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Nonlinear resonance curves of a cylindrical panel with unilateral contact of a discontinuous elastic base

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Abstract. The aim of this work is to analyze the influence of a discontinuous unilateral elastic base and an initial geometrical imperfection on the nonlinear vibrations of a simply supported cylindrical panel. The cylindrical panel is described by the nonlinear shallow shell theory of Donnell and discretized by the Galerkin method, using a reduced order model which is obtained by a perturbation method. The discontinuous elastic base model is described by a Heaviside function and the unilateral contact is defined by the Signum function. The results show the dynamic analysis of the cylindrical panel through the backbone curves, bifurcation diagrams, phase portraits and resonance curves to understanding the influence of the discontinuous unilateral elastic base and the initial geometrical imperfection of the cylindrical panel. An efficient modal solution with two degree-of-freedom is sufficient to describe the nonlinear softening behavior of the cylindrical panel with a discontinuous unilateral elastic base. The influence of the unilateral elastic base and the initial geometrical imperfection on the dynamic stability of the cylindrical panel is demonstrated in the resonance curves, phase planes, Poincaré mappings and bifurcation diagrams, where it is possible to identify important changes in the stable and unstable regions of the resonance curves when compared with a cylindrical panel with a discontinuous bilateral elastic base.

Keywords: cylindrical panel, elastic base, unilateral contact, initial geometrical imperfection.

1 Introduction

Cylindrical panels, or open circular cylindrical shells, are structural elements which have applications in many engineering fields as civil, aerospatial and mechanical engineering, among others. Generally, to prescribe their behavior and stability under static and dynamic loads, the mathematical model must consider their geometric nonlinearities. Reviews of the literature involving cylindrical panels subjected a different hypothesis about their deformation field, reduced order models, applied loads, boundary condition, coupled problems, among so many issues are presented in [1, 2]. The vibration study for cylindrical shells and panels supported on an elastic foundation has been investigated by several authors, which investigated its influence on the natural frequency for different shell geometries, initial stress condition, and foundation parameters [3-5]. In a geometrical nonlinear scenario, [6-8] analyzed the nonlinear dynamic and buckling response of panels supported on an elastic base. The first studies on unilateral foundation can be attributed to Weitsman [9]. The analyzes are for an Euler-Bernoulli beam, supported on a unilateral elastic base, considering the reaction only for compression when subjected to a concentrated mobile load. Modern studies on nonlinear frequency in beams [10, 11] and plates/panels [12-14] supported on unilateral foundation various parameters of the elastic base are found in the literature.
The aim of this work is to analyze the influence of a discontinuous unilateral elastic base and an initial geometrical imperfection on the nonlinear vibrations of a simply supported cylindrical panel. The cylindrical panel is described by the nonlinear shallow shell theory of Donnell and discretized by the Galerkin method, using a reduced order model which is obtained by a perturbation method.

2 Problem formulation

An imperfect simply supported thin-walled circular cylindrical panel with radius R, thickness h, axial length a_x, circumferential length a_θ, and open angle $\Theta[=a_\theta/R]$ is considered, as shown in Fig. 1a. Its material is defined as linear elastic, isotropic, and homogenous with Young's modulus E, Poisson's coefficient ν, and density ρ. In Fig. 1a are represented the displacement fields in the axial, u, circumferential, v, and transversal, w directions, related to the cylindrical coordinates x, θ, and z, respectively. Considering the Donnell's nonlinear shallow shell theory, the nonlinear equilibrium equation and the compatibility equation of the cylindrical panel are given in terms of the transversal displacement field w and the Airy's stress function f:

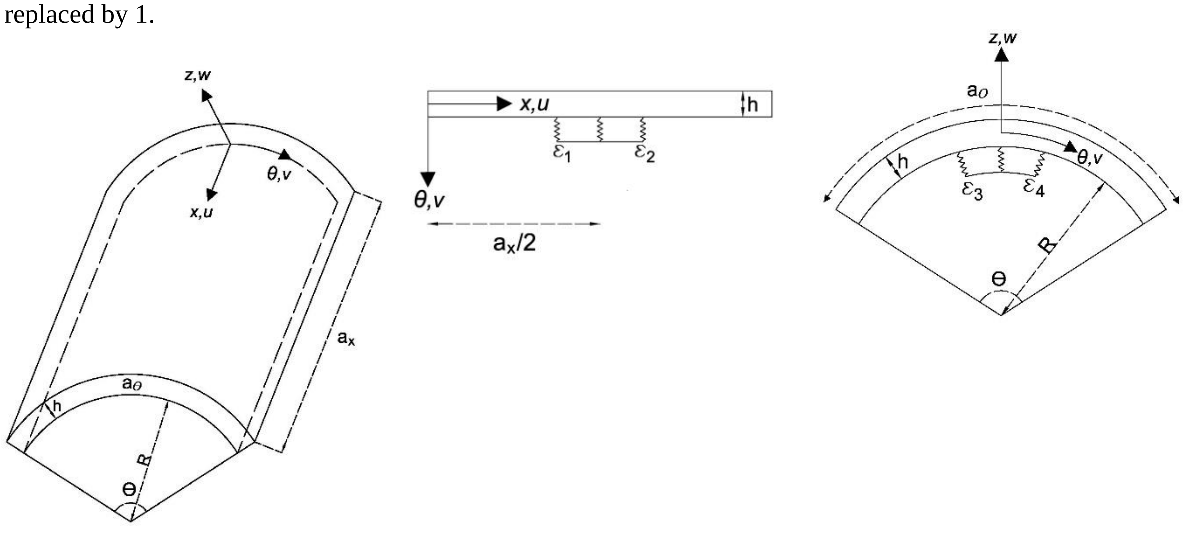
$$\begin{aligned} &\rho h w_{,tt} + 2\eta_1 \rho h \omega_0 w_{,t} + D\left(w_{,xxxx} + \frac{2}{R^2} w_{,\theta\theta xx} + \frac{1}{R^4} w_{x\theta\theta\theta}\right) - f_{,\theta\theta}\left(w_{,x} + w_{0,x}\right)_{,x} \\ &+R f_{,xx} - f_{,xx}\left(w_{,\theta} + w_{0,\theta}\right)_{,\theta} - 2 f_{x\theta}\left(w_{,\theta x} + w_{0,\theta x}\right) + p_k - p(t) = 0 \\ &\nabla^4 f = \frac{Eh}{R^4}\left(w_{,x\theta}^2 - w_{,xx} w_{,\theta\theta} + R w_{,xx} + 2 w_{,x\theta} w_{0,x\theta} - w_{,xx} w_{0,\theta\theta} - w_{,\theta\theta} w_{0,xx} \right). \end{aligned} \tag{1}$$

where ω_0 is the natural frequency of cylindrical panel, η_1 is the viscous damping factor, $D[=Eh^3/12(1-\nu^2)]$ is the flexural stiffness, w_0 is an initial geometrical imperfection, $p(t)$ and p_k are the time-dependent transversal load and the reaction of the discontinuous unilateral elastic base described respectively by:

$$w_0 = W_0^{imp} h \sin\left(\frac{m\pi x}{a_x}\right)\sin\left(\frac{n\pi\theta}{\Theta}\right).$$

$$p(t) = P_L \sin\left(\frac{m\pi x}{a_x}\right)\sin\left(\frac{n\pi\theta}{\Theta}\right)\cos\left(\omega_L t\right). \qquad p_k = \left[K_w w + K_P\left(w_{,xx} + \frac{w_{,\theta\theta}}{R^2}\right)\right] H_x H_\theta \frac{(1-\operatorname{sgn} w)}{2}. \tag{2}$$

where P_L is the magnitude of transversal load, ω_L is the frequency of excitation, K_w and K_P are the Winkler and Pasternak stiffness parameters, respectively; the functions $H_x[=H(x-\varepsilon_1)-H(x-\varepsilon_2)]$ and $H_\theta[=H(\theta-\varepsilon_3)-H(\theta-\varepsilon_4)]$ are Heaviside functions which describes the discontinuous elastic base, in the longitudinal direction in the region defined by $0\leq \varepsilon_1<\varepsilon_2\leq L$, Fig. 1b, and in the circumferential direction in the region defined by $0\leq \varepsilon_3<\varepsilon_4\leq\Theta$, Fig. 1c; and, sgn is the Signum function which controls the unilateral contact of elastic base, i.e., for positive values of w, the reaction p_k becomes zero. For a bilateral contact of a discontinuous elastic base, the term (1-sgn w)/2 in eq. (2) is replaced by 1.



(a) (b) (c)

Figure 1. (a) Geometry and displacement field for a cylindrical panel. (b) Elastic foundation in the longitudinal direction in the region defined by $0\leq \varepsilon_1,\varepsilon_2\leq L$. (c) Elastic foundation in the circumferential direction in the region defined by $0\leq \varepsilon_3,\varepsilon_4\leq\Theta$.

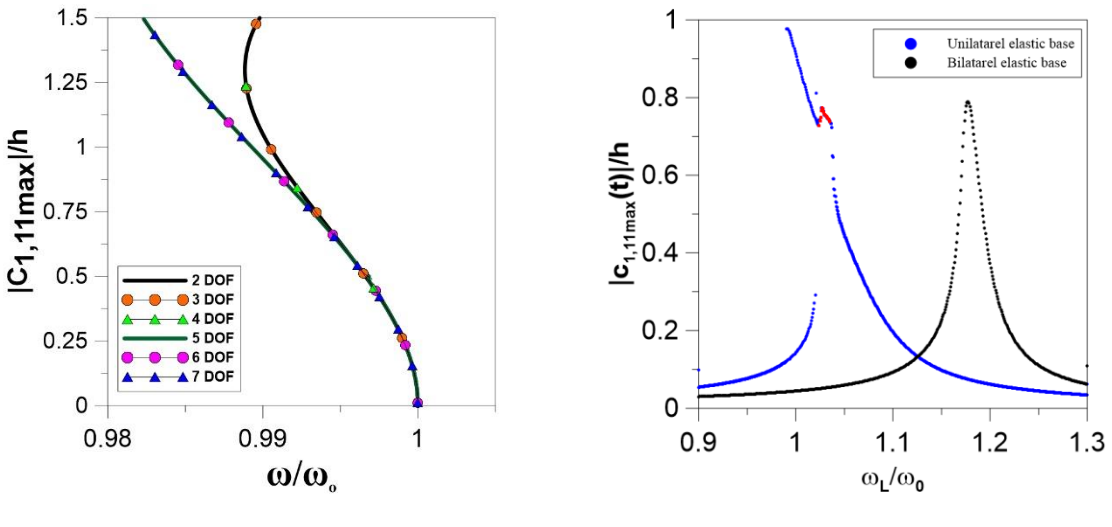
Airy's stress function f is obtained analytically for a particular transversal displacement field w. According to Morais and Silva [15], a consistent transversal displacement field is derived from a perturbation method, obtaining, for a simply supported cylindrical panel, the general modal solution:

$$
\begin{aligned}
w = &\sum_{i=1,3,5}\sum_{j=1,3,5} c_{1,ij}(t)\sin\left(\frac{im\pi x}{a_x}\right)\sin\left(\frac{jn\pi\theta}{\Theta}\right) + \sum_{\alpha=0,1,2,3...}\sum_{\beta=0,1,2,3...} c_{2,(2+6\alpha)(2+6\beta)}(t)\left\{\left[\frac{3+6\alpha}{4+12\alpha}\cos\left(\frac{6\alpha m\pi x}{a_x}\right)\right.\right. \\
&\left. -\cos\left(\frac{(2+6\alpha)m\pi x}{a_x}\right) + \frac{1+6\alpha}{4+12\alpha}\cos\left(\frac{(4+6\alpha)m\pi x}{a_x}\right)\right]\left[\frac{3+6\beta}{4+12\beta}\cos\left(\frac{6\beta n\pi\theta}{\Theta}\right) - \right. \\
&\left.\left.\cos\left(\frac{(2+6\beta)n\pi\theta}{\Theta}\right) + \frac{1+6\beta}{4+12\beta}\cos\left(\frac{(4+6\beta)n\pi\theta}{\Theta}\right)\right]\right\}.
\end{aligned}
\tag{3}
$$

Returning to the nonlinear cylindrical equilibrium equation, with the obtained f and the particular w, it is discretized by Galerkin method, obtaining a set of nonlinear second order differential equations in terms of modal amplitudes $c_{1,ij}(t)$ and $c_{2,(2+6\alpha)(2+6\beta)}(t)$.

3 Numerical results

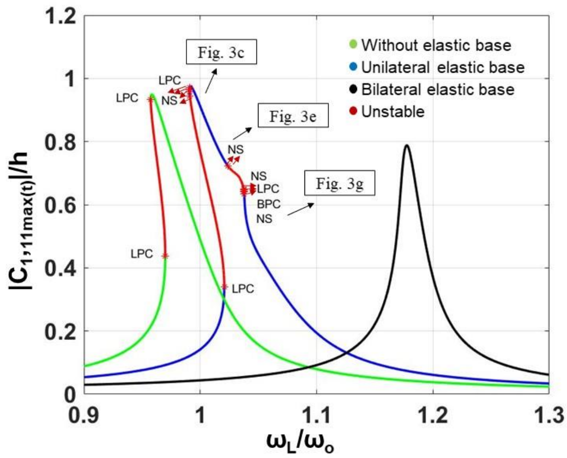
Consider a cylindrical panel with the geometrical and physical parameters: R=8.333 m, h=0.01 m, a_x =1 m, a_θ =1 m, E =210 GPa, ν=0.3 and ρ=7850 kg/m³. In this case, the lowest natural frequency is 437.92 rad/s occurring to wave numbers (m, n) = (1, 1). Also, it is considered a Winkler base with K_w = 46.15 MN/m³ centered in the cylindrical panel (ε_1=0.4, ε_2=0.6, ε_3=0.048 and ε_4=0.072), without Pasternack base K_P = 0 and an initial geometrical imperfection in the shape of the fundamental vibration mode with amplitude equal to 0.05h.



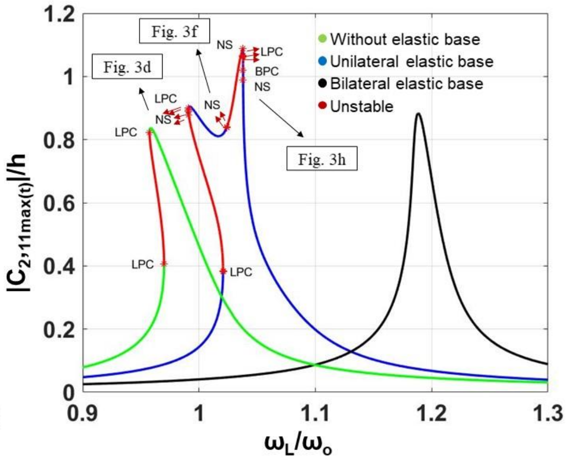
(a) (b)

Figure 2. (a) Frequency-amplitude relation for a perfect cylindrical panel with a discontinuous bilateral elastic base, considering different reduced-order models. 2DOF: $c_{1,11}$, $c_{2,22}$; 3DOF: 2DOF, $c_{2,28}$; 4DOF: 3DOF, $c_{2,82}$; 5DOF: 2DOF, $c_{1,13}$, $c_{1,31}$, $c_{1,33}$; 6DOF: 5DOF, $c_{2,28}$; 7DOF: 6DOF, $c_{2,82}$. (b) Comparison of resonance curves for an imperfect cylindrical panel, considering the contact of elastic base (2DOF system, P_L = 2.5 kN/m², η_1 = 0.01).

Figure 2a displays the frequency-amplitude relation, obtained by the shooting method, for a perfect cylindrical panel in contact with a discontinuous bilateral elastic base. Different modal solutions of w are considered for the analysis, with the aim of determining the number of degrees of freedom (DOF) necessary for the discretization of the transverse displacement field, w, to guarantee the correct representation of the non-linear behavior of the cylindrical panel. It can be seen through the analysis of Fig. 2a that the modal solution with 2 DOF is enough to describe the nonlinear behavior of the cylindrical panel, describing the softening nonlinearity of the backbone curve up to amplitudes around the thickness panel. Considering a time-dependent transversal load with magnitude P_L = 2.5 kN/m² and damping η_1 = 0.01, the resonances curves of an imperfect cylindrical panel are obtained by the brute force method, under the hypothesis of bilateral or unilateral contact, as shown in Fig. 2b. The curves demonstrate how the amplitude varies with the excitation frequency. The resonance peaks are shifted to the right of $\omega_L/\omega_0 = 1$ (resonance region of the perfect cylindrical panel) due to increasing of natural frequencies for both cases of elastic base (470.29 rad/s – unilateral elastic base; 518.47 rad/s – bilateral elastic case). It is observed in Fig. 2b that, for the same transversal load, the resonance curve of the panel with a bilateral elastic base has an almost linear behavior while the resonance curve of panel with unilateral elastic base has a significant softening behavior, that is, there is a loss of stiffness in the structure with increasing of the excitation frequency, features dynamical jumps, bifurcation points and paths with quasi-periodic response (marked in red color).



(a)



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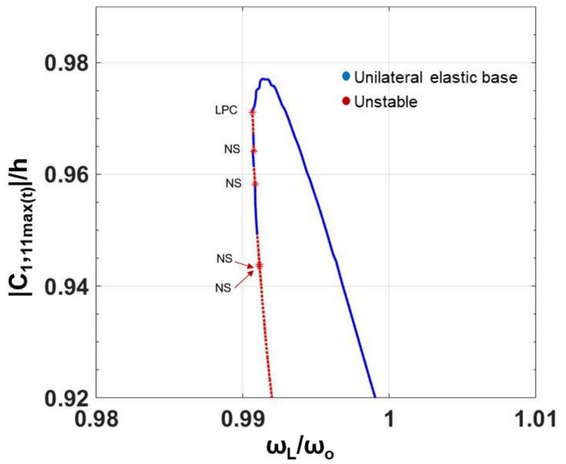
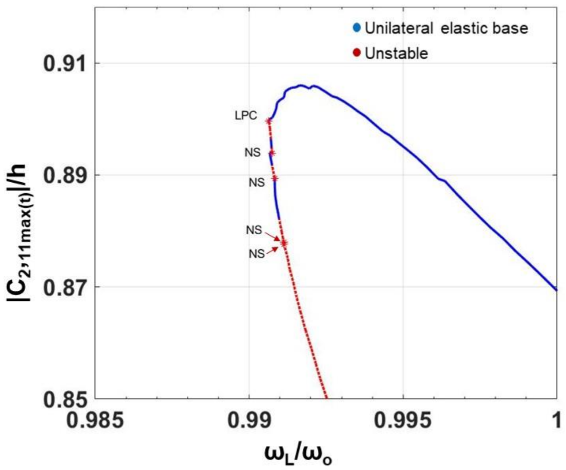
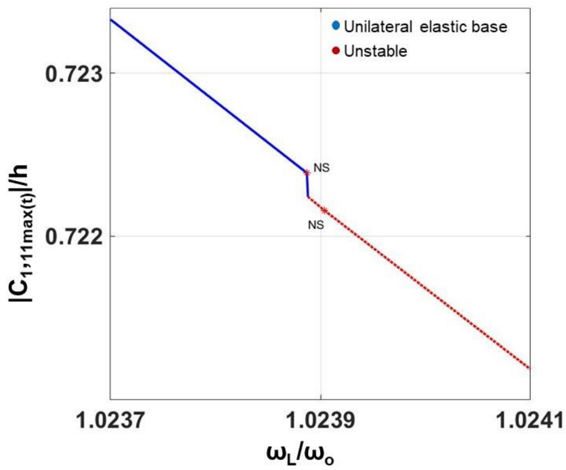


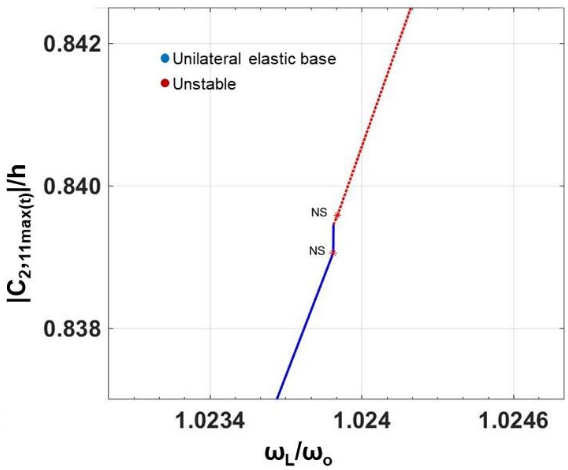
(c)



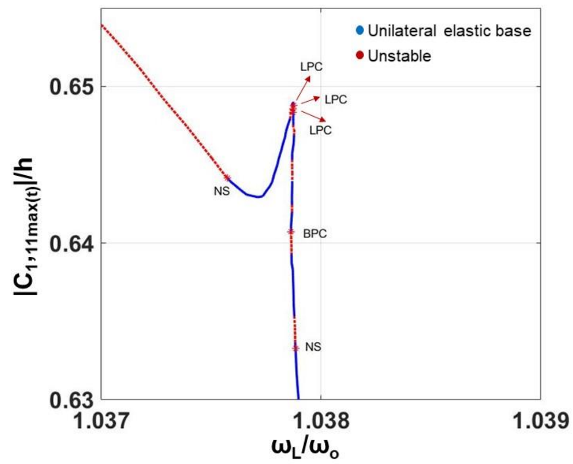
(d)



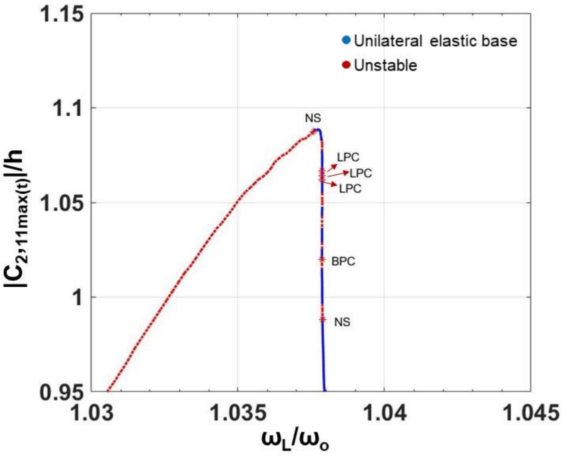
(e)



(f)



(g)



(h)

Figure 3. (a) Bifurcation diagram for cylindrical panels $|C_{1,11max(t)}|/h$ and ω_L/ω_0. (b) Bifurcation diagram for cylindrical panels $|C_{2,11max(t)}|/h$ and ω_L/ω_0. (c) 1º region with bifurcation points $|C_{1,11max(t)}|/h$. (d) 1º region with bifurcation points $|C_{2,11max(t)}|/h$. (e) 2º region with bifurcation points $|C_{1,11max(t)}|/h$. (f) 2º region with bifurcation points $|C_{2,11max(t)}|/h$. (g) 3º region with bifurcation points $|C_{1,11max(t)}|/h$. (h) 3º region with bifurcation points $|C_{2,11max(t)}|/h$.

Figure 3 uses the MatCont, a Matlab software continuation package, for the study of numerical continuation of limit cycles and their bifurcations in the nonlinear system of equations for the same time-dependent load of Fig. 2b. Figs. 3a and 3b illustrate the resonance curves for three types of cylindrical panels: without elastic base (green color), with unilateral elastic base (blue color) and with bilateral elastic base (black color). The resonance curves of the cylindrical perfect panel without elastic base show a bifurcation point LPC (Limit Point of Cycles) increasing the excitation frequency. Then, the resonance curve becomes unstable (red curve) until it finds another LPC bifurcation point, for higher values of vibration amplitude. Soon after, the vibration amplitude shows a smooth decay with increasing excitation frequency, in a stable region, with no bifurcation points. The diagrams for modes $|C_{1,11max(t)}|/h$ and $|C_{2,11max(t)}|/h$ show the same behavior, Figs. 3a and 3b, but the amplitudes for $|C_{1,11max(t)}|/h$ are greater, as it is the mode that has the greatest influence on the nonlinear dynamic behavior of cylindrical panel.

The imperfect cylindrical panel with unilateral elastic base (blue color in Figs. 3a and 3b), is marked by several bifurcation points, Figs. 3c-3h are approximation of these regions. From Figs. 3a and 3b, increasing the excitation frequency, it is found a bifurcation point LPC, following an unstable path (red curve), with a region where it was observed changes in the panel's stability, in addition to several Neimark-Sacker (NS) bifurcation points and one bifurcation point LPC, as presented in Figs. 3c and 3d. After this region, the vibration amplitude decreases and frequency increases continuously for the amplitude $|C_{1,11max(t)}|/h$, for the amplitude $|C_{2,11max(t)}|/h$ the amplitude shows a slight decay, followed by an increase and then decreases continuously as the frequency increases, bifurcation points are still found, first two NS points, Figs. 3e and 3f, and after an unstable path there are several nearby bifurcation points, among them, NS, LPC and BPC, represented prominently in Figs. 3g and 3h. In accordance with Fig. 2b, the imperfect cylindrical panel with bilateral elastic base (black curves) there aren't bifurcation points. The resonance peaks are shifted to the right compared to panels without elastic base and with one-sided elastic base.

For the resonance curve of the imperfect cylindrical panel with unilateral elastic base, Figs. 2b and 3a, plotted in blue, the dynamic behavior and instability of the cylindrical panel were studied through phase plane and Poincaré sections by the fourth-order Runge-Kutta method. Figure 4 investigates some nonlinear responses found in the resonance zone of Figs. 2b and 3a. Figure 4a shows the phase plane and Poincaré section for the ratio ω_L/ω_0 = 1.02. In accordance with Figs. 2b and 3a, for this frequency ratio there are two amplitude values |C1,11max(t)|/h=0.293 and |C1,11max(t)|/h=0.7463, with a stable periodic orbit with a period T. Figures 4b and 4c present the response for the frequency ratio ω_L/ω_0 = 1.0272, with views in the $C_{1,11}/h$ x (dC1,11/dt)/h plane and a 3D view - $C_{1,11}/h$ x (dC1,11/dt)/h x $C_{2,11}/h$, respectively. The analysis of these figures presents a Poincaré map, describing a closed orbit, characterizes a quasi-periodic response which they are representative of a Neimark-Sacker region.

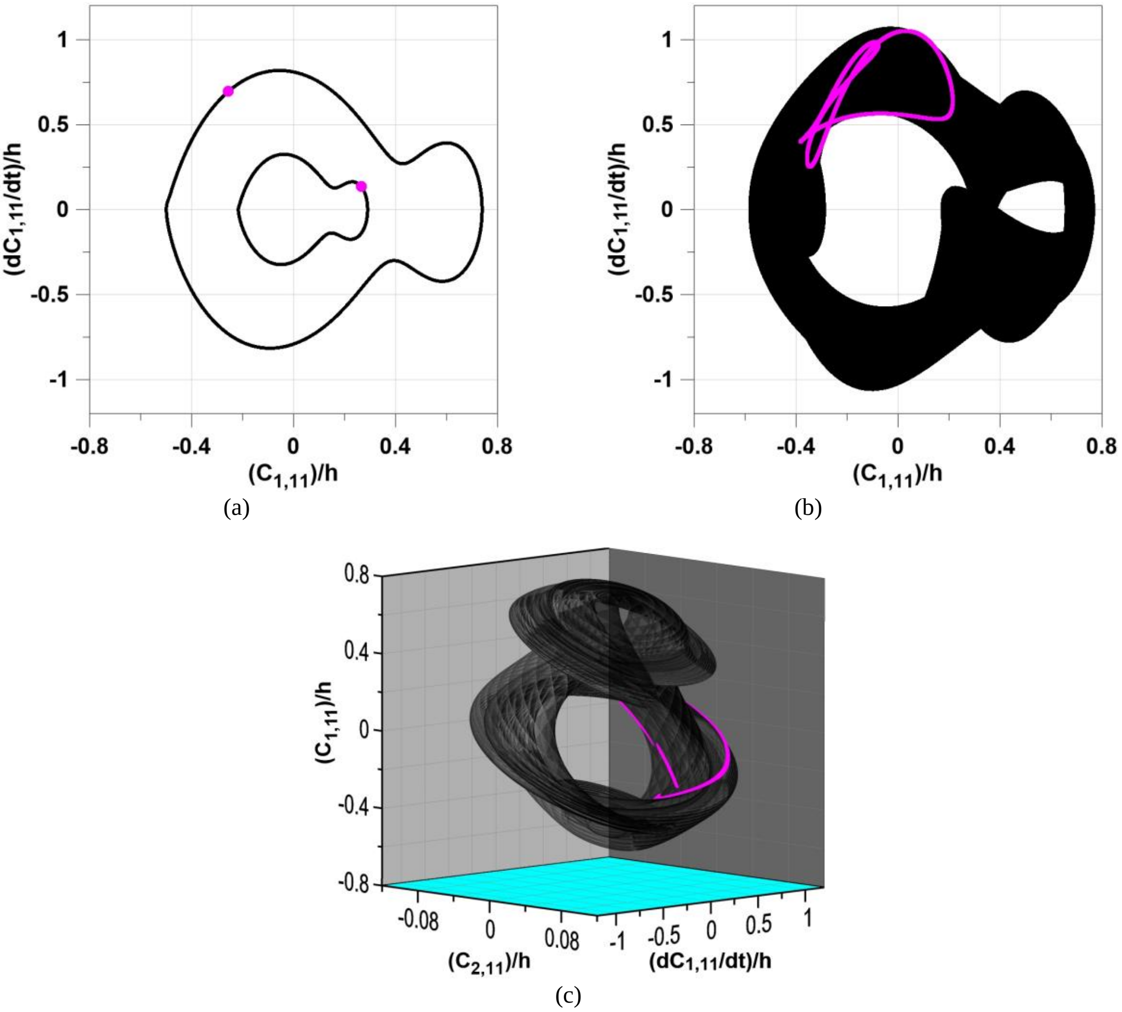


Figure 4. Phase-plane and Poincaré section for $\omega_L/\omega_0 = 1.0200$. (b) Phase-plane and Poincaré section for $\omega_L/\omega_0 = 1.0272$. (c) Phase-plane and Poincaré section for $\omega_L/\omega_0 = 1.0272$ – 3D.

4 Conclusions

In this work, an analytical model for an imperfect cylindrical panel in contact with a discontinuous unilateral elastic base was derived, using a perturbation technique to obtain the modal solution for transversal displacement field. The influence of the unilateral elastic base on the backbone were investigated, the results indicated that the curves show nonlinearity of the softening type and the modal solution with 2 DOF is enough to describe the nonlinear behavior of the cylindrical panel. Resonances curves were studied, the initial geometrical imperfection and unilateral elastic base showed a strong influence on the results. Bifurcation diagrams were investigated, showing unstable regions and bifurcation points. Poincaré mappings and phase planes were analyzed, based on the response of the resonance curve of the cylindrical panel with unilateral elastic base. The numerical results clarify the strong influence of the unilateral elastic base on the panel nonlinear oscillations and dynamic stability, which was affected by several new bifurcation points, specially Neimark-Sacker bifurcation points.

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References

[1] Amabili M., Païdoussis M. P. (2003) Review of studies on geometrically nonlinear vibrations and dynamics of circular cylindrical shells and panels, with and without fluid-structure interaction. Appl. Mech. Rev. 56:349-356.
[2] Alijani F., Amabili M. (2014) Non-linear vibrations of shells: A literature review from 2003 to 2013. Int. J. Non-Linear Mech. 58:233-257.
[3] Tj, H. G., Mikami, T., Kanie, S., & Sato, M. (2006). Free vibration characteristics of cylindrical shells partially buried in elastic foundations. Journal of Sound and Vibration, 290(3-5), 785-793.
[4] Bakhtiari-Nejad, F., & Bideleh, S. M. M. (2012). Nonlinear free vibration analysis of prestressed circular cylindrical shells on the Winkler/Pasternak foundation. Thin-Walled Structures, 53, 26-39.
[5] Bahadori, R., & Najafizadeh, M. M. (2015). Free vibration analysis of two-dimensional functionally graded axisymmetric cylindrical shell on Winkler–Pasternak elastic foundation by First-order Shear Deformation Theory and using Navier-differential quadrature solution methods. Applied Mathematical Modelling, 39(16), 4877-4894.
[6] Duc, N. D., Hadavinia, H., Van Thu, P., & Quan, T. Q. (2015). Vibration and nonlinear dynamic response of imperfect three-phase polymer nanocomposite panel resting on elastic foundations under hydrodynamic loads. Composite Structures, 131, 229-237.
[7] Duc, N. D., Kim, S. E., Manh, D. T., & Nguyen, P. D. (2020). Effect of eccentrically oblique stiffeners and temperature on the nonlinear static and dynamic response of S-FGM cylindrical panels. Thin-Walled Structures, 146, 106438.
[8] Babaei, H., & Eslami, M. R. (2021). On nonlinear vibration and snap-through buckling of long FG porous cylindrical panels using nonlocal strain gradient theory. Composite Structures, 256, 113125.
[9] Weitsman, Y. (1971). Onset of separation between a beam and a tensionless elastic foundation under a moving load. International Journal of Mechanical Sciences, 13(8), 707-711.
[10] Lenci, S.; Clementi, F. (2020) Flexural wave propagation in infinite beams on a unilateral elastic foundation. Nonlinear Dynamics, v. 99, n. 1, 721–735.
[11] Bhattiprolu, U., Bajaj, A. K., & Davies, P. (2016). Periodic response predictions of beams on nonlinear and viscoelastic unilateral foundations using incremental harmonic balance method. International Journal of Solids and Structures, 99, 28-39.
[12] Yang, J., Dong, J., & Kitipornchai, S. (2019). Unilateral and bilateral buckling of functionally graded corrugated thin plates reinforced with graphene nanoplatelets. Composite Structures, 209, 789-801.
[13] Panahandeh-Shahraki, D., Mirdamadi, H. R., Shahidi, A. R. (2013) Nonlinear buckling analysis of laminated composite curved panels constrained by Winkler tensionless foundation. Eur. J. Mech. A/Solids. 39:120-133.
[14] Silveira, R. A., Pereira, W. L., Gonçalves, P. B. (2008) Nonlinear analysis of structural elements under unilateral contact constraints by a Ritz type approach. Int. J. Solids Struct. 45:2629-2650.
[15] Morais J. L., Silva F. M. A. (2019) Influence of modal coupling and geometrical imperfections on the nonlinear buckling of cylindrical panels under static axial load. Eng. Struct. 183:816-829.