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Effects of epistemic uncertainties on truss topology optimization considering progressive collapse

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Abstract. The history of engineering contains many examples of structural failures. Despite being related to diverse causes, these collapses can be attributed to the existence of uncertainties, which are usually classified as aleatory and epistemic. In this context, optimization techniques can be employed in order to obtain optimal structural solutions that are robust to the effects of uncertainty. Additionally, the progressive collapse phenomenon has raised engineers' and researchers' awareness in recent years. However, there are still very few papers addressing the optimal structural design under uncertainty considering progressive collapse. Hence, this paper aims to investigate the effect of aleatory and epistemic uncertainties on truss topology optimization considering progressive collapse. Uncertainties are considered in the optimization problems through the RBDO (Reliability-Based Design Optimization) and RO (Risk Optimization) formulations. Non-structural factors, which are epistemic in nature and can lead to progressive collapse, are considered using a formulation based on the latent failure probability concept. Through a simple six-bar truss problem, the huge impact of epistemic uncertainties on optimal topologies is shown. The variation of the latent failure probability indicates the existence of two transition points in the optimal solutions, named Hyperstatic and Redundancy Thresholds. We conclude that these bounds are mainly controlled by the magnitude of epistemic uncertainties, having a strong effect on the reliability and costs of the optimal solutions. These results reveal something that has already been recognized in practice: engineering structures need to be redundant in order to cope with the effect of epistemic uncertainties. Therefore, despite being an idealized concept, the latent failure probability proves to be a simple tool to impose minimal redundancy in optimal structural solutions.

Keywords: structural reliability, progressive collapse, latent failure probability, risk optimization.

1 Introduction

The occurrence of events that led to large structural collapses, such as the Ronan Point Tower accident and the terrorist attack at the World Trade Center, has raised engineers' and researchers' awareness of the importance of robust design with respect to the progressive collapse phenomenon [1]. Although several studies on progressive collapse have been developed in the last two decades, as reported in Adam *et al.* [1], the study of optimal design under uncertainty with objective consideration of this phenomenon is still recent [2-4].

The formulations used to address uncertainties in optimization problems include Reliability-Based Design Optimization (RBDO) and Risk Optimization (RO). Typically, these formulations address objective, aleatory uncertainties. Epistemic uncertainty is commonly handled through possibilistic approaches, which employ set theory, intervals, fuzzy sets and fuzzy probabilities [5]. However, such formulations are considered of limited usefulness because, despite being appropriate to handle epistemic uncertainties arising from ignorance and vagueness, they are not ideal to take into account gross errors in design, human errors and operational abuse. These non-structural factors are epistemic in nature and may lead to the progressive collapse of structural systems.

In this paper, we investigate the effects of aleatory and epistemic uncertainty in truss topology optimization considering progressive collapse. Given the limitations of possibilistic approaches, epistemic uncertainty associated to non-structural factors such as gross errors in design and manufacturing, human errors, unpredicted load conditions and operational abuse is considered through formulations based on latent failure probability concept [3]. Through a simple six-bar truss example, the huge impact of epistemic uncertainty in reliability-based and risk-based optimization is shown.

2 Formulation

2.1 Systems Reliability

Let $\mathbf{X}$ and $\mathbf{d}$ be, respectively, the vectors of the random and design variables of a structural system. Additionally, let $g_i(\mathbf{X},\mathbf{d})$ be the limit state function associated with the ith failure mode or with the ith element. Analyzing the progressive collapse of hyperstatic systems involves the definition of the possible failure sequences, including the load redistribution after the failure of each element. Hence, the probability of failure for a typical structural system is given by:

$$p_{f_{SYS}}(\mathbf{d}) = P\left[\mathbf{X} \in \Omega_{fsys}\right] = \int_{\Omega_{f_{SYS}}} f_{\mathbf{X}}(\mathbf{x})\, d\boldsymbol{x} \tag{1}$$

where $P[.]$ is the probability operator, $f_{\mathbf{X}}(\boldsymbol{x})$ is the joint probability density function and $\Omega_{f_{SYS}}(\mathbf{d})$ is the system failure domain, written as:

$$\Omega_{f_{SYS}}(\mathbf{d}) = \left\{\mathbf{X} \middle| \cup_k \left[\cap_{\, i \,\in\, C_k} (\mathrm{g}_i(\mathbf{X},\mathbf{d}) \leq 0)\right]\right\} \tag{2}$$

where C_k represents the kth failure sequence and $\mathrm{g}_i(\mathbf{X},\mathbf{d}) \leq 0$ denotes the ith event that composes the failure sequence.

In this study, the probability of failure given in eq. (1) is evaluated using the Monte Carlo method with Stratified Sampling, or simply Stratified Sampling Monte Carlo (SSMC) [6].

2.2 Truss Topology Optimization: RBDO and RO formulations

The latent failure probability concept is employed in RBDO and RO formulations of truss topology optimization problem in order to take into account the impact of epistemic uncertainty. This concept was introduced by Beck (2020) [3] to represent the effects of non-structural factors that may affect system performance. According to the author, any structural design or structural optimization analysis should start with a risk analysis, addressing the location of the structure and its surrounding environment, all possible threats and load cases, the reliability of the manufacturing process, and so on. Many of these factors represent uncertainties of epistemic nature. Beck (2020) [3] postulated that the "environment" would contribute with a fixed probability for each structural element, and/or for the whole structure. This latent failure probability, due to its epistemic nature, does not depend on usual design variables, such as cross-sectional areas or number of elements, but it depends on the above-mentioned non-structural factors.

The RBDO formulation for the minimum weight truss topology optimization problem addressed in this study, considering p_L, is given by:

$$\begin{gathered} \text{given } p_L, \text{ find } \mathbf{d^*} \text{ which minimizes } W(\mathbf{d}) = \textstyle\sum_{i=1}^{n} \rho_i A_i L_i \\ \text{subjected to: } p_{f_{SYS}}(\mathbf{d},\mathbf{X},p_L) \leq p_{fT_{SYS}} \\ A_i^{min} \leq A_i \leq A_i^{max},\ i = 1, \dots, n \end{gathered} \tag{3}$$

where: $\mathbf{d} = \{A_1, A_2, \dots, A_n\}$ is the design vector, containing the cross-sectional areas of all elements; $W(\mathbf{d})$ is the total structural weight; ρ_i, A_i e L_i are, respectively, the specific mass, the cross-sectional area and the length of the i^{th} bar; n is the number of bars; A_i^{min} and A_i^{max} are the lower and upper bounds of the design variables; $\mathcal{R} = (1 - p_{fT_{SYS}})$ is the target system reliability. In conventional RBDO formulation, there is no latent failure probability, or simply $p_L = 0$. This is equivalent to assuming that there are no epistemic uncertainties.

Structures resulting from RBDO are optimal in mechanical sense and respect minimal specified reliability targets. However, the balance between cost and safety is not addressed by this formulation. When the objective is to find the optimal balance between economy and safety, the costs over the life-cycle of the structure must be taken into account. The risk-based formulation makes it possible through the definition of the following total expected cost function $C_{ET}(\mathbf{d},\mathbf{X})$:

$$C_{ET}(\mathbf{d},\mathbf{X}) = C_{construction}(\mathbf{d}) + C_{operation}(\mathbf{d}) + C_{insp.\&maint}(\mathbf{d}) + C_{disposal}(\mathbf{d}) + C_{EF}(\mathbf{d},\mathbf{X}) \quad (4)$$

In this work, only construction costs and expected costs of failure are considered. In order to differentiate failure consequences in progressive collapse analysis, the expected costs of failure are given by:

$$C_{EF}(\mathbf{d},\mathbf{X}) = C_{HF}p_{HF}(\mathbf{d},\mathbf{X}) + C_{PC}p_{PC}(\mathbf{d},\mathbf{X}) + C_{DF}p_{DF}(\mathbf{d},\mathbf{X}) \quad (5)$$

where C_{HF} and p_{HF} are the cost and probability of hyperstatic failures, which do not lead to global collapse; C_{PC} and p_{PC} are the cost and probability of progressive collapse failures; and C_{DF} and p_{DF} are the cost and probability of direct collapse failures. Thus, for the problems addressed in this paper, the total expected cost is:

$$C_{ET}(\mathbf{d},\mathbf{X}) = C_{construction}(\mathbf{d}) + C_{HF}p_{HF}(\mathbf{d},\mathbf{X}) + C_{PC}p_{PC}(\mathbf{d},\mathbf{X}) + C_{DF}p_{DF}(\mathbf{d},\mathbf{X}) \quad (6)$$

Construction cost is assumed proportional to the structural mass: $C_{construction}(\mathbf{d}) = \sum_{i=1}^{n} \rho_i A_i L_i$. For isostatic structures, C_{HF} and C_{PC} are zero. For hyperstatic structures, C_{HF} is assumed proportional to the mass of the g bars of greater mass, where g is the hyperstatic degree of the structure. This represents the cost of substitution for the damaged bars. A factor k_{HF} is introduced to represent the consequences of hyperstatic bar failures, such that: $C_{HF} = k_{HF}\sum_{i=1}^{g} \rho_i A_i L_i$. Also, an important distinction must be made between the costs of the direct collapse and the costs of the progressive collapse of a hyperstatic structure. In theory, when progressive collapse is ductile, the primary element failure provides a warning for the structure to be evacuated and/or its use to be interrupted. Hence, consequences of failure are reduced, in comparison to direct collapse. With such reasoning, global collapse cost terms C_{PC} and C_{DF} are written as $C_{PC} = k\ C_{REF}$ and $C_{DF} = \alpha\ k\ C_{REF}$, where C_{REF} is a reference cost; k is a multiplier for consequences of progressive collapse leading to global collapse; α is another multiplier to account for global collapse with no warning, such that $\alpha = C_{DF}/C_{PC} \geq 1$.

Finally, the RO formulation for the topology optimization problems addressed herein is:

$$\begin{aligned} &\text{given } p_L, \text{ find } \mathbf{d^*} \text{ which minimizes } C_{ET}(\mathbf{d},\mathbf{X},p_L) \\ &\text{subjected to: } A_i^{min} \leq A_i \leq A_i^{max},\ i = 1,\dots,n \end{aligned} \quad (7)$$

In a conventional risk-based formulation, the latent failure probability is simply $p_L = 0$.

2.3 Progressive collapse evaluation

As stated in the previous section, a latent failure probability p_L is introduced in order to represent the effects of epistemic uncertainties. In this paper, the same value of p_L is considered for all bars of the truss. Also, it is important to note that the actual epistemic cause of a bar failure is irrelevant; it is enough to assume that there is a fixed probability p_L that every bar of the truss may fail, losing its load capacity.

In the ground structure approach employed in this study, there is a possibility that isostatic and hyperstatic candidate solutions appear during the optimization process. For hyperstatic candidate solutions, progressive collapse needs to be considered. In order to clearly define and formulate the progressive collapse problem, it is considered that the truss is subject to $g+1$ load applications, where g is the degree of static indeterminacy. In the present formulation, the probability of a bar failing due to epistemic reasons is considered only in the first load application. If $g+1$ bars fail, we have a direct collapse. If the number of epistemic bar failures is g or less, there is a probability that any bar fails due to aleatory uncertainty in loads and bar strengths. If a bar fails, it is removed, load redistribution is considered, and so on, until $g+1$ bars fail, and equilibrium can no longer be warranted.

In the following formulation, only trusses with a maximum hyperstatic degree of $g = 1$ are considered. Unfortunately, the formulation for $g > 1$ becomes quite complicated. Let F_i be the event "Failure of bar i" and $P[F_i]$ be the probability of such an event. The system failure probability for a hyperstatic truss with n bars is:

$$\begin{aligned} p_{fsys}(\mathbf{d},\mathbf{X},p_L) = (1-p_L)^n \Big(\textstyle\sum_{i=1,i\notin iso}^{n} P[F_i] P\big[\bigcup_{j=1,j\neq i}^{n} F_j | F_i\big] + \sum_{i=1,i\in iso}^{n} P[F_i] + \\ P\big[\bigcup_{i=1}^{n}\big(F_i \bigcap_{j=1,j\neq i}^{n} F_j\big)\big]\Big) + p_L(1-p_L)^{(n-1)}\Big(\textstyle\sum_{i=1,i\notin iso}^{n} P\big[\bigcup_{j=1,j\neq i}^{n} F_j | F_i\big]\Big) \\ + \textstyle\sum_{k=2}^{n} \frac{n!}{k!(n-k)!} p_L^k (1-p_L)^{(n-k)} + n_{iso} p_L (1-p_L)^{(n-1)} \end{aligned} \quad (8)$$

where iso is a counter for and n_{iso} is the number of isostatic elements.

In eq. (8), the first term corresponds to no bar failures due to epistemic reasons; the first term in parenthesis is the sequential failure of two bars (bar j conditional on bar i), the second term represents the failure of an isostatic element and the third term is the simultaneous failure of two bars. The second term corresponds to one epistemic bar failure. The last line corresponds to k bar failures due to epistemic reasons.

For an isostatic candidate solution, the system failure probability is:

$$p_{fsys}(\mathbf{d},\mathbf{X},p_L) = (1-p_L)^n (P[\cup_{i=1}^n F_i]) + \sum_{k=1}^n \frac{n!}{k!(n-k)!} p_L^k (1-p_L)^{(n-k)} \tag{9}$$

For the risk-optimization solution, the total expected costs need to be evaluated. For a hyperstatic candidate solution, $C_{ET}(\mathbf{d},\mathbf{X},p_L)$ can be written as:

$$\begin{aligned} C_{ET}(\mathbf{d},\mathbf{X},p_L) = {} & C_{HF}(1-p_L)^n \left(\sum_{i=1,i\notin iso}^n P[F_i]\left(1 - P\left[\cup_{j=1,j\neq i}^n F_j|F_i\right]\right)\right) \\ & + C_{HF} p_L (1-p_L)^{(n-1)} \left(\sum_{i=1,i\notin iso}^n \left(1 - P\left[\cup_{j=1,j\neq i}^n F_j|F_i\right]\right)\right) \\ & + C_{PC}(1-p_L)^n \left(\sum_{i=1,i\notin iso}^n P[F_i] P\left[\cup_{j=1,j\neq i}^n F_j|F_i\right]\right) \\ & + C_{PC} p_L (1-p_L)^{(n-1)} \left(\sum_{i=1,i\notin iso}^n P\left[\cup_{j=1,j\neq i}^n F_j|F_i\right]\right) \\ & + C_{DF}(1-p_L)^n \left(\sum_{i=1,i\in iso}^n P[F_i] + P\left[\cup_{i=1}^n \left(F_i \cap_{j=1,j\neq i}^n F_j\right)\right]\right) \\ & + C_{DF} \left(\sum_{k=2}^n \frac{n!}{k!(n-k)!} p_L^k (1-p_L)^{(n-k)} + n_{iso} p_L (1-p_L)^{n-1}\right) \\ & + \sum_{i=1}^n \rho_i A_i L_i \end{aligned} \tag{10}$$

where: k_{HF} is the hyperstatic bar failure cost factor; $k_{PC} = k$ is the cost factor for progressive collapse; $k_{DC} = \alpha\, k$ is the direct failure cost factor; k and α are the multiplying factors presented in section 2.3.

For a candidate isostatic solution, the total expected costs are:

$$C_{ET}(\boldsymbol{d},\boldsymbol{X},p_L) = C_{DF}(1-p_L)^n (P[\cup_{i=1}^n F_i]) + C_{DF}\left(\sum_{k=1}^n \frac{n!}{k!(n-k)!} p_L^k (1-p_L)^{(n-k)}\right) + \sum_{i=1}^n \rho_i A_i L_i \tag{11}$$

The optimization problems discussed in this paper are nonlinear, non-convex and discontinuous. Hence, the use of gradient-based optimization methods becomes impractical. On the other hand, heuristic algorithms are suitable for problems involving nonlinear and non-differentiable functions, with multiple local minima. The main drawback of such methods is their computational cost, which grows in proportion to the population size. Nevertheless, for the problems presented in this study, these methods prove to be advantageous. The optimal solutions for the problems formulated in the previous section are obtained using the Particle Swarm Optimization (PSO) method. In order to better control exploration and exploitation, an inertia weight strategy proposed by Cekus and Skrobek [7] is employed. Also, as PSO does not yield any guarantees that the global optimal is found, every problem variant is solved 10 replications with different initial population sets.

The truss topology optimization performed in this study follows the ground structure approach [8]. The bars are eliminated from the ground structure following the criteria proposed by Deb and Gulati [9]. The cross-sectional areas are compared to a small value ϵ, called critical area. If the element area is smaller than the critical value, the bar is eliminated from the ground structure. Note that the value of ϵ and the lower ($\mathbf{A}^{min}$) and upper ($\mathbf{A}^{max}$) bounds of the cross-sectional areas must be selected so that an unnecessary element has a considerable probability of being removed from the final topology. Besides, in order to avoid singular stiffness matrices associated to unstable solutions, the displacements of unconnected nodes are set equal to zero.

3 Numerical example: Six-bar truss

The truss studied in this paper is presented in Figure xx. It is a 6-bar 4 node truss with a horizontal force F applied at node D. The force follows a Gumbel distribution with mean intensity of 500 kN and a coefficient of variation of 10%. The maximum allowable stress is 250 MPa in tension and compression. The elasticity modulus is 200 GPa and material density is 7850 kg/m^3. Critical are for bars is 0.10 cm^2. Topology optimization is performed first using the RBDO formulation, then the RO formulation. In both cases, the conventional solutions ($p_L = 0$) are compared to those obtained considering the effect of epistemic uncertainties.

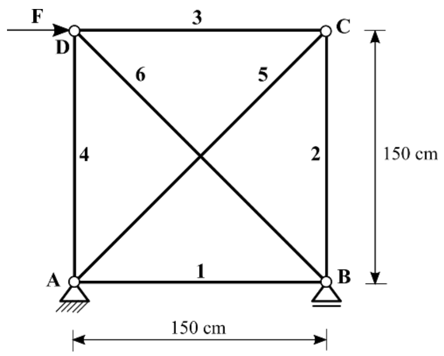

Figure 1 – Six-bar truss with one loaded node.

3.1 Results for system RBDO

RBDO solutions were computed for $\beta_T = 3$ and $\beta_T = 4$, for increasing values of p_L. For each p_L value, 10 replications were made. The best solutions for $\beta_T = 3$ are presented in Table 1. The optimal system reliability indexes β_{sys} were obtained by post-processing, using simple Monte Carlo simulation with 10^6 samples. In order to interpret the results presented in Table 1, we need to consider the maximum reliability that the isostatic 3-bar (3B) system can achieve, given the presence of epistemic uncertainty, which is written as:

$$\beta_{max3B} = \Phi^{-1}[(1 - p_L)^3] \tag{12}$$

Table 1 - Optimal RBDO solutions for $\beta_T = 3$.

Design Variables	**Latent Failure Probability**											
	0	**5×10⁻⁷**	**10⁻⁶**	**5×10⁻⁶**	**10⁻⁵**	**5×10⁻⁵**	**10⁻⁴**	**5×10⁻⁴**	**10⁻³**	**5×10⁻³**	**10⁻²**	**5×10⁻²**
A_1 (cm^2)	29.31	29.29	29.39	0.12	29.36	29.48	29.68	19.97	20.89	24.78	35.66	37.52
A_2 (cm^2)	0.00	0.00	0.00	29.35	0.00	0.00	0.00	19.69	21.47	24.07	35.66	37.49
A_3 (cm^2)	0.00	0.00	0.00	29.34	0.00	0.00	0.00	19.69	21.41	24.00	35.66	37.49
A_4 (cm^2)	29.31	28.29	29.39	0.00	29.36	29.48	29.68	19.94	21.06	24.74	35.66	37.48
A_5 (cm^2)	0.00	0.00	0.00	41.49	0.00	0.00	0.00	27.84	30.28	33.95	50.44	53.02
A_6 (cm^2)	41.45	41.42	41.56	0.17	41.52	41.69	41.97	28.21	29.69	34.98	50.43	53.00
Obj. Fun. (kg)	138.06	137.95	138.42	138.62	138.27	138.86	139.79	186.70	199.75	229.69	335.94	353.15
Type of system	**Iso**	**Iso**	**Iso**	**Iso**	**Iso**	**Iso**	**Iso**	**Hyper**	**Hyper**	**Hyper**	**Hyper**	**Hyper**
$\beta_{sys} \cong \beta_T$	2.99	2.99	2.98	2.97	3.01	2.97	3.00	3.00	3.00	3.00	2.98	1.84
β_{max3B}	∞	4.67	4.53	4.17	4.01	3.62	3.43	2.97	2.74	2.17	1.89	1.07

Analyzing the first seven columns of Table 1, we note that, for small values of p_L, the optimal trusses are isostatic structures. Most of these solutions are three-bar trusses. We notice that these isostatic solutions are obtained when $\beta_{sys} \cong \beta_T < \beta_{max3B}$. Hence, the reliability constraint allows the truss system to be less reliable than the background reliability imposed by p_L (eq. (12)). This can be accomplished by non-redundant isostatic structures. We also observe that, when $\beta_{sys} \cong \beta_T > \beta_{max3B}$, the optimal solutions become redundant 6-bar hyperstatic structures. By redundant, we mean that the additional bars have enough capacity to withstand load redistribution, following failure of a hyperstatic bar. Therefore, we note that the reliability constraint requires the truss system to be more reliable than the background reliability imposed by p_L. To achieve this, the system must become redundant. Also, the value of p_L around which the optimal design changes from isostatic to redundant is called redundancy threshold herein.

Figure 2 shows the optimal objective function values for the RBDO solutions with $\beta_T = 3$ and $\beta_T = 4$, as a function of p_L. As can be observed, the total weight is not very sensitive to p_L, as p_L increases below the redundancy threshold. At the redundancy threshold, the optimal solution becomes hyperstatic and the weight starts to increase. For large values of p_L, we note that the optimal solutions become independent of the target reliability. This suggests that when non-structural factors have large influence on system reliability the safety level is controlled by the epistemic uncertainties.

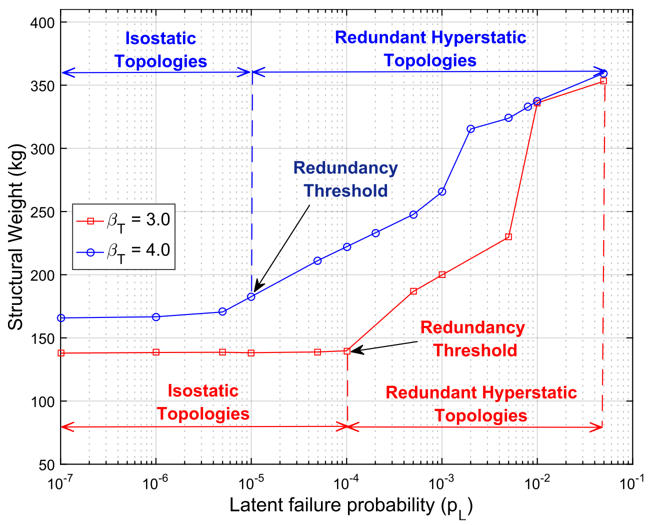

Figure 2 – Optimal RBDO results as a function of p_L.

3.2 Results for Risk Optimization

Results for RO were computed for $k_{HF} = 2$ and considering several values of p_L. Table 2 presents the results considering $k = 10$ and $\alpha = 5$. Similar results were also obtained for $k = 20$ and $\alpha = 5$. Analyzing the results in Table 2, we can divide the optimal solutions in three groups, as indicated in the 9th line of the table. Group A, corresponding to small values of p_L, is formed by isostatic optima; Group B, corresponding to intermediate values of p_L, is formed by hyperstatic but non-redundant structures, since some bars do not have enough capacity to withstand load redistribution, in case of failure of a main bar; and Group C, corresponding to large values of p_L, formed by hyperstatic redundant trusses. The extreme variants (A and C) are similar to that observed for RBDO.

Table 2 - Optimal solutions for risk optimization, k = 10, α = 5.

Design Variables	**Latent failure probability**											
	0	**5×10^{-7}**	**10^{-6}**	**5×10^{-6}**	**10^{-5}**	**5×10^{-5}**	**10^{-4}**	**5×10^{-4}**	**10^{-3}**	**5×10^{-3}**	**10^{-2}**	**5×10^{-2}**
A_1 (cm^2)	31.71	31.50	31.82	31.76	31.53	4.67	1.36	4.82	2.45	3.70	23.58	25.96
A_2 (cm^2)	0.00	0.00	0.00	0.00	0.00	27.41	30.81	27.40	29.12	28.11	23.58	25.74
A_3 (cm^2)	0.00	0.00	0.00	0.00	0.00	27.41	30.81	27.40	29.12	28.11	23.58	25.74
A_4 (cm^2)	31.71	31.50	31.82	31.76	31.53	3.71	1.24	5.00	2.51	3.74	23.57	25.96
A_5 (cm^2)	0.00	0.00	0.00	0.00	0.00	38.76	43.58	38.75	41.18	39.76	33.35	36.39
A_6 (cm^2)	44.84	44.55	45.00	44.91	44.59	5.88	1.61	6.48	3.54	5.05	33.34	36.72
Weight (kg)	149.35	148.38	149.86	149.57	148.51	148.76	150.87	151.41	148.89	149.59	222.10	243.50
C_{ET}	157.31	157.34	156.68	157.65	158.16	158.25	158.83	164.55	171.13	235.77	275.21	1063.21
Group	A	A	A	A	A	B	B	B	B	B	C	C
Type of system	**Iso**	**Iso**	**Iso**	**Iso**	**Iso**	**Hyper**	**Hyper**	**Hyper**	**Hyper**	**Hyper**	**Hyper**	**Hyper**
β_{sys}	3.45	3.37	3.45	3.41	3.37	3.28	3.25	2.92	2.71	2.16	2.59	1.80
β_{max3B}	∞	4.67	4.53	4.17	4.01	3.62	3.43	2.97	2.74	2.17	1.89	1.07

The optimal system reliability indexes shown in Figure 3 help us understand why these three groups exist. Also, two limiting behavior curves are included in the figure to aid interpretation of results: the maximum reliability that an isostatic three-bar structure can achieve (β_{max3B}, eq. (18)); and the maximum reliability that the hyperstatic six-bar truss can achieve (β_{max6B}); both due to presence of epistemic uncertainty.

In Figure 3 we observe that the optimal isostatic designs of Group A do not change as a function of p_L. Hence, we can infer that β_{sys} for this group is mainly a function of the aleatory uncertainties. In the region corresponding to Group B, the optimal reliability asymptotically approaches the limiting value β_{max3B}. Therefore, we note that the optimal designs are directly dependent or controlled by p_L. Yet, in Group B the optimal designs are less reliable than β_{max3B}, as they approach this curve from below. In the boundary between groups B and C there is a jump, where optimal system reliability becomes greater than β_{max3B}. This occurs when the optimal structure become redundant, as observed for RBDO. Hence, we can interpret that the optimal design becomes hyperstatic, and then redundant, to cope with the increasing effect of epistemic uncertainties. Finally, as p_L becomes very large, the epistemic uncertainties completely dominate system reliability, which asymptotically approaches the limiting curve β_{max6B}.

The boundary between the optimal solutions in Groups A and B is called hyperstatic threshold. On the other hand, the boundary between the optimal solutions in Groups B and C is called redundancy threshold, as in RBDO. In Figure 3 we see that this point varies significantly with the failure cost multipliers k and α. When k changes from 10 to 20, the redundancy threshold moves to the left, making the hyperstatic redundant solutions occur for smaller values of p_L. When α changes from 5 to 10, the hyperstatic threshold also moves to the left.

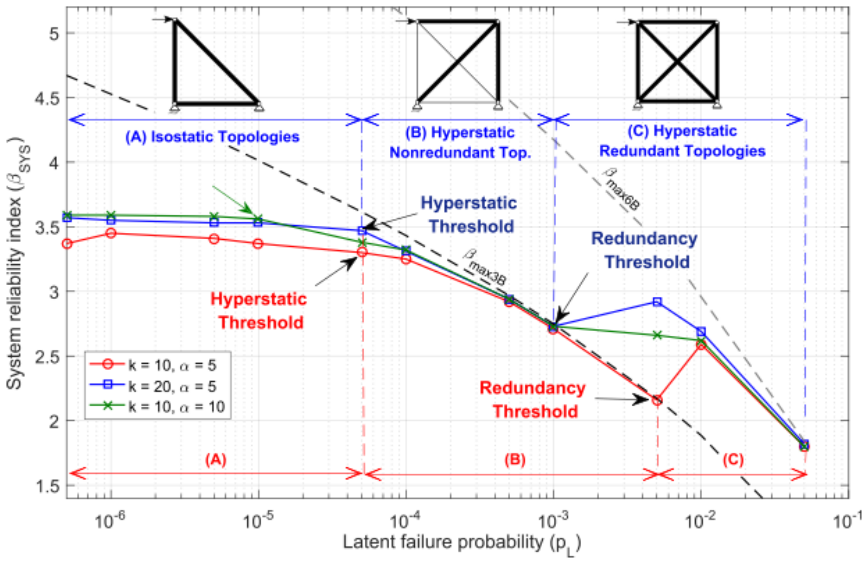

Figure 3 - Optimal RO results as a function of p_L.

4 Conclusions

This paper addressed the study of the effects of epistemic uncertainties in the optimal structural design. The latent failure probability concept was employed in reliability-based and risk-based truss topology optimization. This concept represents the impacts of non-structural factors such as gross errors in design and manufacturing, human errors, operational abuse and unanticipated loading. Through a simple six-bar truss example, it is shown that optimal designs resulting from risk-based analysis can be divided in three groups. When the effect of epistemic uncertainties is negligible, the optimal solutions are isostatic structures. As the epistemic uncertainty increases, optimal solutions become hyperstatic and then redundant. The boundary between the isostatic and hyperstatic optimal topologies was called hyperstatic threshold, while the boundary between the hyperstatic and redundant solutions was called redundancy threshold. The results presented herein show something that was already recognized in practice: structural systems should be redundant to cope with epistemic uncertainty. Finally, the results show that the latent failure probability, despite being an idealized concept, is a simple tool to impose minimal redundancy in reliability-based and risk-based topology optimization.

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