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A three degrees of freedom model of the support structure of a non-ideal motor

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Abstract. n rotating-unbalance-machine/structure systems, stable regime is when angular velocity is constant, all energy supplied by the motor consumed by internal friction and energy dissipated by damping of structure. Energy beyond this accelerates the system. Each energy level provided by the motor corresponds to stable constant angular speed. H is the torque consumed by the motor internal friction. Summed to the R torque to overcome damping forces of the support structure gives the S torque. L is the active motor torque provided. In an ideal system, the motor provides power to go over the resonance peaks of the structure and achieve the rated motor speed. In a non-ideal system, with limited power supply, an available motor torque level L can intercept the torque curve S consumed by friction and the structure at a constant rotation point before or after the resonance peak. If before, is a stable point of capture at resonance. Angular velocity no longer increases, stagnating before the peak, not reaching higher rotation speed. Jumps happens when more energy is supplied to overcome this stagnation. There comes a point the torque curve goes over the peak and intercepts the consumed torque curve further ahead in steady higher angular velocity, no intermediate stable steady states. There is really no difference between ideal and non-ideal systems. Only the available power level. A model considering mutual interaction support-structure/machine, should always be used. In that sense, all systems are non-ideal. In practice, if the motor has enough power this effect is negligible and we only consider interaction between the motor and the structure but not the reverse interaction, simplifying the model. Literature is available on 1-DOF support structure models, with 2 coupled autonomous equations of motion, 1 for the structure, 1 for the motor. In this paper we present a better representation of a machine foundation moving in horizontal and vertical directions, with mass, stiffness and damping for each one. Thus, we study 2 possible occurrences of the Sommerfeld Effect in the same model

Keywords: nonideal systems, Sommerfeld effect, capture at resonance, unbalanced motor foundation.

1 Introduction

In a rotating unbalance-machine/structure system, a stable rotation regime is one in which the angular velocity is constant (zero angular acceleration) corresponding to a situation in which all energy supplied by the motor is fully consumed by the internal friction of the equipment and by the energy dissipated by the damping of the structure. Any energy beyond this causes non-zero acceleration of the system, changing the speed.

For each energy level provided by the motor, there is a stable regime corresponding to it, at a certain constant angular speed.

Let's write a *H* torque function consumed by the motor internal friction. Summed to the *R* torque needed to overcome the damping forces of the vibrating support structure we obtain the S torque function. Let's also write the L active torque functions provided by the motor, the *L* characteristic curves. Their unities are Newton x meter.

In an "ideal" system, the motor would be able to provide a power level such that it can go over the resonance peaks with the structure and achieve a steady speed of rotation that can be called the "rated motor speed".

In a "not ideal" system, with limited power supply, each available torque level L curve provided by the motor can intercept the torque curve S consumed by the engine and the structure at a point with constant rotation speed which can be before or after the resonance peak. If before, it would be a stable point of capture or stagnation at resonance. The angular velocity no longer increases, stagnating before the resonance peak, and cannot reach higher rotation regimes.

A “jump” would take place when more energy is supplied to overcome this stagnation of rotations before the resonance. There comes a point where the torque curve goes over the peak and will intercept the consumed torque curve much further ahead in steady, much higher angular velocity, regimes, with no intermediate stable steady states.

Thus, there is really no difference between "ideal" and "non-ideal" systems. Only the available power level.

The most complete model, considering the mutual interaction between the support structure and the machine, should always be used. In that sense, all systems are "non ideal".

But in practice, when the motor has enough power, this effect is negligible and we can only consider the interaction between the machine and the structure, without considering the reverse interaction between the structure and the machine, what simplifies the mathematical model.

Some literature is available on the case of a single degree of freedom support structure model. In this case, we have two coupled autonomous equations of motion, one for the structure itself and one for the rotation of the motor (references).

In this paper we present a more realistic representation of a machine foundation that can move in the horizontal and vertical axis, with mass, stiffness and damping properties in both directions. Thus, we are able to study two possible occurrences of the Sommerfeld Effect in the same model.

2 Mathematical Model

Figure 1 displays a block of mass M supported by spring K_1 and damper C_1 in the horizontal direction, and spring K_2 and damper C_2 in the vertical direction. A motor is mounted on this block, whose mass is included in M. Its rotor moment of inertia is J. We consider unbalancing represented by a small mass m_0 at a distance e of the machine axis.

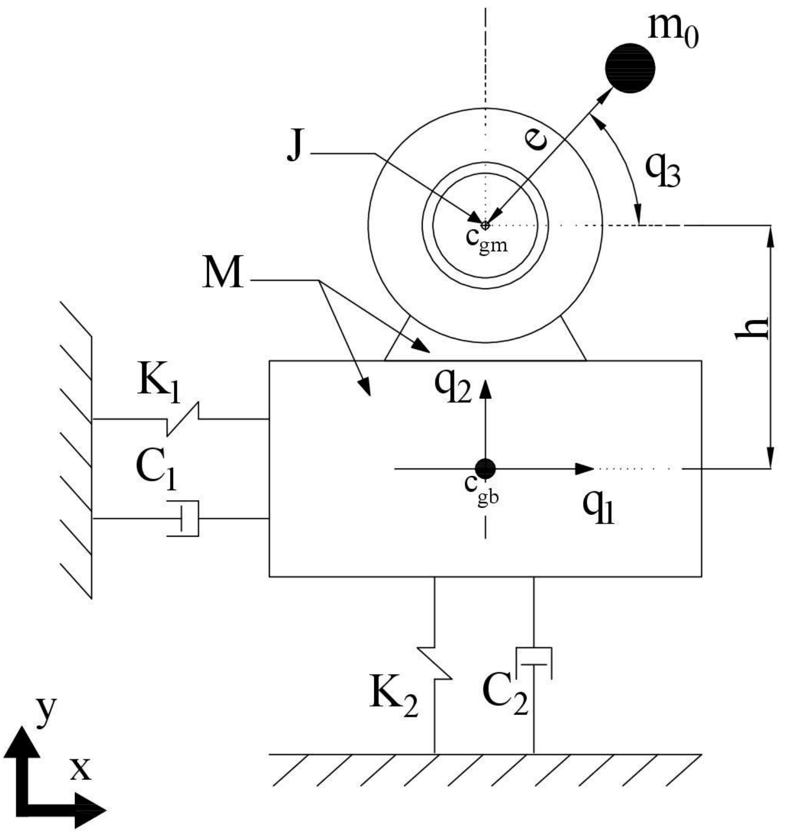

Figure 1: 2-D-O-F unbalanced motor foundation model

The adopted generalized coordinates of this system are the q_1 and q_2 motions of the block, in meters, and the q_3 rotation of the motor rotor, in radians. Thus, the coordinates and velocities (denoted by a superposed dots) of the considered two masses are:

$$x_M = q_1 \quad \dot{x}_M = \dot{q}_1 \quad y_M = q_2 \quad \dot{y}_M = \dot{q}_2 \tag{1}$$
$$x_0 = q_1 + e\cos q_3 \quad \dot{x}_0 = \dot{q}_1 - e\dot{q}_3 \sin q_3 \tag{2}$$
$$y_0 = q_2 + e\sin q_3 \quad \dot{y}_0 = \dot{q}_2 + e\dot{q}_3 \cos q_3 \tag{3}$$

The Kinetic Energy is

$$T = \frac{1}{2}\left[M(\dot{x}_M^2 + \dot{y}_M^2) + m_0(\dot{x}_0^2 + \dot{y}_0^2) + J\dot{q}_3^2\right] \tag{4}$$

The Strain Energy is

$$U = \frac{1}{2}(K_1 q_1^2 + K_2 q_2^2) \tag{5}$$

The work of the weight forces is

$$W = -g[Mq_2 + m_0(e\sin q_3)] \tag{6}$$

The Total Potential Energy is

$$V = U - W \tag{7}$$

Applying Lagrange's Equation,

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = N_i \qquad i = 1,2,3 \tag{8}$$

where $\mathcal{L} = T - V$ is the Lagrangian and N_i are the generalized nonconservative forces, such as damping forces, we obtain the following three second order Equations of Motion:

$$(M + m_0)\ddot{q}_1 + C_1\dot{q}_1 + K_1 q_1 = -m_0 e(\ddot{q}_3 \sin q_3 + \dot{q}_3^2 \cos q_3) \tag{9}$$
$$(M + m_0)\ddot{q}_2 + C_2\dot{q}_2 + K_2 q_2 = m_0 e(\ddot{q}_3 \cos q_3 - \dot{q}_3^2 \sin q_3) - (M + m_0)g \tag{10}$$
$$(J + m_0 e^2)\ddot{q}_3 + H(\dot{q}_3) = L(\dot{q}_3) - m_0 e(\ddot{q}_1 \sin q_3 - \ddot{q}_2 \cos q_3 - g\cos q_3) \tag{11}$$

where $L(\dot{q}_3)$ is the liquid torque provided by the motor. and $H(\dot{q}_3)$ is the motor internal friction torque. In matrix notation:

$$[A]\{\ddot{q}\} = \{b\} \tag{12}$$

where

$$[A] = \begin{bmatrix} M + m_0 & 0 & m_0 e\sin q_3 \\ 0 & M + m_0 & -m_0 e\cos q_3 \\ m_0 e\sin q_3 & -m_0 e\cos q_3 & J + m_0 e^2 \end{bmatrix} \tag{13}$$

$$\{\ddot{q}\} = \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{Bmatrix} \tag{14}$$

$$\{b\} = \begin{Bmatrix} -C_1\dot{q}_1 - K_1 q_1 - m_0 e\dot{q}_3^2 \cos q_3 \\ -C_2\dot{q}_2 - K_2 q_2 - m_0 e\dot{q}_3^2 \sin q_3 - (M + m_0)g \\ L(\dot{q}_3) - H(\dot{q}_3) + m_0 e(\ddot{q}_1 \sin q_3 + \ddot{q}_2 \cos q_3 - g\cos q_3) \end{Bmatrix} \tag{15}$$

3 Torque Relationships

A steady state constant motor frequency condition is given by the torque relationship

$$L(\dot{q}_3) = H(\dot{q}_3) + R(\dot{q}_3) \tag{16}$$

In Eq. (16), it is defined the function $R(\dot{q}_3)$, the torque dissipated by damping of the support structure, given by

$$R(\dot{q}_3) = \sum_{i=1}^{2} \frac{c_i}{2q_3} \omega_i^2\, a_i^2 \tag{17}$$

where

$$\omega_i = \sqrt{\frac{K_i}{M+m_0}} \tag{18}$$

are the two undamped frequencies of vibration of the support structure (rad/s). The amplitudes of vibration of these two modes are

$$a_i = \frac{m_0 e}{M+m_0} D_i \beta_i^2 \tag{19}$$

where the nondimensional Coefficients of Dynamic amplification are

$$D_i = \frac{1}{\sqrt{\left(1-\beta_i^2\right)^2 + (2\xi_i \beta_i)^2}} \tag{20}$$

defining the nondimensional relationships

$$\beta_i = \frac{q_3}{\omega_i} \qquad \xi_i = \frac{c_i}{2(M+m_0)\omega_i} \tag{21}$$

4 Numerical Simulations

Next, numerical parameters are adopted: M = 2 t, K_1 = 50,000 KN/m, K_2 = 100,000 KN/m, m_0 = 0.1 t, e = 0.01 m, $\xi_1 = \xi_2 = 0.05$, J = 1.7x10-4 tm², $H(\dot{q}_3) = 4\text{x}10^{-4}\dot{q}_3$ KNm. Figure 2 displays the $S(\dot{q}_3)$ curve of this system (in black), from Eq. (16), and two possible $L_k(\dot{q}_3)$ liquid torque characteristic curves of the motor (in red and blue), for two different available energy levels, considered as linear, in KNm,

$$L_1(\dot{q}_3) = 0.25 - 0.0009\dot{q}_3 \qquad L_2(\dot{q}_3) = 0.35 - 0.0009\dot{q}_3$$

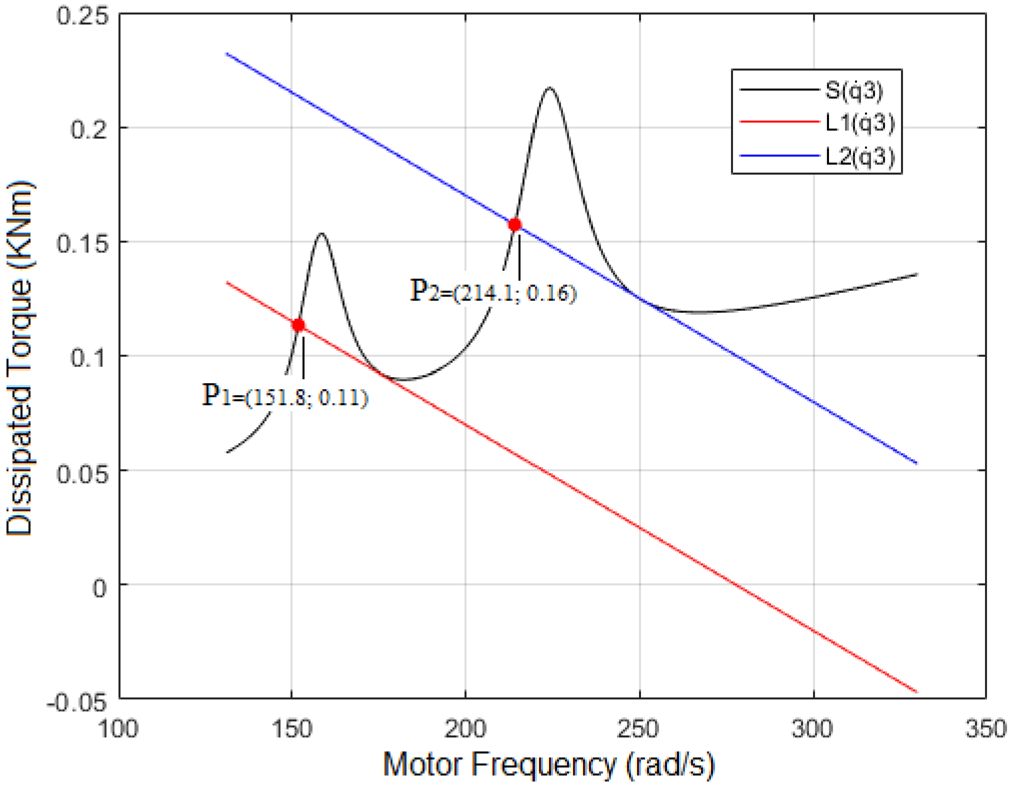

Figure 2: $S(\dot{q}_3)$ curve, in black, $L_1(\dot{q}_3)$ line, in red, and $L_2(\dot{q}_3)$ line, in blue

The computed stable steady state constant motor frequencies (rad/s) and corresponding torques, for positive increase of the motor power (KNm) are: P1= (151.8; 0.111), P2= (214.1; 0.16).

Next, it is performed a time step-by-step numerical integration of Eq. (12), using Runge-Kutta's 4th and 5th order algorithm, implemented in MATLAB software. The first steady state regime, P1, is displayed in Fig. 3, the second, P2, is displayed in Fig. 4.

In Figures 3 and 4, a fairly good agreement with Fig. 2 results is obtained. As expected, the effect of gravity on m_0 leads to a complex steady state behavior, as is possible to see in Fig. 5, a zoom of part of Fig. 3.

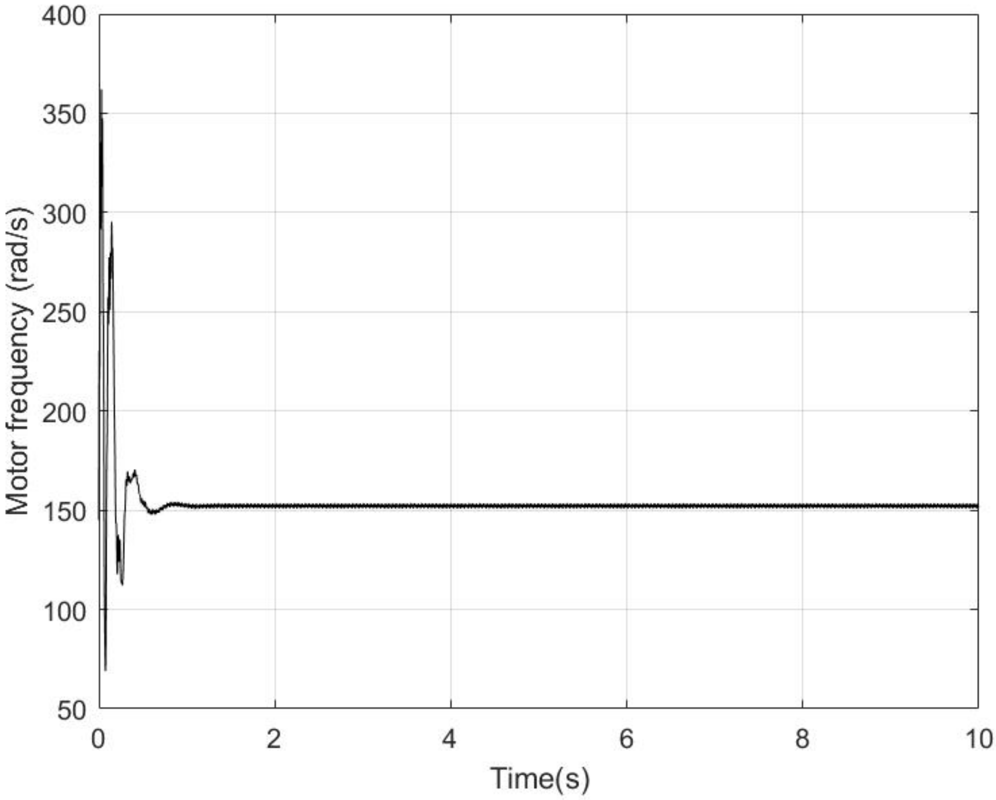

Figure 3: First steady state constant motor frequency regime, P1

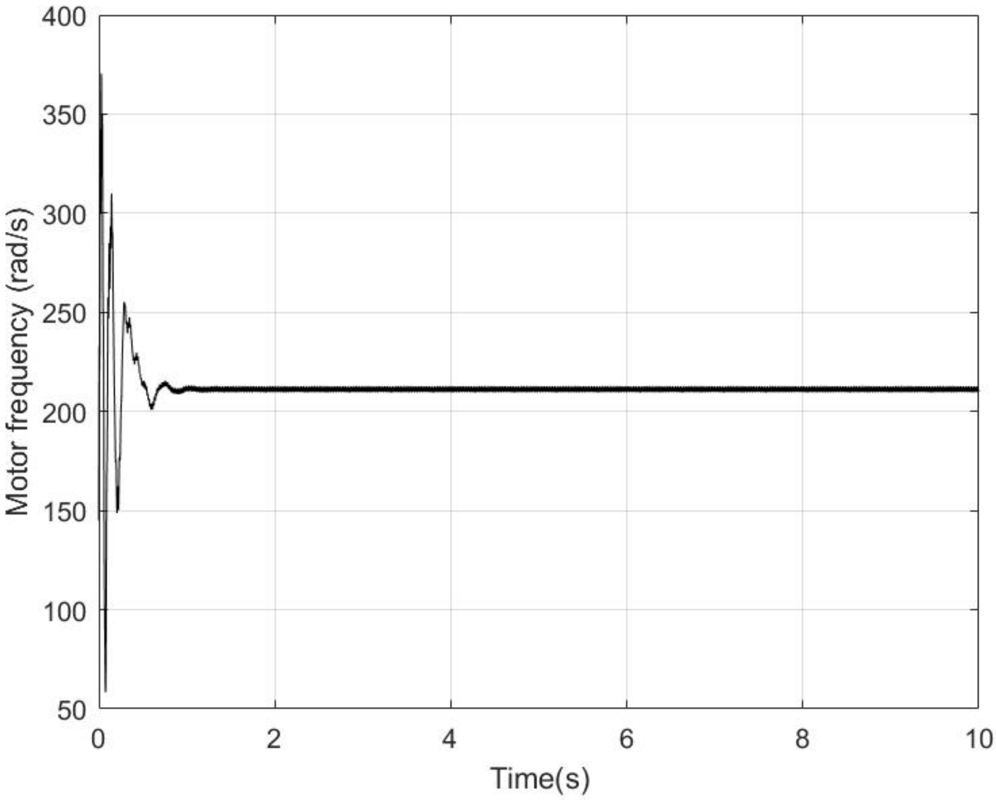

Figure 4: Second steady state constant motor frequency regime, P2.

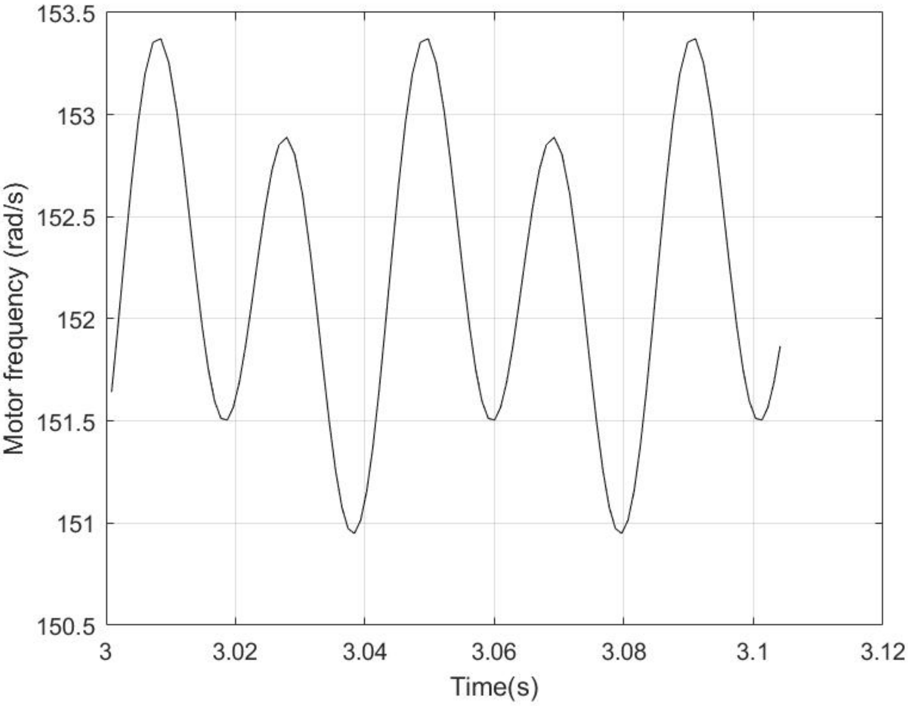

Figure 5: Zoom of first steady state constant motor frequency regime, P1

5 Conclusions

A study of non-ideal behavior of a two degrees of freedom support structure for a limited power unbalanced motor was presented. The expected Sommerfeld Effect of rotation frequency stagnation near resonances was observed. We conclude that all such systems are non-ideal, in the sense that mutual interaction between motor and support structure should always be considered.

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