Comparative analysis between different integration methods for frames subjected to earthquakes

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Abstract. The choice of the type of structure model, the duration time of the dynamic loading, the size of the time step, as well as the inclusion or not of nonlinearity are factors that should be taken into consideration when deciding on different solution methods for the dynamic analysis, as they are able to influence the stability and accuracy in the response of the structure. This paper presents a comparison of dynamic analysis results for different integration methods in the time domain. This comparison is evaluated using a steel plane frame subjected to a Tohoku earthquake that occurred in 2011 in Japan. The structure is modeled in two-dimensional finite elements and discretized into bar elements. The entire analysis is developed by implementing a numerical routine in Python 3 language. It is expected to find distinct results regarding the accuracy in displacements, velocities and accelerations due to the spurious oscillations inserted due to the characteristic of the finite element method.

Keywords: Dynamic analysis, seismic excitation and numerical integration.

1 Introduction

The earthquake is an arbitrary dynamic loading in time. Thus, the solution of the general equation of motion for this type of loading can be solved in the frequency domain or in the time domain.

According to [1], solving the general equation of motion in the frequency domain is advantageous due to its speed in obtaining the transfer function for responses focused on stationary excitation problems (e.g., However, with respect to seismic excitations, the method has the following disadvantages: earthquakes are not stationary and therefore, it is necessary to select a long period of time for the solution of the finite-length earthquake to be completely damped; the transformation of the result from the frequency domain to the time domain, even with the use of Fast Fourier Transform (FFT) methods, requires considerable computational effort; moreover, it is limited to linear problems.

The solution in the time domain under arbitrary forces can be done using the modal superposition method or integration methods.

The modal superposition method works with the general equation of motion in an uncoupled manner in which the response is expressed as a serial expansion of the vibration modes and the contribution of modal participation. Thus, this is the main advantage of the method since it allows obtaining the response of only the predominant fundamental modes, and its effectiveness is remarkable, although it is limited only to linear cases [2].

According to [3] integration methods provide the only complete general approach for response analysis in the linear and nonlinear cases regardless of the structural behavior. This is due to the fact that the analyses in the time domain are performed step by step. In addition, they allow the contribution of high frequencies to the response to be taken into account directly. The integration methods are considered "direct" because unlike the modal superposition method, the equations of motion are not transformed before the numerical integration process, i.e., the equation of motion is solved in coupled form. According to [4] direct numerical integration is based on two main concepts, namely: the first concept, the equilibrium equation (elastic forces, damping, and inertia) must be satisfied at discrete points in the solution interval; in the second concept, the interpolation function must be assumed for the displacements, velocities, and accelerations within each time interval (dt). These two concepts as well as the size of the time interval determine the accuracy, consistency, and stability of the method. Depending on the type of analysis, excitation duration, and system model, integration methods can result in a high computational cost, which is a possible drawback of the method. However, it is up to the analyst to choose the most appropriate and advantageous method for each type of problem to be solved, and in the literature there are several integration methods proposed, from traditional ones such as central finite differences, Newmark's method, Wilson-θ, HHT-α, to more recent methods such as the methods proposed by [5] and [6].

Integration methods can be explicit, i.e., when the current solution depends on the current and previous step, or implicit, when the current solution depends on the future step as well. Moreover, they can be considered conditionally or unconditionally stable, where the limiting factor is the minimum size of the time interval (dt).

Regarding seismic analysis, knowing that the duration time of the event can be long and that the sampling frequency of the detection devices delimit a constant and small time interval, as pointed out by [7] and [8], classical methods such as the Newmark method, because it is an implicit method and unconditionally stable depending on its parameters, has been widely used in research involving excitations of long duration as is the case of earthquakes, bringing good accuracy, stability and consistency in the results, while still maintaining its value.

2 Verification of integration methods

To verify the dynamic response of the structure using a Python programmed routine, the Newmark (mean acceleration), Wilson-θ, HHT-α and central finite difference integration methods were developed and compared with the analytical solutions for the mass-spring-damper system employed with 4Dof (degree of freedom) and subjected to a harmonic external force with non-zero initial conditions exposed by [9], The results were evaluated at durations T = 0s, and T = 7 seconds. The system is shown in Figure 1, and the data are described in Table 1.

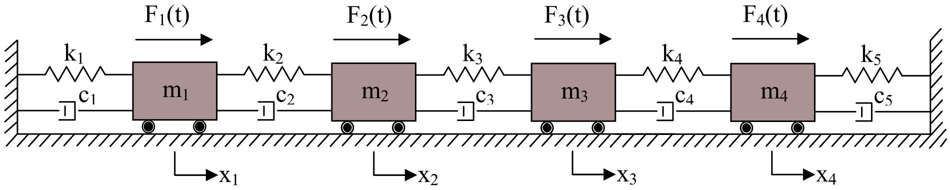

Figure 1. Forced Harmonic Vibration System with 4DoF

Table 1. Mass-spring-damper system data with 4 DoF

DoF	m [kg]	k [N/m]	c [Ns/m]	x_0 [m]	$\dot{x}_0$ [m/s]	$F(t)$ [N]
1	8	30	6	-0.500565	0.750247	$+20sen(5t)+80\cos(5t)$
2	9	45	9	-0.055132	-0.410529	$-50sen(5t)+60\cos(5t)$
3	5	50	10	-0.814934	-0.411299	$+70sen(5t)+35\cos(5t)$
4	6	20	4	0.450169	0.480426	$-45sen(5t)-25\cos(5t)$
-	-	25	5	-	-	-

The analytical solution of the equation of motion of the system with MDoF (multi degree of freedom) externally excited with harmonic forces (Equation 1) is presented in its matrix form in Equation 2, and the exact particular solution is depicted in Equation 3.

$$\mathbf{M}\ddot{x}+\mathbf{C}\dot{x}+\mathbf{K}x=\vec{F}_A\mathrm{sen}(5t)+\vec{F}_B\cos(5t). \tag{1}$$

$$\begin{bmatrix} \left(-\mathbf{M}\omega^2+\mathbf{K}\right) & -\mathbf{C}\omega \\ \mathbf{C}\omega & \left(-\mathbf{M}\omega^2+\mathbf{K}\right) \end{bmatrix} \begin{bmatrix} \vec{A} \\ \vec{B} \end{bmatrix} = \begin{bmatrix} \vec{F}_A \\ \vec{F}_B \end{bmatrix}. \tag{2}$$

$$x_p\left(t\right)=\begin{bmatrix} 0.150049 \\ -0.08211 \\ -0.08226 \\ 0.096085 \end{bmatrix} \operatorname{sen}\left(5t\right)+\begin{bmatrix} -0.50056 \\ -0.05513 \\ -0.81493 \\ 0.450169 \end{bmatrix} \cos\left(5t\right). \tag{3}$$

Thus, the responses in terms of displacements of the analytical and numerical solutions for the mass spring damping system with 4 DoF were compared for $dt = 0.02s$, shown in Table 2.

Table 2. Mass-spring-damper system data with 4 DoF

DoF	Analytical		Newmark		Central F. Dif.		Wilson-θ		HHT-α	
	T = 0s	T = 7s	T = 0s	T = 7s	T = 0s	T = 7s	T = 0s	T = 7s	T = 0s	T = 7s
1	-0.501	0.388	-0.501	0.388	-0.501	0.388	-0.501	0.389	-0.501	0.388
2	-0.055	0.085	-0.055	0.084	-0.055	0.085	-0.055	0.083	-0.055	0.084
3	-0.815	0.772	-0.815	0.771	-0.815	0.773	-0.815	0.773	-0.815	0.771
4	0.450	-0.448	0.450	-0.447	0.45	-0.449	0.450	-0.448	0.450	-0.447

After the analyses of the responses in terms of displacements, it is concluded that the computational routine developed in Python is verified, and therefore can be applied to solve more sophisticated dynamic problems, e.g., earthquakes and MDoF structures.

3 Studied case

Due to its long duration time (i.e., T = 300 seconds) with dt=0.01s, being considered one of the earthquakes with the highest intensity, which occurred at 14:46 (JST, GMT þ 9) on March 11, 2011, the Tohoku earthquake was used as an excitation in a plane frame structure in the dynamic analysis for evaluation of different integration methods was Tohoku. The accelerogram can be seen in Figure 2, and the earthquake characteristics are shown in the Table 3.

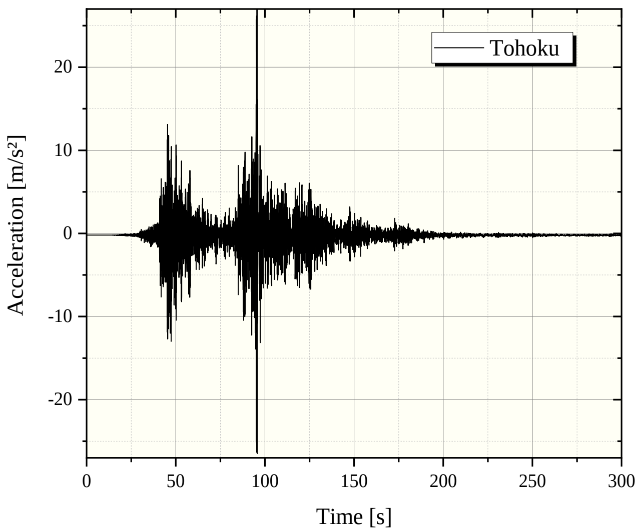

Figure 2. Tohoku accelerogram

Table 3 – Earthquake characteristic

Fault	Name	Data	St.	Country	Hip. [Km]	Mw	PGA [g]	dt [s]	Database
Far	Tohoku	11/03/2011	MYG 004	JP	131.0	9.1	2.82	0.005	K-NET

The frame under study is a four-story building (Figure 3), where the first floor has a height of 4.57 meters and the others 3.66 meters. The total length of the building is 27.42 meters, with three spans of 9.14 meters. The structure itself is made of steel with properties equal to $\rho = 7850 kg/m^3$, $E = 200GPa$ and $\nu = 0.3$. The properties of the steel profiles employed are shown in Table 4.

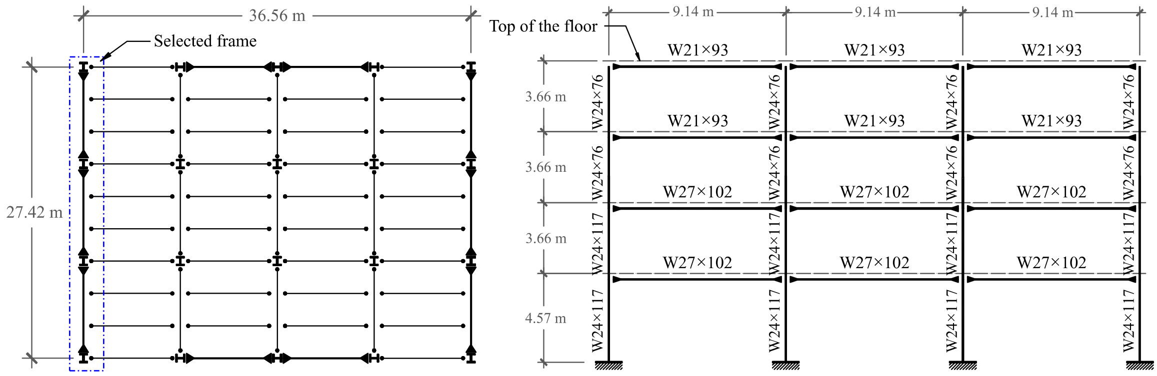

Figure 3. Floor plan and elevation of the selected frame structure

Table 4. Locating and geometric properties of the profiles

Element Type	Location	Section	A [$\times 10^{-2}$ m^2]	I [$\times 10^{-4}$ m^4]
Column	1st and 2nd floor	W24×117	2.2194	14.7346
	3rd and 4th floor	W24×76	1.4452	8.7409
Beam	1st and 2nd floor	W27×102	1.9355	15.0676
	3rd and 4th floor	W21×93	1.7613	8.6160

The response of structures subjected to dynamic loading is totally linked to their dynamic properties that are obtained through modal analysis, i.e., the natural frequencies and their respective vibration modes. Unlike superposition methods, the integration methods used in this research do not require a previous modal analysis for the dynamic solution. However, it is understood that this previous step is essential for a better understanding and analysis of the responses of the structures that will later be installed control systems in accordance with the external excitation and modal shape.

Aiming to verify the correct assembly of the mass and stiffness matrix, the frame of the building presented in the work of [10] was selected. Thus, the natural frequencies and vibration modes were calculated through the routine developed in Python, and therefore compared with the results obtained in the finite element software, Abaqus®.

The frame was considered as a multiple degree of freedom system (MGDL), with 28 elements and 20 nodes. The structure was modeled in plane frame bar elements, i.e., element with 2 nodes and 3 GDL per node. Regarding the Abaqus® software, the element used to represent each member of the plane frame was the finite element B23, i.e., two-dimensional, elastic-linear Euler-Bernoulli beam element that uses cubic interpolation. The B23 element considers the consistent mass formulation and has three degrees of freedom at each node, being two translational degrees of freedom X (1) and Y (2) and one rotational degree of freedom (3) around the Z axis [11].

Table 5. Comparison of natural frequencies between Abaqus® software and the Python computational routine

Vibration Modes	Natural frequencies	
	Abaqus®	Python
1	6.151	6.151
2	17.525	17.517
3	34.899	34.925

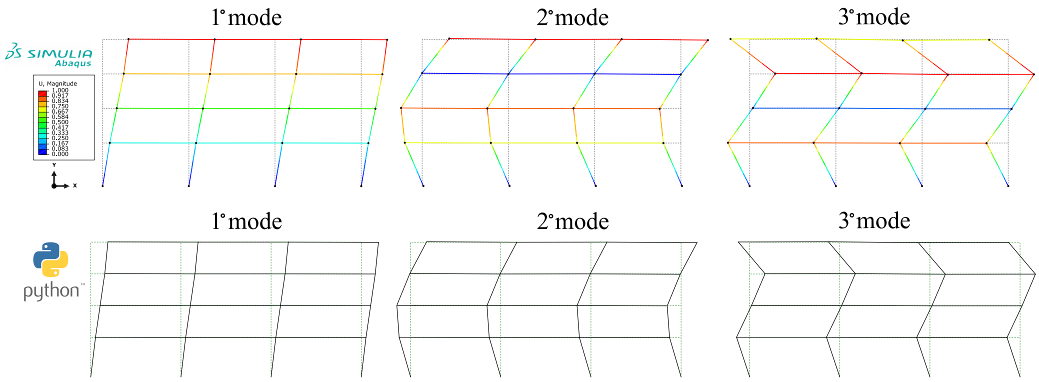

Figure 4. Comparison of vibration modes between Abaqus® software and Python computational routine

The response of the structure at the par displacement times for the last floor is presented in Figure 5. The Newmark, Wilson-θ and HHT-α integration methods were evaluated.

It should be noted that the central finite difference method was not evaluated because it is considered a conditionally stable method. Therefore, as the excitation is of long duration, and the time step is considered small (dt=0.01s), the computational cost became very high.

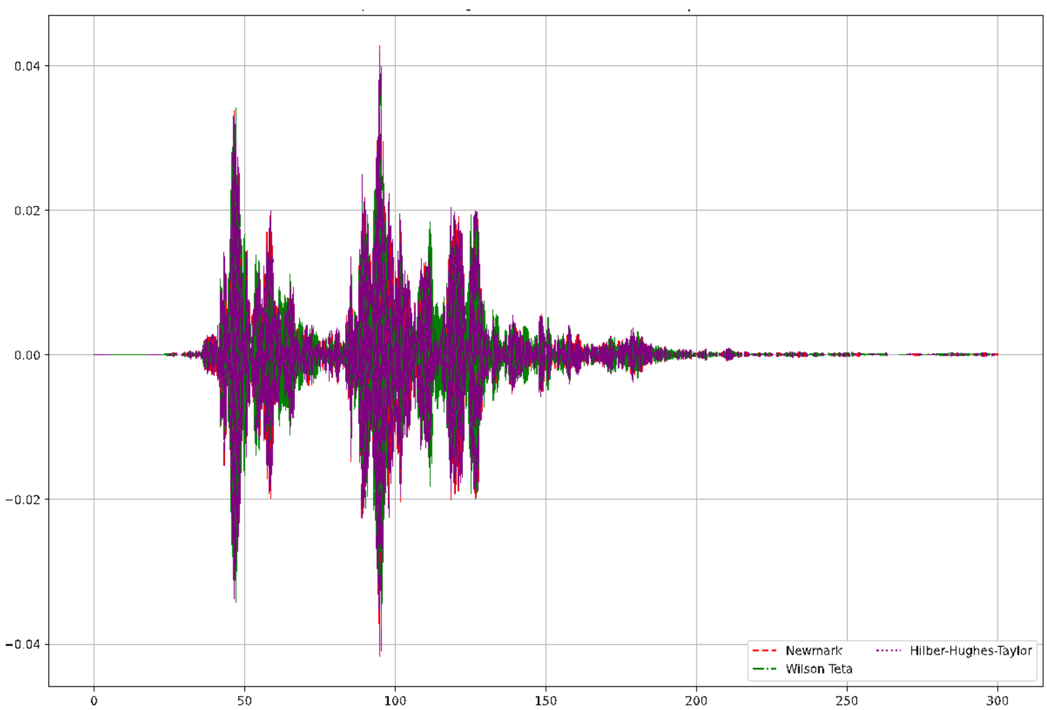

Figure 5. Comparison of integration methods in terms of displacement

4 Conclusions

Integration methods are the most general way of solving the differential equation of motion, and are valid for arbitrary excitations in time, for both linear and nonlinear systems.

However, as far as the methods are concerned, they have differences in accuracy and stability. This is because of the interpolation functions that determine stability, consistency and accuracy.

Overall the HHT-α method attenuated the response due to the numerical damping included in the method. For systems with MDoF where the frequencies in the high modes are relevant, this method is an excellent choice, due to the spurious oscillation that occurs in the finite element method.

Overall the Newmark method is widely used in seismic analysis due to its accuracy and performance.

The Wilson-θ method proved to be inferior to the two methods mentioned above.

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