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Structural Damage Identification based on the Curvature Matrix of the Accelerance

Juan C. Herrera

Civil & Industrial Engineering Department, Pontificia Universidad Javeriana-Seccional Cali
Calle 18 No. 118-250, 760031, Cali, Colombia
juancherrera@javerianacali.edu.co

Abstract. In this work, a damage identification method based on the curvature-matrix of the Frequency Response Function-FRF is proposed. This is used in the formulation of two indices to predict the damage location and to estimate the severity of the damage in a structure. To evaluate the effectiveness of the method, numerical simulations are performed in different damage scenarios simulated in a one-storey, one-bay frame using Euler-Bernoulli beam elements. The structural damage is simulated by reducing the flexural stiffness of selected elements. The FRF-Accelerance for the undamaged and damaged frame is numerically obtained with the frequencies and mode shapes of the lower modes. The curvature of the FRF is calculated numerically using finite differences, based on an expansion of the Taylor series. The results of the simulations indicate that the proposed method can localize and estimate the damage severity in a one-storey, one-bay frame. The proposed damage indices could be an alternative to the traditional modal analysis methods.

Keywords: Damage identification, FRF, Taylor series, Finite Element.

1 Introduction

The assessment of structural condition of existing civil infrastructures is essential to prevent potential catastrophic events and to the planning future investments for their rehabilitation and reparation. The presence of damage in a structure can be produced by the application of higher loads different to the specified in the original design or due to that its mechanical properties have deteriorated by aging or environmental impact. The structural deterioration leads to changes in the modal parameters (i. e., natural frequencies, mode shapes, modal damping ratios, etc.) of the structure and therefore these changes could be theoretically used to detect the presence of damage in actual structures. In the last two decades a variety of methods based on Frequency Response Function (FRF) data have been proposed in the technical literature (Thyagarajan et al. [1], Lee and Shin [2]. The use of FRFs for damage detection have definite advantages over the traditional methods using modal analysis data (He [3]).

In this paper, a methodology for structural damage identification based on the FRF-curvature is formulated. First, the concept of FRF-curvature matrix is formulated. Next, two damage indices based on the variations of this matrix over the structure are proposed. The damage indices are conceived to indicate the damage location and to estimate the severity of the damage in a structure directly from the measured FRF-Accelerance. Finally, numerical simulations are performed to evaluate the effectiveness of the proposed damage indices to localize and quantify the severity of damage in a one-storey, one-bay frame.

2 Formulation

In this work, a damage index based on the Accelerance-curvature matrix is proposed. The index was conceived to predict the damage location and to estimate the severity of the damage in a structure directly from the measured FRF-Accelerance. The damage indices are based on the change of the FRF-Curvature at the members of the structure for a given excitation frequency.

2.1 The Frequency Response Function (FRF)

The general mathematical representation of a single degree of freedom (SDOF) system can be expressed by

$$m\ddot{x}(t)+c\dot{x}(t)+k\dot{x}(t)=F(t) \tag{1}$$

Assuming that the forcing function is harmonic of the form $F(t)=Fo*e^{i\,\Omega t}$ and the damping is linear and viscous, the Frequency Response Function (FRF)-Accelerance for multiple degree of freedom (MDOF) systems with classic damping, the FRF between the degrees of freedom r and s is defined as

$$\Lambda_{rs}(\Omega)=\sum_{j=1}^{n}\frac{\Omega^2\phi_{rj}\phi_{sj}}{\omega_j^2-\Omega^2+(2\zeta_j\omega_j\Omega)i} \tag{2}$$

where, ϕ_{ij} is the row "*i*" of the vibration mode "*j*"; ω_j *y* ξ_j are the natural frequency and the damping ratio of the "*j*" mode, respectively. $\Lambda_{rs}(\Omega)$ is the acceleration of the DOF *r* due to a single harmonic force excitation of unit amplitude applied at the DOF *s*. This FRF is defined as the Accelerance function.

The functions $\Lambda_{rs}(\Omega)$ defined by the Equation (2) can be arranged in matrix form. This leads to the Accelerance matrix defined as

$$\left[\Lambda(\Omega)\right]=\begin{bmatrix}\Lambda_{11} & \Lambda_{12} & \cdots & \Lambda_{1n}\\ \Lambda_{21} & \Lambda_{22} & \cdots & \Lambda_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \Lambda_{n1} & \Lambda_{n2} & \cdots & \Lambda_{nn}\end{bmatrix} \tag{3}$$

The curvature matrix of the FRF-Accelerance $\left[\Lambda''(\Omega)\right]$ is defined as:

$$\left[\Lambda''(\Omega)\right]=\begin{bmatrix}\Lambda''_{11} & \Lambda''_{12} & \cdots & \Lambda''_{1n}\\ \Lambda''_{21} & \Lambda''_{22} & \cdots & \Lambda''_{22}\\ \vdots & \vdots & \ddots & \vdots\\ \Lambda''_{n1} & \Lambda''_{n2} & \cdots & \Lambda''_{nn}\end{bmatrix} \tag{4}$$

The matrix for damage identification based on the Accelerance curvature, is defined by

$$\left[\Theta\right]_{nxn}=\left[\Lambda''(\Omega)\right]_{nxp}\left[\Lambda''(\Omega)\right]_{pxn}^{T} \tag{5}$$

where,

$\left[\Lambda''(\Omega)\right]$: curvature matrix of the FRF-*Accelerance* for a frequency Ω.

n: number of points where the FRF is obtained
p: number of points where the input force is applied.

If the input force is applied at a single point "*k*", the diagonal of $\left[\Theta\right]$ is defined as

$$\left\{\alpha\right\} = \left\{(\Lambda''_{1,k})^2 \quad (\Lambda''_{2,k})^2 \quad \cdots \quad (\Lambda''_{n,k})^2\right\} \tag{6}$$

The Accelerance-curvature for each frequency can be calculated numerically by using a central finite-divided difference based on an expansion of the Taylor series. For the Accelerance FRF measured at location *j* due to a excitation force at position *k*, the curvature of the Accelerance $\Lambda_{jk}(\Omega)$ can be calculated by:

$$\Lambda''_{jk}(\Omega) = \frac{\left|\Lambda_{j+1,k}(\Omega)\right| - 2\left|\Lambda_{j,k}(\Omega)\right| + \left|\Lambda_{j-1,k}(\Omega)\right|}{h^2} \tag{7}$$

where h is the distance between two consecutive measurement points: *(j) and (j+1)* or *(j) and (j-1)*

For the damaged structure, the FRF curvature matrix is defined by

$$\left[\Theta_*\right]_{nxn} = \left[\Lambda''_*(\Omega)\right]_{nxp}\left[\Lambda''_*(\Omega)\right]^T_{pxn} \tag{8}$$

where $\Lambda''_*(\Omega)$ is the curvature of the FRF-Accelerance for the damaged structure.

If the input force is applied at a point "*k*", the diagonal of $\left[\Theta_*\right]$ is defined as

$$\left\{\alpha_*\right\} = \left\{(\Lambda''_{*1,k})^2 \quad (\Lambda''_{*2,k})^2 \quad \cdots \quad (\Lambda''_{*n,k})^2\right\} \tag{9}$$

In terms of the FRF curvature matrices, the index for damage location expresses the relationship between the states of a structure element with damage and without damage, defined by:

$$\left\{\chi\right\} = \left\{\alpha_*\right\}./\left\{\alpha\right\} \tag{10}$$

In the Equation (10) the symbol (./) is used to indicate that the division of the vectors is done element by element. The proposed damage severity index is defined by

$$\left\{\Gamma\right\} = \lambda\left[1 - \{\alpha\}./\{\alpha_*\}\right] \tag{11}$$

where,

$$\lambda = \frac{\sum_{i=1}^{n} (\Lambda''_{i,k})^2}{\sum_{i=1}^{n} (\Lambda''_{*i,k})^2} \tag{12}$$

After the damage index Γ_j is computed, the value of the indicator is normalized according to the rule formulated by Kim and Stubbs [4]:

$$Z_j = \frac{\Gamma_j - \mu}{\sigma} \tag{13}$$

where μ and σ are, respectively, the mean value and the standard deviation of the damage index Γ_j.

The damage is assigned to the elements by using a statistical-pattern-recognition technique that utilizes hypothesis testing, proposed by Kim and Stubbs [4]:

The null hypothesis, referred to as H_0, corresponds to the structure *not* damaged at the element *j* and the alternate hypothesis, denoted as H_1, means that the structure *is* damaged at the element *j*. To assign damage to a specific location, the following decision rule is used: (a) select H_0 if $Zj < 1{,}9$ or (b) select the alternate H_1 if $Zj \geq 1{,}9$. This test corresponds to a confidence level of 97 %.

3 Numerical simulations

To demonstrate the effectiveness of the proposed damage indices to locate and estimate the structural damage, the one-storey frame shown in Figure 1 is considered. Five damage scenarios are analyzed. The FRF and the FRF-curvature of the damaged and undamaged frames were generated numerically. The following dimensions and material properties are used for the frame: span length L= 2.5 m, height of the frame H=2.5 m, height of the cross section h=0.125 m, width of the cross section b= 0.05 m, elastic modulus E = 200000 MPa and mass density ρ= 20.21 kg/m3. For the numerical simulation the beam was divided into 20 finite elements of equal length. Five damage scenarios were studied and are listed in Table 1. The structural damage is simulated by reducing the flexural stiffness of an element near the beam's mid-span.

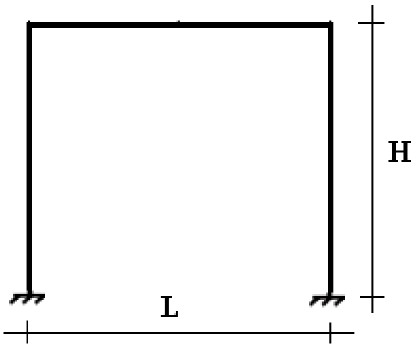

Figure 1. One-bay frame.

To calculate the proposed indices, the values of the FRF-Accelerance were obtained at 19 locations equally spaced along the longitudinal axis of the beam. The FRF-Accelerance was calculated with Equation (2) for the second and fourth modes and was assumed that the modal damping ratio ξ_j was constant for all the modes and equal to 0.05. Using the values of the FRF-Accelerance, the FRF-curvatures were generated numerically via a central difference approximation by using Equation (7).

Table 1. Damage scenarios.

Damage scenario	Location	EI reduction
DS1	0.5L	0.45
DS2	0.5L	0.40
DS3	0.5L	0.35
DS4	0.5L	0.30
DS5	0.5L	0.25

The damage location index χ is shown in Figure 2. The indices were calculated for a frequency of 225 rad/s. The estimated location of damage and the severity index values for cases DS1 to DS4 are shown in Figure 3. The estimated severity of damage is listed in Table 2. As can be seen, for the damage scenarios analyzed, the proposed indices are able to indicate the location of the damaged regions of the frame´s beam and the severity of the defects.

Table 2. Damage severity.

Damage Scenario	Simulated Damage	Estimated Damage	Error (%)
DS1	0.45	0.40	10.0
DS2	0.40	0.38	5.0
DS3	0.35	0.348	0.5
DS4	0.30	0,31	3,3
DS5	0.25	0,27	8,0

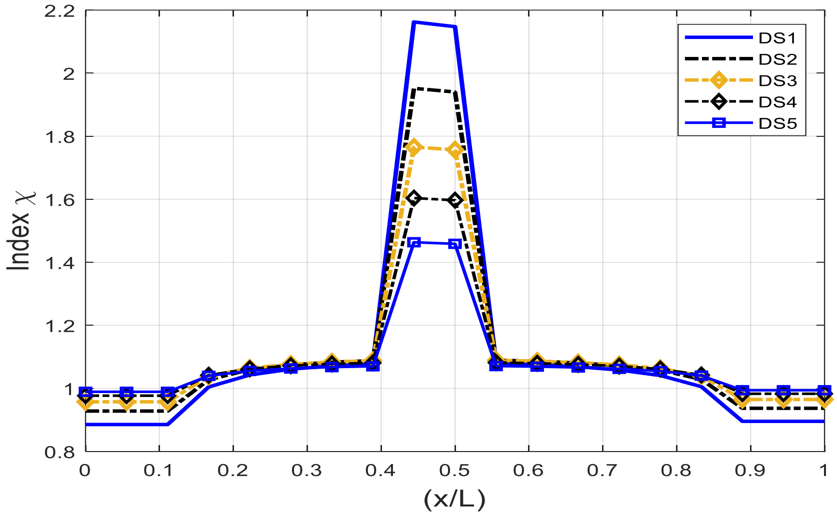

Figure 2. Damage location.

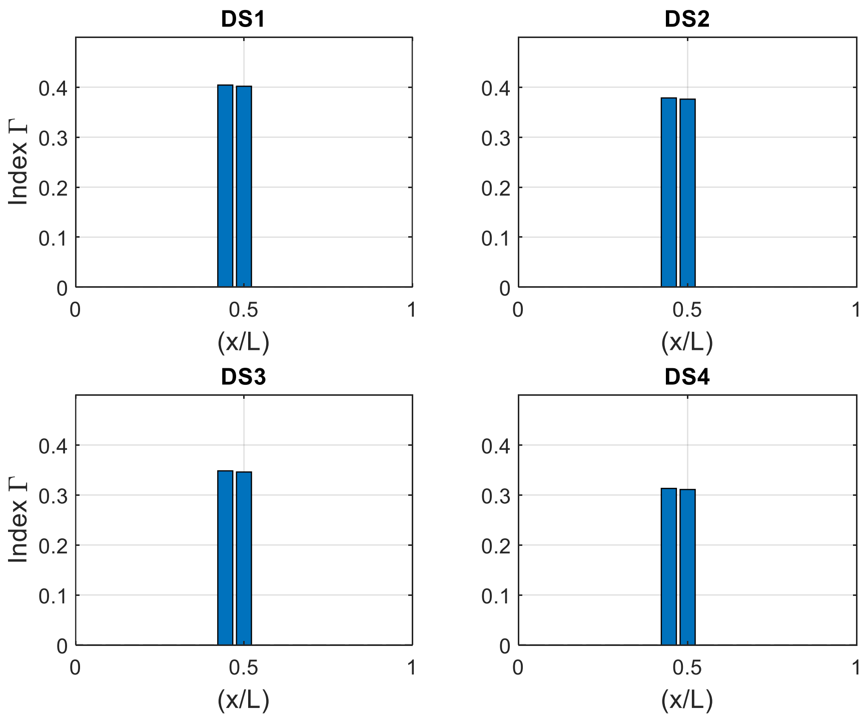

Figure 3. Damage severity.

4 Conclusions

This article presents the theoretical formulation of two indices to locate and quantify the severity of the damage in a structure directly from the Frequency Response Function, based on the matrix of FRF-Accelerance curvatures. The indices were evaluated for different damage scenarios, varying the severity of the damage in a one-bay frame using Euler-Bernoulli beam theory. Errors in estimating the damage severity ranged from 0.5% to 10%. The results obtained from the numerical simulations indicate that the proposed indexes can locate and estimate the damage severity for single damage scenarios in the analyzed frame. Although errors in the quantification of damage severity were obtained, for practical engineering purposes these could be acceptable. The proposed damage indices could serve as an alternative to the traditional techniques of damage identification through modal analysis.

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