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A continuous-discontinuous strategy to represent the crack process in concrete structures

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Abstract. The study of fracture in quasi-brittle materials such as concrete has significant importance since it is one of the main causes of material failure. There are two numerical approaches to representing fracture: smeared and discrete models, and both techniques have pros and cons. Among the smeared approaches, damage models are used to reproduce the degradation of a continuum media. These models are appropriate for describing the first stages of degradation, identifying damaged regions, and replacing original mechanical properties with damaged ones. This strategy assumes that the cracks are spread over an area known as the fracture process zone. However, this phenomenological approach cannot represent the crack path properly since the discontinuities are not geometrically described. In contrast, the discrete methods are the most indicated to characterize the fracture explicitly. Such methods frequently deal with remeshing, an alternative that has been avoided because of the high computational cost. Although, based on the efficiency of modern computers, it is now possible to evaluate the viability of coupling continuous and discontinuous models to reproduce fracture in its entirety, from nucleation to collapse. In this context, it is proposed a combined strategy that associates nonlocal damage models to represent the smeared aspects of crack propagation with a discrete crack description based on nodal duplication to capture the crack discontinuity. Finally, numerical simulations were performed to analyze the efficiency of this strategy in representing the degradation processes from smeared cracks to geometric discontinuities.

Keywords: Continuous-discontinuous strategy, Concrete crack representation, Nonlocal damage models.

1 Introduction

The prediction of a material collapse is a relevant subject in Engineering. Among the materials adopted in civil construction, the fracture is one of the most common failure modes. To prevent such phenomenon and its consequences, at the beginning of the sec. XX engineers and researchers realized that classical mechanics was not enough to describe the behavior of the quasi-brittle material like concrete. To fulfill this demand, the Fracture Mechanics emerged to study the transition between the continuum and the discrete behavior according to the material degradation (Gopalaratnam et al. [1]).

In order to represent the structural behavior of concrete, Griffith [2] was the pioneer with the Linear Elastic Fracture Mechanic (LEFM). One of the firsts to apply this theory in analyses using the Finite Element Method (FEM) were Ngo and Scordelis [3] and Nilson [4], using the process of crack propagation based on nodal duplication. Besides the simplicity, the discrete methods have been avoided because of the high computational cost of remeshing.

As an alternative, the smeared crack methods appeared with Rashid [5]. This author presents a new way to reproduce degradation, treating the media as a continuum while changes in the mechanical properties of the material represent the crack effects. Another theory that evaluates the degradation as a continuum phenomenon is the Continuum Damage Mechanics (CDM). Initially proposed by Kachanov [6], the CDM quantifies defects and voids using damage variables. Although widely used, smeared models present strain-localization problems as a limiting factor. In this scenario, nonlocal approaches have been developed to overcome such restrictions.

However, it is necessary to associate the smeared and the discrete strategies to represent the fracture process in total, from the initial cracks nucleation to structural collapse. The smeared models are adequate to reproduce the first levels of degradation when the microcracks are dominating. On the other hand, the discrete models are better models to represent macroscopic defects once the geometry discontinuities are explicitly represented. Nowadays, with more powerful computers, it is possible to analyze the viability of associating both theories. Based on this requirement, the present paper proposes an associate model that couples a nonlocal damage model with the nodal duplication strategy, aiming to represent the whole process of fracture.

2 Nonlocal damage model

The volumetric damage model proposed by Penna [7] was chosen to represent the initial stage of fracture. This model adopts different load functions to regions of tensile and compression, separating the volumetric and the deviatoric parcels of strain tensor ε.

The local equivalent strains measures for tensile and compression are given, respectively, by

$$tr^{+}\varepsilon = \bar{\varepsilon}_t^v = \frac{tr\varepsilon + |tr\varepsilon|}{2}; \quad tr^{-}\varepsilon = \bar{\varepsilon}_c^v = \frac{tr\varepsilon - |tr\varepsilon|}{2}. \tag{1}$$

Then, the secant constitutive relation is written as

$$\boldsymbol{E}^s = 2\mu\frac{\partial \mathbf{e}}{\partial \varepsilon} + \left[K_{+}\left[\mathcal{H}(tr\varepsilon)\right]^2 + K_{-}\left[\mathcal{H}(-tr\varepsilon)\right]^2\right]\mathbf{I}\otimes\mathbf{I}, \tag{2}$$

where

$$\mu = \mu_0(1-D_t^v)(1-D_c^v); \quad K_{+} = K^0(1-D_t^v); \quad K_{-} = K^0(1-D_c^v). \tag{3}$$

K^0 is the volumetric modulus, written as $K^0 = \frac{E^0}{3(1-2\nu)}$; $\mathcal{H}(x)$ represents the Heavyside function, with $\mathcal{H}(x) = 1$ para $x > 0$ and $\mathcal{H}(x) = 0$ para $x < 0$; $\mathbf{e}$ is the deviatoric strain tensor; $\mathbf{I}$ is the identity tensor; μ_0 is the shear modulus; D_t^v is the tensile damage; D_c^v is the compressive damage.

The nonlocal strategy adopted to avoid localization problems is the integral formulation proposed by Jirásek [8]. The local strain measure (eq. (1)) is replaced by a nonlocal variable

$$\bar{\epsilon}_k = \sum_{l=1}^{N_{PG}} \left(w_l J_l \alpha_{kl} \epsilon_l\right), \tag{4}$$

where w_l is the weight attributed to the Gauss point l, J_l is the Jacobian of the finite element in this point, and α_{kl} is the weight of the interaction between the points k and l. N_{PG} indicates the total number of Gauss points of the model. It is essential to highlight that α_{kl} is not null only for points whose distance is equal to or smaller than the nonlocal ratio ℓ (eq. (5)).

$$\alpha_{kl} = \frac{\alpha_0 \left\|\mathbf{x_k} - \mathbf{x_l}\right\|}{\sum_{m=1}^{N_{PG}} \left(w_m J_m \alpha_0 \left(\left\|\mathbf{x_k} - \mathbf{x_m}\right\|\right)\right)}. \tag{5}$$

The variable α_0 indicates the distribution function of the nonlocal domain. In the present work, the Gaussian function is adopted (eq. (6)), where k is responsible for the function shape.

$$\alpha(x) = \exp\left(-k\frac{x^2}{\ell^2}\right). \tag{6}$$

3 Nodal duplication model

A nodal duplication algorithm was developed to deal with remeshing without many interventions in the original mes. Such methodology is less computationally onerous than the ones that are based on adaptive refinement.

An algorithm was created to go through all the model nodes, evaluating the tensile damage in which node. Once the tensile damage variable reaches a crack limit previously defined, the nodal duplication algorithm is triggered.

The nodal duplication creates a copy of the original node and updates the incidence of the elements which contain the original node. Using the local crack system as a reference, the elements on the left of the original node do not have their incidence changed, while the elements on the right have the original node replaced by its copy. Then, the geometric discontinuity is inserted into the mesh (Fig. 1). The crack local system definition is based on tracking the maximum damage value in the nodes connected with the duplicated node.

The element erosion is an strategy that remove elements that do not contribute to the model stiffness from the original mesh because they present a tensile degradation bigger than the maximum accepted. Such strategy was also implemented as an optional resource of the nodal duplication process.

This whole procedure is summarized in the code of algorithm 1.

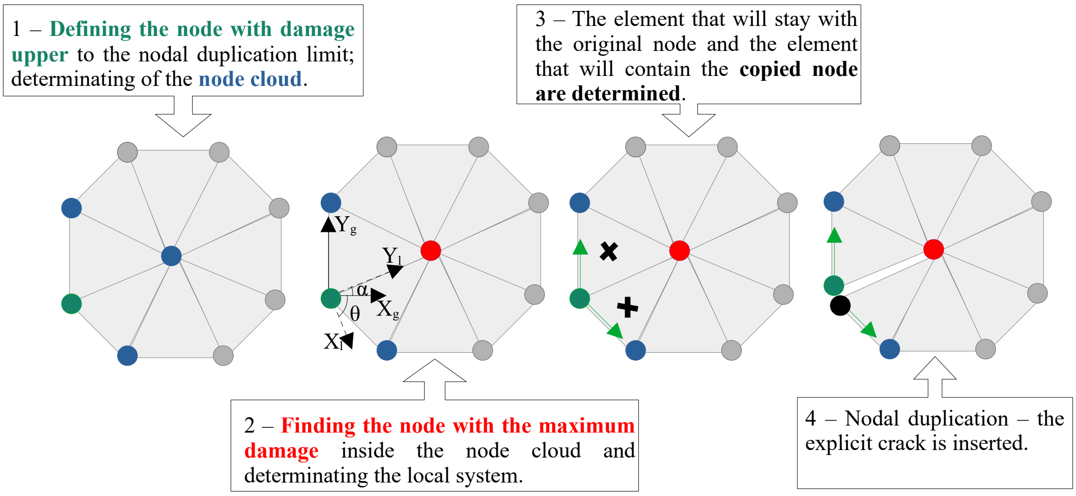

Figure 1. Nodal duplication process.

Algorithm 1 Nodal duplication.

Require: D; E; Model ▷ D - duplication limit; E - erosion limit
Ensure: Updated mesh
$i = 1$
while $i \leq$ nºElements **do** ▷ Loop: elements list
  elm$_i$ ▷ Current element
  D_i ▷ Damage average in elm$_i$
  if $D_i \geq$ E **then**
    element remotion ▷ Remove elements without stiffness
  end if
  $i \leftarrow i + 1$
end while
$j = 1$
while $j \leq$ nºNodes **do** ▷ Loop: node list
  node$_j$ ▷ Current node
  if NodeRestriction$_j$ $==$ false **then** ▷ Evaluate if the node$_j$ is restrained
    if $D_j \geq$ D **then**
      $l = 1$
      while $l \leq$ nºElmCloud **do** ▷ Loop: element cloud of a node$_j$
        elm$_l$ ▷ Current element connected to node$_j$
        NodeD_{max} ▷ Node into the cloud with the major value of damage
        θ ▷ Local system (LS) angle
        if xelm$_l$ $>$ xnode$_j$ **then** ▷ Analyze if elm$_l$ is on the right of node$_j$ in LS
          node'$_j$ ▷ New node: copy of node$_j$
          IncidenceElm$_l$ ▷ Replace node$_j$ by node'$_j$
        end if
        $l \leftarrow l + 1$
      end while
    end if
  end if
  $j \leftarrow j + 1$
end while
Update the node list

4 Numerical simulations

The proposed model has been implemented in the INteractive Structural ANalysis Environment (INSANE). The INSANE is an open-source code developed at the Department of Structural Engineering of the Federal University of Minas Gerais (`https://www.insane.dees.ufmg.br/en/home/`). Four numerical simulations were performed to analyze the response of the continuous-discontinuous model using the L-shaped panel experimentally tested by Winkler et al. [9, 10] as a reference. The geometry of this structure and the crack pattern found by Winkler are represented in Fig. 2. Two different meshes were adopted to study the mesh sensitivity of the nodal duplication process(Fig. 3). The first one is composed of 4-node quadrilateral finite elements, and the second is composed of 6-node triangular finite elements and a higher level of refinement.

The tests performed by Winkler et al. [9, 10] provide the material parameters: $E_0 = 25850(\pm 1381)$ N/mm^2, $f_t = 2.7$ N/mm^2, $f_c = 31.0$ N/mm^2, $G_c = 0.065$ N/mm and $\nu = 0.18$. For a better representation of the experimental curve, the numerical simulations of this current section use the minimum Young modulus verified by Winkler, given as $E_0 = 24469$ N/mm^2. The concrete adopted in the experiments was made with a coarse aggregate with the maximum diameter of $d_{max} = 8$ mm. Considering the characteristic length of the material between $3 \times d_{max}$ and $5 \times d_{max}$, it was determined as $h = 28$mm.

The selected constitutive model was the nonlocal volumetric damage model with a polynomial damage law. The parameters were determined based on the parametrization presented by Penna [7], whose values are specified in Table 1. The Gaussian weight function (eq. (6)) was applied, with the local ratio of $\ell = 12$ mm, and the constant $k = 1.0$.

For the solution process was adopted the direct displacement control method proposed by Batoz and Dhat [11] and secant equilibrium. The vertical displacement of the node highlighted in Fig. 2 was controlled, using the increment of 0.001 mm and convergence tolerance in the displacement of $1,0 \times 10^{-4}$.

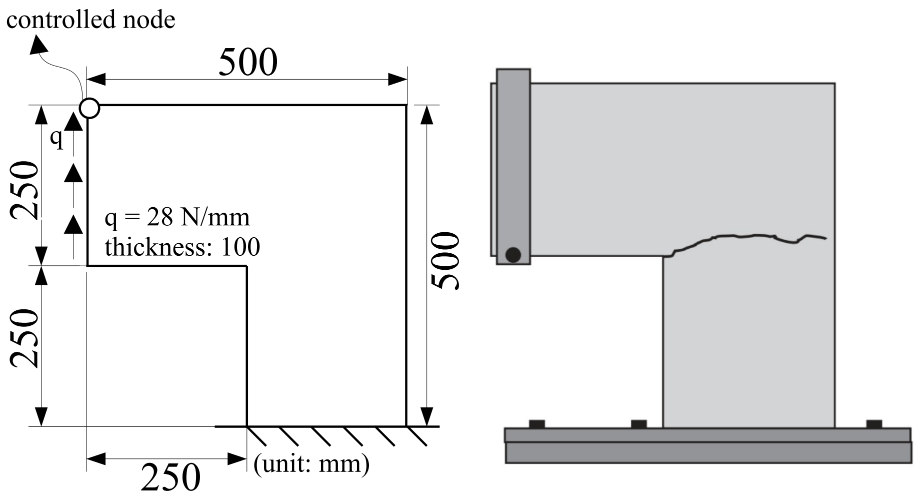

Figure 2. L-shaped panel: geometry and experimental crack pattern (Adapted from Winkler et al. [10]).

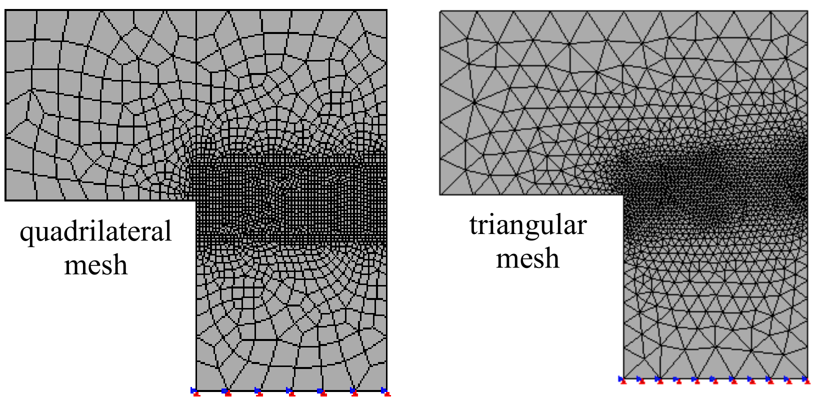

Figure 3. L-shaped panel: proposed meshes.

Table 1. Polynomial damage law parameters

Tensile	Compression
$f_e = 1.43$ N/mm^2	$f_e = 16.0$ N/mm^2
$\kappa_0 = 0.000215$	$\kappa_0 = 0.0022$
$\tilde{E} = 13463.0$ N/mm^2	$\tilde{E} = 13463.0$ N/mm^2

4.1 First analysis: nodal duplication

The first analysis evaluates how the continuous-discontinuous model performs when only the nodal duplication is activated, without element erosion. The damage crack limit was defined as $D = 0.9$. As shown in Fig. 4, when the degradation starts to assume significant values, the numerical results diverge from the experimental curve. It is possible to verify a stiffening in the softening branch of both meshes. This response can be explained by the deformed shape of the panel, illustrated in Fig. 5, where a group of elements intercept the crack propagation path and deviate the crack in a different direction from the one observed experimentally.

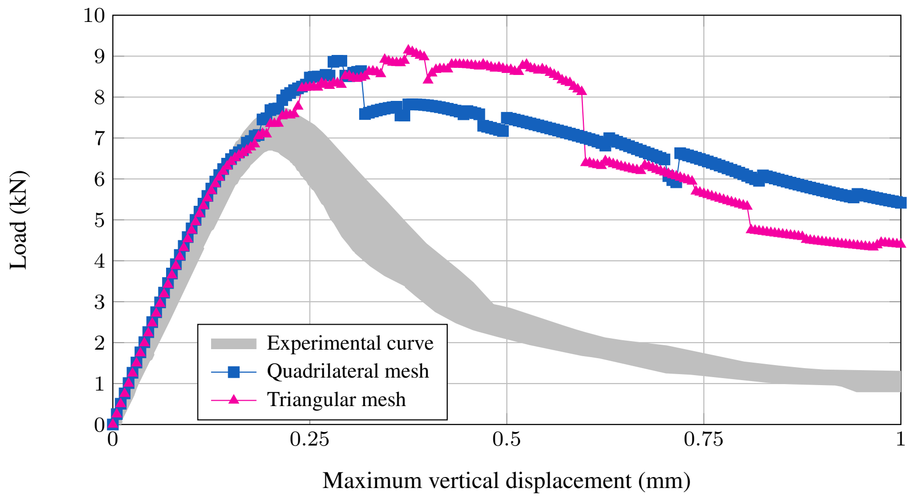

Figure 4. L-shaped panel: nodal duplication with crack limit $D = 0.9$.

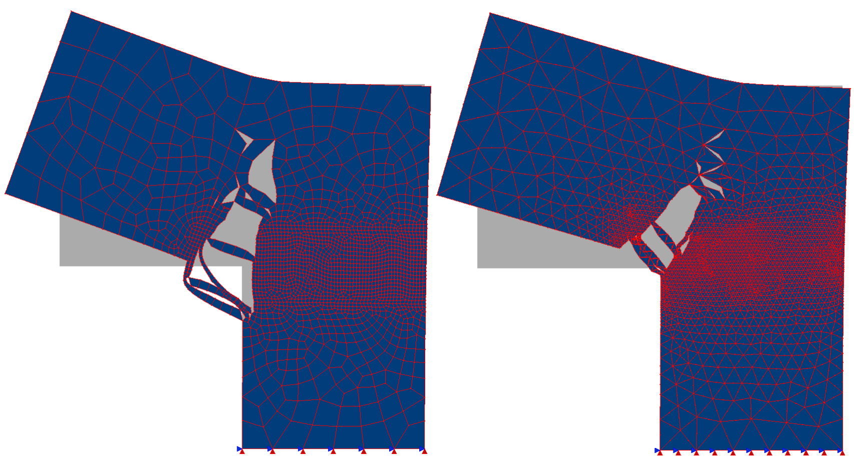

Figure 5. Deformed configuration - crack limit $D = 0.9$ without erosion.

4.2 Second analysis: element erosion

The element erosion was introduced in the analyses to remove the elements with a high level of degradation, using the whole code presented in algorithm 1. In this case, the erosion limit was established as $E = D = 0.9$.

Fig. 6, Fig. 7, and Fig. 8 represent the improvements related to the element erosion. The stiffening in the softening branch was corrected, and so the crack propagation path.

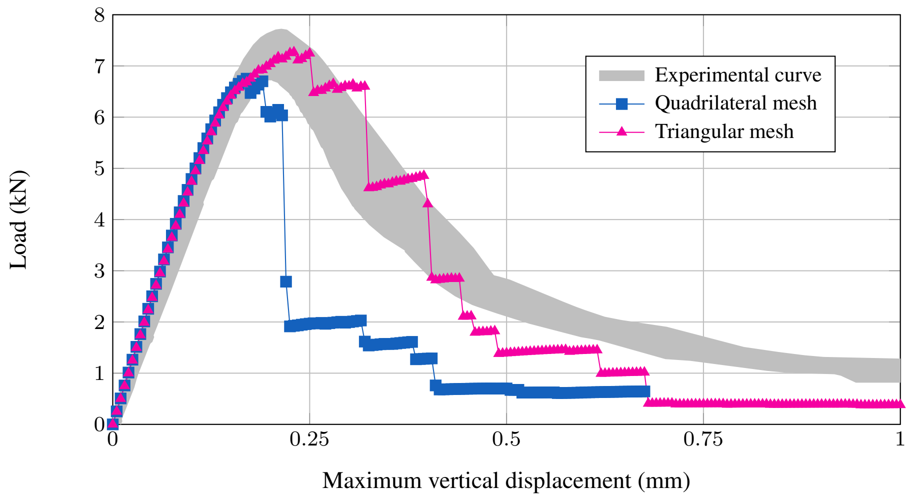

Figure 6. L-shaped panel: nodal duplication with crack limit and erosion limit $D = E = 0.9$.

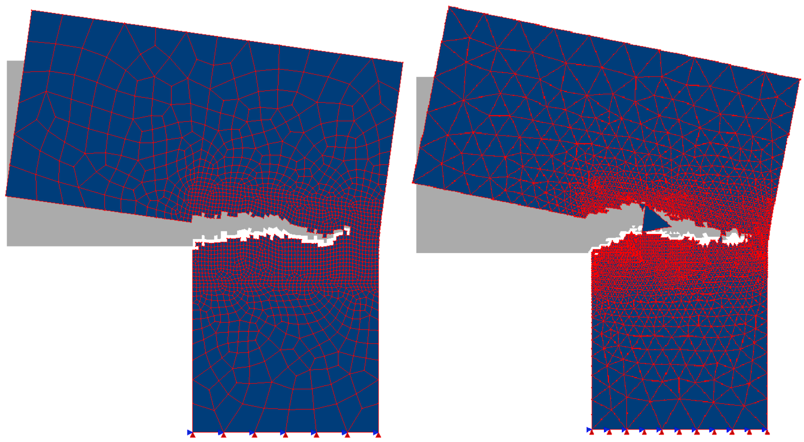

Figure 7. Deformed configuration - crack and erosion limits $D = E = 0.9$.

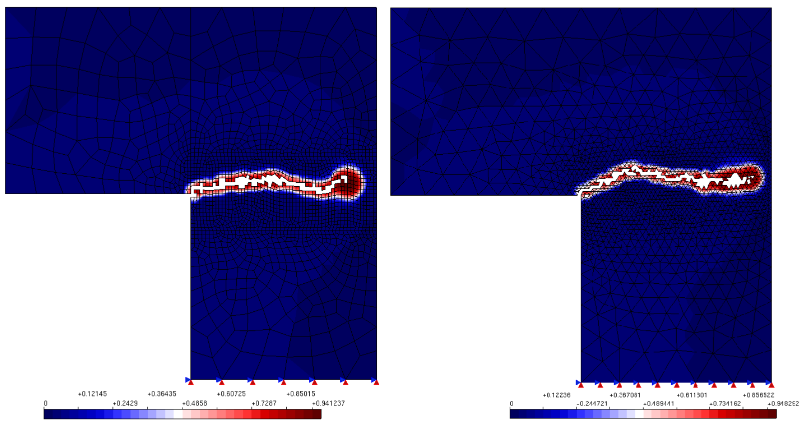

Figure 8. Damage distribution - crack and erosion limits $D = E = 0.9$.

It is necessary to highlight that the agreement between the experimental curve and the numerical simulation in Fig. 6 is limited because a discrete method was adopted to insert the geometric discontinuity. As a consequence, we have a more abrupt softening observed in the numerical curves. It is intended to study different values of D

and E and perform analyses including cohesive forces to correct this phenomenon. It is also evident that the mesh refinement interferes with the proposed model response.

Besides this disparity, the qualitative description of the crack propagation (Fig. 7) fits the experimental pattern with great coherence. The damage spectrum presented in Fig. 8 follows such pattern either. Even using a nonlocal model, coupled with the discrete crack method, the damage did not spread along with the crack neighborhood.

5 Conclusions

This research proposed a continuous-discontinuous model. Based on the results, the main conclusions are:

- The association between the nonlocal damage model and the discrete crack method performed well, and both worked satisfactorily to represent the continuum and the discontinuous material behavior, respectively;
- The nodal duplication, alone, can not represent a discrete crack properly because of stiffening;
- Good results were achieved by associating the nodal duplication with the element erosion;
- The representation of the equilibrium path needs to be improved since the discrete method accelerates the degradation process;
- There are evident signs of mesh dependence in the numerical equilibrium paths. However, the deformed shape of the structure did not present a relevant difference for the meshes analyzed.

Further studies will include:

- Improvement of the nodal duplication technique;
- Analyses of different values of D and E;
- Adoption of other parameters to control the crack propagation, besides the damage;
- Develop a continuous-discontinuous model including cohesive forces.

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