \tag{5}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control vector, $w \in \mathbb{R}^m$ is the vector of exogenous signals (e.g., disturbances) and $z \in \mathbb{R}^n$ is the vector of "error" signal which is to be minimized in some sense to meet control objectives. Furthermore, the additional functions are $C^0(\mathbb{R}^w)$.

Consider such state-space model, eq. (5), described by a transfer function G. Now consider the stabilization of plant G which has a normalized left coprime factorization [20, 21]:

$$G = M^{-1}N \tag{6}$$

then a perturbed plant model G_p can be written as [21]:

$$G_p = (M + \Delta_M)^{-1}(N + \Delta_N) \tag{7}$$

where Δ_M, Δ_N are stable unknown transfer functions that represent the uncertainty in the nominal plant G.

The objective of robust stabilization H_∞ is to stabilize not only the nominal plant G, but a family of perturbed plants defined by [20, 21]:

$$G_p = \{(M + \Delta_M)^{-1}(N + \Delta_N) :: ||[\Delta_M \ \Delta_N]||_\infty < \epsilon\} \tag{8}$$

where $\epsilon > 0$ is the stability margin. To maximize this stability margin is the problem of H_∞ robust stabilization of normalized coprime factor plant descriptions [20]. For the positive feedback of Fig. 1, the perturbed plant is robustly stabilizable if and only if the nominal feedback is stable and [20, 21]

$$|| \begin{bmatrix} K \\ I \end{bmatrix} (I - GK)^{-1}M^{-1}||_\infty \leq \epsilon^{-1} \tag{9}$$

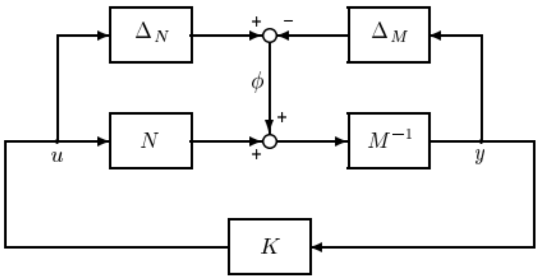


Figure 1. H_∞ robust stabilization problem with left coprime factorization [21].

The maximum stability margin ϵ and the corresponding minimum γ are given as [20]:

$$\gamma_{min} = \epsilon_{max}^{-1} = (1 + \rho(XZ))^{\frac{1}{2}} \tag{10}$$

where ρ denotes the spectral radius (maximum eigenvalue) and for the initial state-space realization Z and X are solutions of AREs.

Z and X are the solutions to the AREs [20, 21]:

$$\begin{aligned} (A - BS^{-1}D^TC)Z + Z(A - BS^{-1}D^TC)^T - ZC^TR^{-1}CZ + BS^{-1}B^T = 0 \\ (A - BS^{-1}D^TC)^Tx + X(A - BS^{-1}D^TC) - XBS^{-1}B^TX + C^TR^{-1}C = 0 \\ R = I + DD^T \\ S = I + D^TD \end{aligned} \tag{11}$$

A controller which guarantees that [20, 21]:

$$|| \begin{bmatrix} K \\ I \end{bmatrix} (I - GK)^{-1}M^{-1}||_\infty \leq \gamma \tag{12}$$

for a specified $\gamma > \gamma_{min}$, is given by:

$$\begin{aligned} K_{H_\infty} = \begin{bmatrix} A + BF + \gamma^2(L^T)^{-1}ZC^T(C + DF) & \gamma^2(L^T)^{-1}ZC^T \\ B^TX & -D^T \end{bmatrix} \\ F = -S^{-1}(D^TC + B^TX) \\ L = (1 - \gamma^2)I + XZ \end{aligned} \tag{13}$$

Therefore, regarding the combination of SDRE and H_∞ the procedure to compute the controller that maximizes the stability margin for the perturbed plants in each step is:

1. Reconstruct the matrices using the SDC form;
2. Solve the two ARES of eq. (11) computing X and Z;
3. Compute γ_{min} using eq. (10);
4. Define a state-space model (A,B,C,D) using X, Z and a $\gamma > \gamma_{min}$ by eq. (13);
5. Solve the third ARE that results from state-space model described by eq. (13), which leads to $P_{K_{H_\infty}}$ as the unique, symmetric, positive-definite solution of such ARE;
6. Compute the controller K for the original system using $K(x) = R^{-1}(x)B_2(x)P_{K_{H_\infty}}(x)$.

It is known that if a controller can be found using that procedure, the exogenous signal will be locally attenuated by γ [4, 20, 21] in each step.

3 Satellite Physical Modeling

The simulator is designed based on a typical mission developed by INPE, in which the AOCS must stabilize a satellite in three-axes so that the optical payload can point to the desired target. Therefore, the satellite model available in the simulator is defined to be a three-axes stabilized attitude-maneuvering satellite, therefore, it is basically a *zero-bias-momentum* system. In the sense that, a major control requirement is to remove the unwanted accumulated angular momentum, consequently, an active control system is needed. Note the existence of angular momentum in the satellite would cause control difficulties when attitude maneuvers in space would be executed since this superfluous momentum would provide the spacecraft with unwanted gyroscopic stability [7].

Since an active control system is required, the simulator uses momentum exchange actuators - they do not change the inertial angular momentum; or, a symmetrical rotating body produces torque when accelerated about its axis of rotation, since such change in the momentum is internal to the satellite, it transfers the momentum change to the satellite with negative sign (*angular momentum is a conservative quantity*) [7]. The type of the momentum exchange actuator used is the reaction wheel, a rotating machine which is commonly applied for very accurate control and for moderately fast maneuvers since it allows continuous and smooth control with the lowest possible disturbing torques [7]. In particular, reaction wheels are often used in satellites that carry optical payloads, as in the previously discussed typical mission of the INPE. The basic technical features of a reaction wheel are: maximum torque, maximum momentum storage capacity (or maximum angular velocity), torque noise and friction [7]. The reaction wheel provided in the simulator models the first two technical features.

Focusing on the sensors, there are two principal types of attitude determination hardware: attitude sensors and angular velocity sensors [7]. The simulator models these two types of sensors: (1) a set of attitude sensors, the set of sun sensors (quite-common on earth-orbiting satellites [7]), and (2) an angular velocity sensor, a gyroscope. The sensors, available in the simulator, are ideal and simplified, in the sense that, they can read the physical quantities at any moment with perfect accuracy and no noise. Additionally, the set of sun sensors provides through the entire simulation the same measure of the solar versor so there is no eclipse, the sun is not moving in the Earth-centered inertial (ECI) reference frame and the sun is always visible by each individual sensor. Indeed, ECI is a quasi-inertial reference frame generally used in AOCS [6, 7].

Next subsections explore the kinematics and the rotational dynamics of the satellite attitude available in the simulator.

3.1 Kinematics

Given the ECI reference frame ($\mathfrak{F}_i$) and the frame defined in the satellite with origin in its centre of mass (the body-fixed frame, $\mathfrak{F}_b$), then a rotation $R \in SO(3)$ ($SO(3)$ is the set of all attitudes of a rigid body described by 3×3 orthogonal matrices whose determinant is one) represented by a unit quaternion $Q = [q_1\ q_2\ q_3 \mid q_4]^T$ can define the attitude of the satellite.

Defining the angular velocity $\vec{\omega} = [\omega_1\, \omega_2\, \omega_3]^T$ of $\mathfrak{F}_b$ with respect to $\mathfrak{F}_i$ measured in the $\mathfrak{F}_b$, the kinematics can be described by eq. (14) [6].

$$\dot{Q} = \frac{1}{2}\Omega(\vec{\omega})Q = \frac{1}{2}\Xi(Q)\vec{\omega}$$

$$\Omega(\vec{\omega}) \triangleq \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \tag{14}$$

$$\Xi(Q) \triangleq \begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix},$$

where the unit quaternion Q satisfies the following identity:

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1 \tag{15}$$

eq. (14) allows the prediction of the satellite's attitude if it is available the initial attitude and the history of the change in the angular velocity ($\dot{Q} = F(\omega, t)$). Nonetheless, it is worthy to mention that although the definition of the unit quaternion is global in the sense that it can represent all attitudes, each physical attitude $R \in SO(3)$ is represented by a pair of unit quaternions $\pm Q \in \mathbf{S}^3$ [22]. This characteristic can produce undesirable effects as *unwind*, in which the trajectories of the closed-loop system start close to the desired attitude and yet travel a large distance before returning to the desired attitude [22].

Another possible derivation of the eq. (14) is using the vector g (Gibbs vector or Rodrigues parameter) as $Q = [g^T|q_4]$.

$$\dot{Q} = -\frac{1}{2}\begin{bmatrix}\omega^{\times}\\ \omega^{T}\end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q_3\end{bmatrix} + \frac{1}{2}q_4\begin{bmatrix}1_{3\times 3}\\ 0\end{bmatrix}\vec{\omega} \tag{16}$$

where $\omega^{\times}$ is the cross-product skew-symetric matrix of ω and 1 is the identity matrix. Note the Gibbs vector is geometrically singular since it is not defined for $180\circ$ of rotation [22], nonetheless, the eq. (16) is global.

3.2 Rotational Dynamics

In order to know the history of the change in the angular velocity, it is necessary to understand the history of the change in the angular acceleration ($\dot{\vec{\omega}} = G(\tau, t)$) of the satellite. According to the Euler-Newton formulation of the rotational motion, angular acceleration is caused by torques, in other words, the change in the angular momentum $\dot{\vec{h}}$ is equal to the net torques $\vec{g}$ applied in the satellite, see eq. (17) (the present subsection is derived based on the centre of mass of the satellite, for the general case, see [6]).

$$\dot{\vec{h}} = \vec{g} \tag{17}$$

The angular momentum is also known as the moment of momentum since it defines the moment of a given momentum $\vec{p}$ ($\vec{p} \triangleq m\vec{v}$) about a given point P_{cm}. See eq. (18), in which r locates a given point p with respect to P_{cm}.

$$\vec{h} = \vec{r} \times \vec{p} \tag{18}$$

Taking into account the motion of the body-fixed frame $\mathfrak{F}_b$ with respect to the ECI $\mathfrak{F}_i$ and the angular velocity $\vec{\omega}$, the derivative of the angular momentum in $\mathfrak{F}_b$ is defined by eq. (19).

$$\dot{\vec{h}} = \vec{g} - \vec{\omega} \times \vec{h} \tag{19}$$

Furthermore, $\dot{\vec{h}} = \vec{I}.\dot{\vec{\omega}}$ and $\vec{h} = \vec{I}.\vec{\omega}$, where $\vec{I}$ is the time-invariant inertia tensor. The combination of this definition and eq. (19) results in eq. (20).

$$\vec{I}.\dot{\vec{\omega}} = \vec{g} - \vec{\omega} \times (\vec{I}.\vec{\omega}) \tag{20}$$

Recall the satellite has a set of 3 reaction wheels, each one aligned with its principal axes of inertia, moreover, such type of actuator, momentum exchange actuators, does not change the angular momentum of the satellite. Consequently, it is mandatory to model their influence in the satellite, in particular, the angular momentum of the satellite is defined by eq. (21).

$$\vec{h} = (\vec{I} - \sum_{n=1}^{3} I_{n,s} a_n a_n^T)\vec{\omega} + \sum_{n=1}^{3} h_{w,n}\vec{a_n} \tag{21}$$

where $I_{n,s}$ is the inertia moment of the reaction wheels in their symmetry axis $\vec{a_n}$, $h_{w,n}$ is the angular momentum of the n reaction wheel about its centre of mass ($h_{w,n} = I_{n,s}a_n^T\omega + I_{n,s}\omega_n$) and ω_n is the angular velocity of the n reaction wheel.

One can define I_b using eq. (22).

$$I_b = \vec{I} - \sum_{n=1}^{3} I_{n,s} a_n a_n^T \tag{22}$$

Using I_b, the motion of the satellite can be modeled by eq. (23) (expanded until the version used in the simulator).

$$\begin{gathered} I_b\dot{\omega} = g_{cm} - \omega^{\times}(I_b\omega + \sum_{n=1}^{3} h_{w,n}\vec{a_n}) - \sum_{n=1}^{3} g_n\vec{a_n} \implies \\ \dot{\omega} = I_b^{-1}g_{cm} - I_b^{-1}\omega^{\times}I_b\omega - I_b^{-1}\omega^{\times}\sum_{n=1}^{3} h_{w,n}\vec{a_n} - I_b^{-1}\sum_{n=1}^{3} g_n\vec{a_n} \end{gathered} \tag{23}$$

where g_{cm} is the net external torque and g_n are the torques generated by the reactions wheels ($\dot{h}_{w,n} = g_n$).

4 Controller Design

In a *zero-bias-momentum* system, there are two dynamics states that must be controlled: (1) the attitude (perhaps described by unit quaternions Q) and (2) its stability ($\dot{Q}$, in other words, the angular velocity ω of the satellite). These high-level requirements lead to the control loop showed in Fig.2.

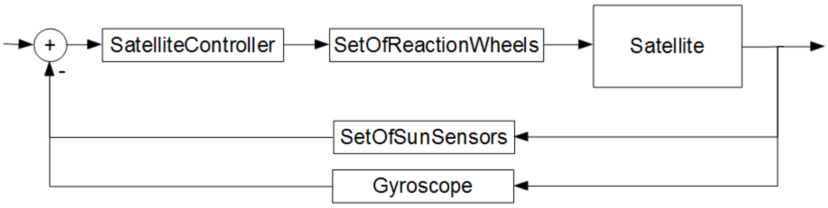


Figure 2. Satellite control.

The state and the control vectors, for the control loop, can be defined by eq. (24).

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} Q \\ \omega \end{bmatrix}$$
$$\begin{bmatrix} u_1 \end{bmatrix} = \begin{bmatrix} T_c = \sum_{n=1}^{3} g_n \vec{a_n} \end{bmatrix} \tag{24}$$

The control regulator problem requires that errors in the attitude and angular velocity must be obtained. The error in the angular velocity is directly obtained from the gyroscope readings, nonetheless, the error in the attitude must be computed. The applied approach to the computation of the error in the attitude is: given two versors, namely (a) the actual Sun versor s_b in the satellite frame obtained by the proper transformation of the Sun versor in the ECI (s_i); and (b) the reference versor in the satellite frame; to compute a rotation (there are many) from the actual Sun versor to the reference versor. The computed rotation can be described by a unit quaternion Q.

The following subsections explore the state-space modeling and the controllers' synthesis.

4.1 Nonlinear Control based on State-Dependent Riccati Equation (SDRE) Controller

For small and slow maneuvers a linear controller can be used, however, for large and fast maneuvers, the linearized equations do not hold and discontinuities compromise the system (e.g., saturation of the actuators) [7]. In order to avoid linearization, the SDRE method is applied. SDRE is based on the arrangement of the system model in a pseudo-linear form given by eq. (1) and then solving an ARE in each sampling step [2].

Assuming that there are no net external torques ($g_{cm} = 0$), the rotational dynamics defined in eq. (23) can be rearranged as defined by eq. (25) (applying the property $v^{\times} w = -w^{\times} v$).

$$\dot{\omega} = (-I_b^{-1} \omega^{\times} I_b + I_b^{-1} (\sum_{n=1}^{3} h_{w,n} a_n)^{\times}) \omega - I_b^{-1} \sum_{n=1}^{3} g_n a_n \tag{25}$$

Taking into account the state and control vectors defined in eq. (24), the state space model can be defined using eq. (14) (Ω) and (25), however, the SDC matrices do not fulfill the SDRE requirements, in particular, the pair (A,B) is not pointwise stabilizable. Another option for the definition of the SDC matrices is to use eq. (14) based on Ξ, once more, the SDC matrices do not fulfill the SDRE requirements, in particular, the pair (A,B) is not pointwise stabilizable.

Table 1. Satellite characteristics, initial conditions and references.

Name	Value
Satellite Characteristics	
inertia tensor ($kg.m^2$)	$\begin{bmatrix} 310.0 & 1.11 & 1.01 \\ 1.11 & 360.0 & -0.35 \\ 1.01 & -0.35 & 530.7 \end{bmatrix}$
Actuators Characteristics - Reaction Wheels	
inertia ($kg.m^2$)	0.01911
inertia tensor of 3 reaction wheels ($kg.m^2$)	$\begin{bmatrix} 0.01911 & 0 & 0 \\ 0 & 0.01911 & 0 \\ 0 & 0 & 0.01911 \end{bmatrix}$
maximum torque ($N.m$)	0.075
maximum angular velocity (RPM)	6000
References for the controller	
solar vector in the body (XYZ)	$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$
angular velocity ($radians/second$, XYZ)	$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$

An alternative option for the definition of the SDC matrices is to use eq. (16), which leads to eq. (26).

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} Q \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \begin{bmatrix} \omega^\times \\ \omega^T \end{bmatrix} & 0 & \begin{bmatrix} \frac{1}{2} q_4 I_{3\times3} \\ 0 \end{bmatrix} \\ 0 & 0 & -I_b^{-1}\omega^\times I_b + I_b^{-1}(\sum_{n=1}^{3} h_{w,n} a_n)^\times \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -I_b^{-1} \end{bmatrix} \begin{bmatrix} u_1 \end{bmatrix} \tag{26}$$

$$\begin{bmatrix} y \end{bmatrix} = I \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Equation (26) has shown to satisfy SDRE conditions described in Section 2, moreover, in it only A is a function of the state vector, consequently, $A(x)$.

4.2 Nonlinear Control based on State-Dependent Riccati Equation (SDRE) with H_∞ Controller

Although the SDRE with H_∞ controller uses the eq. (26), it follows the procedure defined in Subsection 2.1. Such a procedure requires the solving of three AREs in each step, instead of one ARE as usual in the SDRE controller.

5 Simulation Results

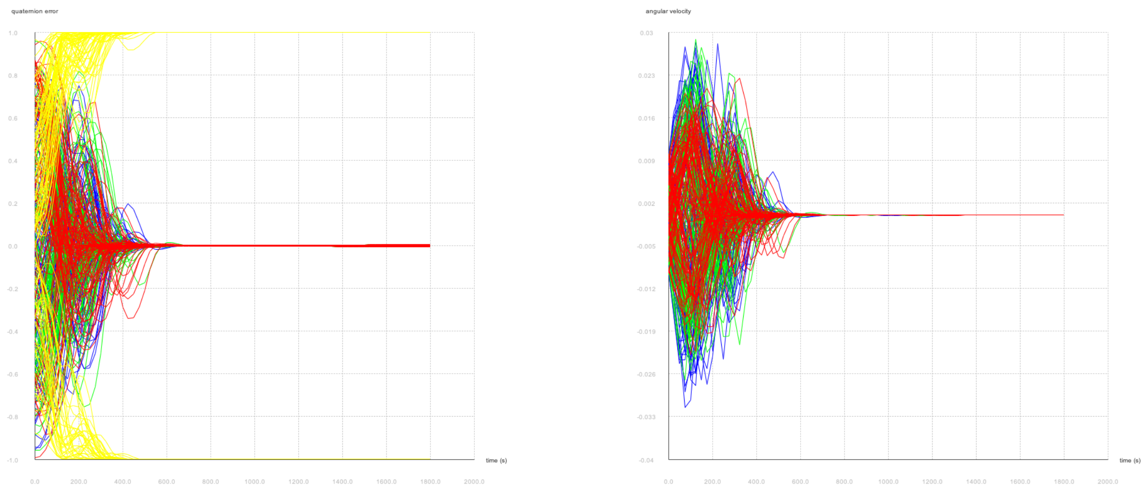
Table 1 shows the satellite characteristics, and references used in the simulation results.

In order to compare the performance of the controllers, a simulation test was conducted with the full Monte Carlo perturbation model described as follows: (1) the initial Euler angles of the nonlinear spacecraft system are randomly selected using independent uniform distributions ($minimum = -180°$, $maximum = 180°$); (2) the initial angular velocity errors are randomly selected using independent uniform distributions ($minimum =$

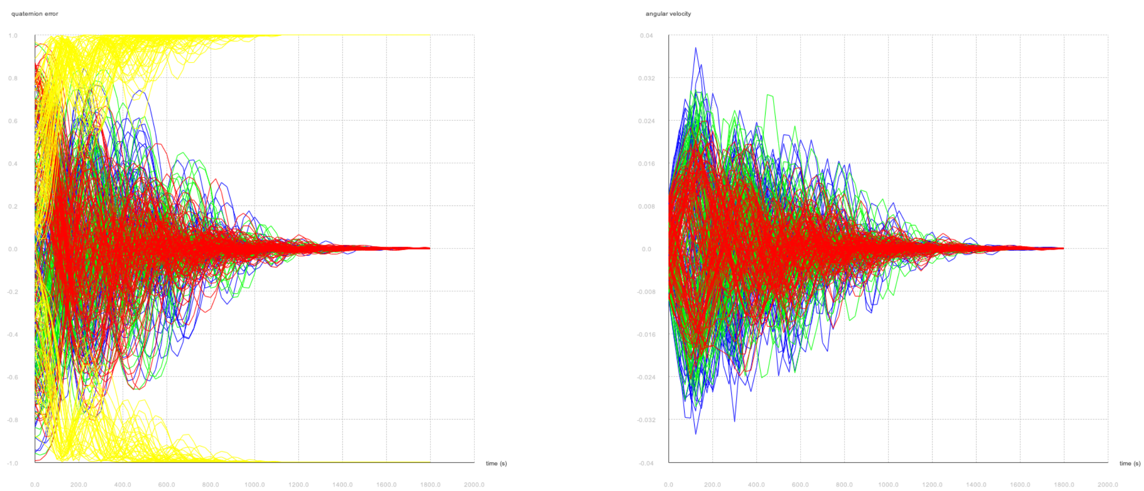
$-0.01\ rad/s$, $maximum = 0.01\ rad/s$); and, (3) each diagonal element of the inertia tensor defined in Table 1 is changed accordingly a uniform distribution $\pm 20\%$.

The Monte Carlo model ran 100 times and in each time one simulation of the controllers were executed. Such executions used simulation time 1800 seconds, fixed step 0.05 seconds, the data presented in Table 1 and the following controllers, both defined by eq. (26) and (3): (1) SDRE controller ($R = 1$ and $Q = 1$) and (2) SDRE with H_∞ controller ($R = 1$ and $Q = 1$).

Each graph in Fig. 3 shows the respective collection of all quaternion errors and angular velocities computed during simulations for a given controller.



(a) SDRE Gibbs.



(b) SDRE Gibbs with H_∞.

Figure 3. Comparison between controllers' performance.

The SDRE with H_∞ controller, Fig.3(b), has the worst performance (regarding time) since it uses more time to stabilize the initial conditions defined by the Monte Carlo perturbation model. Furthermore, the results showed that SDRE controller as well as SDRE with H_∞ are robust to $\pm 20\%$, at least, variations in the inertia tensor of the satellite.

6 Conclusion

As the results are based on analysis through simulations, they are neither valid for general cases nor scenarios out of the range of the Monte Carlo perturbation models due to the underlining nonlinear dynamics.

The SDRE controller as well as SDRE with H_∞ were robust to $\pm 20\%$, at least, variations in the inertia tensor of the satellite Amazonia-1.

The appropriately tackling of uncertainties for nonlinear systems in the context of SDRE and SDRE with H_∞ as applied by this paper is a contribution per se. In fact, this work provides a path for missing rationales of Çimen [2] about the robustness of SDRE.

The approach for SDRE with H_∞ based on left coprime factorizations is also a contribution in the realm of SDRE since the attention was moved to the size of error signals and away from the size and bandwidth of selected closed-loop transfer function [17]. Additionally, left coprime factorizations enable the finding of the suboptimal controller solving exactly three AREs, whereas the literature suggests the γ-iteration in each step in order to solve the general H_∞ problem [4].

Finally, a relevant aspect that would require further investigation is to confirm the observed slower response of SDRE with H_∞ due to its extended capability of disturbance rejection [17].

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