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Evaluation of Structural Dynamic Modification by Viscoelastic Neutralizers based on Response Reanalysis Methods

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Abstract. Structural modifications can occur for several reasons, such as unsatisfactory product for the customer, structural defects, updating of obsolete parts, generation of intense noise from the equipment to the operator, and unwanted vibrations. When there is some prior knowledge of the dynamic characteristics of the mechanical system of interest, particularly in association with the region where the changes will occur, a practical and convenient approach is to analyze the effects of these changes using reanalysis techniques. These techniques aim to predict the dynamic behavior of the system after implementing modifications, from a compact and specific set of data related to the system and the modifications. The present work aims to investigate the use of two response reanalysis techniques - in matrix formulation - to evaluate the effects of inserting viscoelastic dynamic neutralizers as localized structural modifications. Viscoelastic neutralizers are considered in the context of future implementation of vibration control on a cantilever steel beam. In modeling the neutralizers, concepts of generalized equivalent parameters are used to maintain the same dimension in the system matrices before and after inserting the devices. To validate the use of the employed techniques, their predictions are compared to the results obtained by an established computational program dedicated to the optimal design of dynamic neutralizers in mechanical systems, named LAVIBS-ND®. It is shown that the investigated techniques do not differ, as far as their predictions are concerned, from the technique currently used in the above mentioned program, which is employed as a benchmark to assess the accuracy of the evaluated methods. It is then concluded that the focused techniques are actually capable of accurately predicting the effects of structural modifications by viscoelastic dynamic neutralizers for vibration control purposes.

Keywords: Response Reanalysis, Structural Modification, Vibration control, Viscoelastic material, Viscoelastic neutralizer.

1 Introduction

When a mechanical system of interest undergoes structural modifications and there is some prior knowledge of the dynamic characteristics of the system, particularly in association with the region in which the changes occur, a practical and convenient approach to analyze the effects of such modifications consists of using reanalysis techniques. These techniques aim to predict the dynamic response of the system after the changes have been implemented, on the basis of a compact and specific set of data related to the system and to the changes themselves.

A frequent cause for making changes in mechanical systems is the existence of unwanted vibrations. These vibrations can lead, among other effects, to early component wear, failure due to fatigue, excessive noise, loosening of bolted connections, and unsatisfactory manufacturing of parts. An effective technique for such situations is the use of dynamic neutralizers, which are auxiliary (secondary) devices that, when inserted into a vibrating mechanical system (then called primary system), seek to reduce vibrations by redistributing the motion energy along frequency. When dampened, these devices also promote the dissipation of such energy.

The insertion of dynamic neutralizers can be considered a structural modification and, as such, can be analyzed through reanalysis methods, as observed in He and Fu [1]. This is also extended to viscoelastic dynamic neutralizers, which, by containing elements of viscoelastic materials, will act to redistribute and dissipate motion energy along frequency. In general, viscoelastic materials are widely used in vibration control due to their manufacturing versatility and low cost combined with their energy storage and dissipation properties, which are frequency and temperature dependent.

In order to analyze its dynamic behavior, a mechanical system of interest can be described through its modal parameters or its frequency response functions (FRFs). While in the first case the modeling is based on natural frequencies, damping ratios, and vibration modes, in the second case system functions are used, such as receptances, mobilities, and inertances. Since modal models and response models can be constructed either exactly or approximately, the corresponding reanalysis methods can be classified based on the above mentioned characteristics, as seen in Brandon [2]. In the present work, exact response reanalysis methods are considered.

Exact response reanalysis methods were employed in the design of viscoelastic neutralizers in Lopes [3] and Rodrigues et al. [4]. In Lopes [3], the matrix product method - described below - was adopted for analyzing modifications by viscoelastic devices in an aluminum frame, considering both numerical and experimental responses. On the other hand, in Rodrigues et al. [4], using the matrix partition method - also described below -, a multi-degree-of-freedom viscoelastic neutralizer was designed for reducing vibration in a wide frequency range in a cantilever beam.

More recently, Caixu et al. [5] referred to reanalysis when reviewing the subject of typical vibrations of machine tools in milling operations (known as chatter). It is considered the assessment of the mass loading effect of sensors which are used for obtaining FRFs to predict the dynamic behavior of the machines of concern. In that context, the modification is the additional presence of force and vibration transducers.

This paper aims to investigate two matrix response reanalysis methods, namely, matrix partition and matrix product. The objective is to evaluate their respective efficiencies in predicting the effects of inserting a dynamic viscoelastic neutralizer for passive vibration control into frequency bands of a simple structure, namely, a cantilever beam. For the purposes of proving the effectiveness of the methods, the modal approach implemented in the LAVIBS-ND® computer program is taken as a reference for it is already clearly consolidated for the design of viscoelastic neutralizers, as shown in Voltolini et al. [6].

2 Response Reanalysis

2.1 Motion Equations

Considering a linear mechanical system with multiple degrees of freedom, one has, in the time and frequency domain, the following equations of motion ([2], [3], [7], [8]):

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{f(t)\} \rightarrow \{\bar{X}(\omega)\}\big(-\omega^2[M] + i\omega[C] + [K]\big) = \{\bar{F}(\omega)\} \tag{1a}$$

$$[M]\{\ddot{x}(t)\} + \big([K] + i[H]\big)\{x(t)\} = \{f(t)\} \rightarrow \{\bar{X}(\omega)\}\big(-\omega^2[M] + i[H] + [K]\big) = \{\bar{F}(\omega)\} \tag{1b}$$

In the above equations, $[M]$ is the mass matrix, $[C]$ is the viscous damping matrix, $[H]$ is the hysteretic damping matrix, and $[K]$ is the stiffness matrix. Also in those equations, $\{x(t)\}$, $\{\dot{x}(t)\}$, $\{\ddot{x}(t)\}$ and $\{f(t)\}$ are, respectively, the generalized displacement, velocity, acceleration, and force vectors in the time domain, while $\{\bar{X}(\omega)\}$ and $\{\bar{F}(\omega)\}$ are the generalized displacement and force vectors in the frequency domain.

Equation 1a refers to the viscous damping case, and Equation 1b refers to the hysteretic damping case. If proportional viscous damping is assumed, it follows that $[C] = \alpha[M] + \beta[K]$, where α and β are constants, to be usually determined by experiments ([8], [9]).

Matrices relating displacements $\{\bar{X}(\omega)\}$ to forces $\{\bar{F}(\omega)\}$ in Equations 1a and 1b are known, in this context, as dynamic stiffness matrices. They consist of the mass, stiffness, and damping matrices and are denoted by $[\bar{S}(\omega)]$, where the overbar indicates their character as complex matrices. The inverses of these matrices are the receptance matrices, so that $[\bar{R}(\omega)] = [\bar{S}(\omega)]^{-1}$.

2.2 Structural Change Representation

When a structural modification is performed, by using with viscoelastic devices under constant temperature, one can change - in the equations above - the mass matrix by $[\Delta M(\omega)]$, the stiffness matrix by $[\Delta K(\omega)]$, and/or the damping matrix by $[\Delta C(\omega)]$ or $[\Delta H(\omega)]$. Thus, according to the damping model employed and for the sake of generality, one has ([3], [6])

- Dynamic stiffness matrix for a general modification

$$\big[\Delta\bar{S}(\omega)\big] = -\omega^2\big[\Delta M(\omega)\big] + i\omega\big[\Delta C(\omega)\big] + \big[\Delta K(\omega)\big] \tag{2a}$$

$$= -\omega^2\big[\Delta M(\omega)\big] + i\big[\Delta H(\omega)\big] + \big[\Delta K(\omega)\big] \tag{2b}$$

- Dynamic stiffness matrix for a localized modification

$$[\Delta\bar{S}(\omega)]_{n\times n} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & 0 \\ 0 & 0 & \Delta\bar{S}(\omega)_{r\times r} \end{bmatrix} \tag{3}$$

where n is the number of degrees of freedom of the system, and r is the number of degrees of freedom of the modification.

It follows then that the modified receptance matrix, denoted by $[\bar{R}(\omega)]^*_{n\times n}$, is given by

$$[\bar{R}(\omega)]^*_{n\times n} = \Big([\bar{S}(\omega)]_{n\times n} + [\Delta\bar{S}(\omega)]_{n\times n}\Big)^{-1} \tag{4}$$

2.3 Response Reanalysis Methods

Matrix Partition

As seen in Rodrigues et al. [4] and Soares [10], the elements of the receptance matrix of the modified (composite) system in the partition associated with the degrees of freedom of the modification can be obtained by

$$[\bar{R}(\omega)]^*_{jj} = \Big([\bar{R}(\omega)]^{-1}_{jj} + [\Delta\bar{S}(\omega)]_{jj}\Big)^{-1} \tag{5}$$

where j indicates the partition of interest, with dimension $r \times r$. This method is called 'Matrix Partition Method', since its focus lies on the partition of the modified receptance matrix that is associated with the degrees of freedom in which the structural modification takes place - as illustrated in Equation 6. The demonstration of Equation 5, which results from the application of matrix inversion concepts from Linear Algebra, can be found in Soares [10].

$$[\bar{R}(\omega)]^*_{n\times n} = \tag{6}$$

$$\begin{bmatrix} \bar{R}(\omega)^*_{1,1} & \bar{R}(\omega)^*_{1,2} & \cdots & \bar{R}(\omega)^*_{1,n} \\ \bar{R}(\omega)^*_{2,1} & \bar{R}(\omega)^*_{2,2} & \vdots & \vdots \\ \vdots & \vdots & \ddots & \\ \bar{R}(\omega)^*_{n,1} & \bar{R}(\omega)^*_{n,2} & \cdots & [\bar{R}(\omega)]^*_{22} \end{bmatrix} = \begin{bmatrix} \bar{R}(\omega)^*_{1,1} & \bar{R}(\omega)^*_{1,2} & \cdots & \bar{R}(\omega)^*_{1,n} \\ \bar{R}(\omega)^*_{2,1} & \bar{R}(\omega)^*_{2,2} & \vdots & \vdots \\ \vdots & \vdots & \ddots & \\ \bar{R}(\omega)^*_{n,1} & \bar{R}(\omega)^*_{1,2} & \cdots & \begin{bmatrix} \bar{R}(\omega)^*_{1,1} & \cdots & \bar{R}(\omega)^*_{1,r} \\ \vdots & \ddots & \vdots \\ \bar{R}(\omega)^*_{r,1} & \cdots & \bar{R}(\omega)^*_{r,r} \end{bmatrix}^*_{22} \end{bmatrix}$$

Matrix Product

Now consider - as in Brandon [2], Lopes [3], and Soares [10] - that the dynamic stiffness matrix of the modification can be expressed by the following product:

$$[\Delta S(\omega)_{n\times n}] = [U(\omega)]_{n\times r} \times [V(\omega)]_{r\times n} \tag{7}$$

Then, the modified receptance matrix $[R(\omega)]^*$ can be given by

$$[R(\omega)]^*_{n\times n} = [R(\omega)]_{n\times n} - \Big[[R(\omega)]_{n\times n}[U(\omega)]_{n\times r}[W(\omega)]^{-1}_{r\times r}[V(\omega)]_{r\times n}[R(\omega)]_{n\times n}\Big] \tag{8}$$

where

$$[W(\omega)]_{r\times r} = [I_r] + \Bigg[[V(\omega)]_{r\times n}[R(\omega)]_{n\times n}[U(\omega)]_{n\times r}\Bigg] \tag{9}$$

This method is called 'Matrix Product Method' since the modification is represented as a product of two matrices. The demonstration of Equation 8, which also derives from the application of matrix inversion concepts from Linear Algebra, can be found in Brandon [2], Lopes [3], and Soares [10].

For modifications in which the number of degrees of freedom of the modification is much smaller than the number of degrees of freedom of the system, i.e., $r \ll n$, the application of any of the methods is much more convenient than a new thorough analysis of the composite mechanical system (original mechanical system + modification). If the interest is restricted to the region in which the modification occurs, the matrix partitioning method is the most suitable, in view of its focus. However, if points in other regions need to be considered, the matrix product method is recommended because it provides all the elements of the modified receptance matrix.

3 Methodology

3.1 Primary System and Modification

To study the reanalysis methods presented above, the modification of a cantilever beam - shown on the left in Figure 1 - by a single-degree-of-freedom viscoelastic dynamic neutralizer (VDN) - illustrated in its typical form on the right side of Figure 1 - was considered. The beam was modeled through the Finite Element Method (FEM), with 10 nodes in transverse translation, and also experimentally investigated through experimental modal analysis (EMA) at the same points. The nominal dimensions of the beam were 1500 mm long, 50 mm wide, and 7.94 mm thick.

The VDN was optimally designed using the LAVIBS-ND® software. This design was based on the modal parameters of the beam, and LAVIBS-ND® provided the device's optimal mass and characteristic frequency. The chosen viscoelastic material was EAR™ C-1002, from Aearo Technologies LLC, the frequency range for vibration reduction was 150 to 350 Hz, and the operation temperature was 20 ^{o}C (293 K).

Figure 1. Cantilever Beam + Viscoelastic Dynamic Neutralizer

3.2 Update and Validation of the Numerical Model

In order to update and validate the numerical model of the beam based on the performed experimental measurements, a hybrid optimization procedure was employed, in which two non-linear optimization techniques were sequentially associated ([11]). Initially, the genetic algorithm technique was applied (to approximate the values of the variables in the design vector), and then, the SQP (Sequential Quadratic Programming) technique was applied (to refine the values of the variables) from their respective ga and fmincon implementations in the MATLAB® computational environment. In the second optimization technique, constraints were used for the relevant variables. Due to the great discrepancy among the orders of magnitude of the design variables, a linear normalization scheme was adopted throughout the optimization procedure.

Through the above mentioned procedure, it was sought to minimize the difference (error) among parameters from experimental and numerical FRFs (inertances) and, thus, determine values for the following quantities: α (constant that multiplies the mass matrix for the insertion of proportional viscous damping), β (constant that also multiplies the stiffness matrix for the insertion of proportional viscous damping), t (beam thickness), E (Young's modulus of the beam material) and ρ (beam material density). The minimization of the difference among parameters contemplated the following actions:

1. Bringing the numerical natural frequencies closer to the experimental ones;
2. Bringing the numerical damping ratios closer to the experimental ones;

3. Bringing the numerical amplitudes at resonance closer to the experimental ones.

Through these actions, the aim was to approximate the experimental and numerical inertances using a compact volume of information and to update the numerical model with the values found for the design vector variables.

Designating the natural frequencies by ω_n, the damping ratios by ζ, and the amplitudes at resonance by A_r, in addition to using the superscripts $^{\text{exp}}$ and $^{\text{num}}$ to indicate, respectively, experimental and numerical values, the addressed optimization problems can be expressed, for clarity and convenience, as follows:

To minimize the objective function $f(x)$ given by

$$f(x) = \sum_{j=1}^{7} \left(\frac{\omega_{n,j}^{\text{exp}} - \omega_{n,j}^{\text{num}}}{\omega_{n,j}^{\text{exp}}} \right)^2 + \sum_{j=1}^{7} \left(\frac{\zeta_j^{\text{exp}} - \zeta_j^{\text{num}}}{\zeta_j^{\text{exp}}} \right)^2 + \sum_{j=1}^{7} \left(\frac{\mathrm{A}_{r,j}^{\text{exp}} - \mathrm{A}_{r,j}^{\text{num}}}{\mathrm{A}_{r,j}^{\text{exp}}} \right)^2 \tag{10}$$

where the vector of design variables x is such that

$$x = \left(\alpha, \beta, t, E, \rho\right)^T \tag{11}$$

whereas in the second part of the hybrid optimization procedure the problem is subject to the constraints

$$\alpha_{\text{min}} \leq \alpha \leq \alpha_{\text{max}} \qquad \beta_{\text{min}} \leq \beta \leq \beta_{\text{max}} \qquad t_{\text{min}} \leq t \leq t_{\text{max}}$$
$$E_{\text{min}} \leq E \leq E_{\text{max}} \qquad \rho_{\text{min}} \leq \rho \leq \rho_{\text{max}}$$

where subscripts $_{\text{min}}$ and $_{\text{max}}$ refer, respectively, to the minimum and maximum values of the variables. Although not explicitly stated in Equation 10, the parameters listed in it, as a whole, are dependent on the variables listed in Equation 11.

The obtained experimental inertance related the force applied via impact hammer near the beam clamp - at node 1 - to the acceleration measured via accelerometer at the free end - at node 10. As the natural frequencies of the first seven vibration modes of the beam were far apart, it was possible to estimate them by the peak picking method, as well as to estimate the damping ratios by the half power bandwidth method ([8]).

3.3 Inserting Modification

It was assumed that the viscoelastic dynamic neutralizer would be inserted at the free end of the beam. Therefore, for the application of the response reanalysis methods, the point inertance at node 10 was employed, which associates, at that point, applied force and corresponding acceleration. Based on this FRF - converted to receptance - and on the characteristics of the viscoelastic neutralizer provided by LAVIBS-ND®, the modified receptances were determined, both by the Matrix Partition Method and the Matrix Product Method. The results obtained were compared with one another and also with the result provided by LAVIBS-ND®, for the purpose of confirming the prediction indicated by the reanalysis methods.

4 Results and Discussions

This section presents the results found in both the validation of the numerical model and the application of the response reanalysis methods.

4.1 Validation of the Numerical Model

The values obtained for the variables of the design vector using hybrid optimization are shown below. In Figure 2, a comparison is made between the experimental point inertance at the end of the beam and its numerical counterpart, obtained through the Finite Element Model and updated as explained in the previous section.

The values of the variables of the design vector are the following:

- $\alpha = 0.2300$
- $\beta = 7.682 \times 10^{-6}$
- $t = 7.985$ mm ($t_{\text{reference}} = 7,938$ mm)
- $E = 1.900 \times 10^{11}$ Pa ($E_{\text{reference}} = 2 \times 10^{11}$ Pa)
- $\rho = 7939$ kg/m^3 ($\rho_{\text{reference}} = 7860$ kg/m^3)

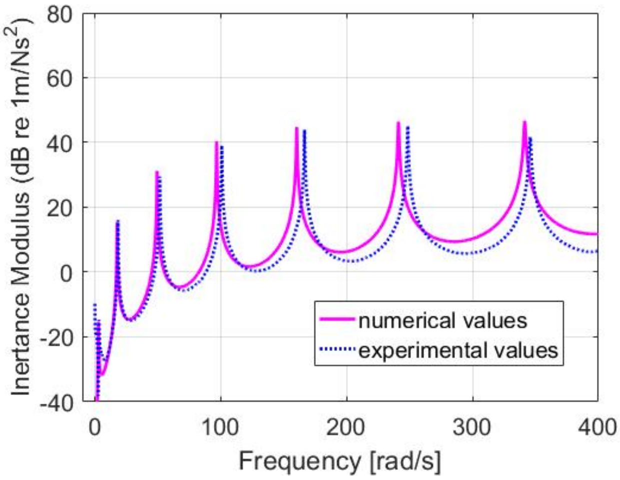

Figure 2. Comparison of Experimental and Numerical Inertances at Beam End

4.2 Characteristics of the Viscoelastic Neutralizer

The optimal design of the viscoelastic neutralizer by means of the LAVIBS-ND® software resulted in the characteristics listed in Table 1.

Table 1. Mass and Optimal Frequency of the Viscoelastic Neutralizer

frequency range	mass	characteristic frequency
150 to 350 Hz	179 g	122, 6 Hz

4.3 Primary and Composite Systems

Figure 3 shows the graphs of the FRFs (receptances) in the frequency range of the neutralizer action. The figure shows: in light blue, the original FRF of the primary system through the numerical model of the beam; in green, the original FRF of the primary system through the LAVIBS-ND® modal method; in black, the modified FRF of the composite system through the matrix partition method; in red, the modified FRF of the composite system through the matrix product method; and finally, in pink, the modified FRF of the composite system through the LAVIBS-ND® modal method.

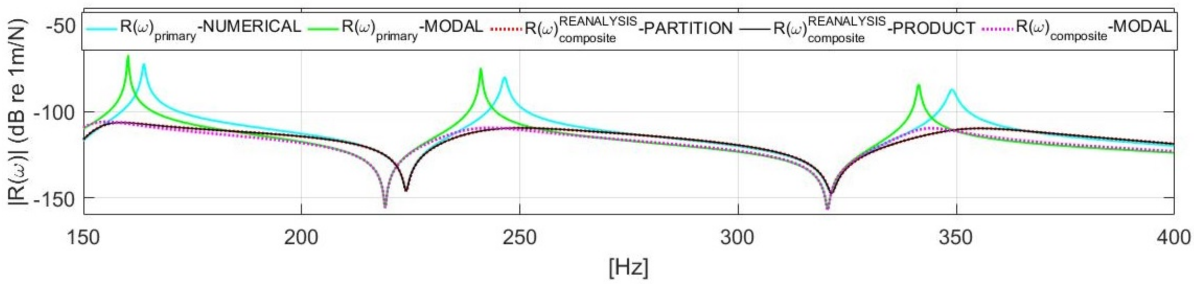

Figure 3. Receptances of the Primary and Composite Systems (Primary + Neutralizer)

As already expected, Figure 3 shows that the investigated reanalysis methods present identical responses for the modified FRFs. The slight discrepancy of the modified FRF of the composite system (primary system + neutralizer) provided by the modal method, relative to the modified FFRs provided by the reanalysis methods, is explained by the fact that the original FRF used by LAVIBS-ND® is different from the original FRF employed by the reanalysis methods. It should be noted that the FRFs are built in LAVIBS-ND® using the beam modal parameters, whereas the reanalysis methods use FRFs built from the updated numerical model of the same beam.

In Figure 4, the modified FRFs are determined based on the same original FRF of the primary system used by LAVIBS-ND®. It can then be seen that there is no difference between the modified FRFs.

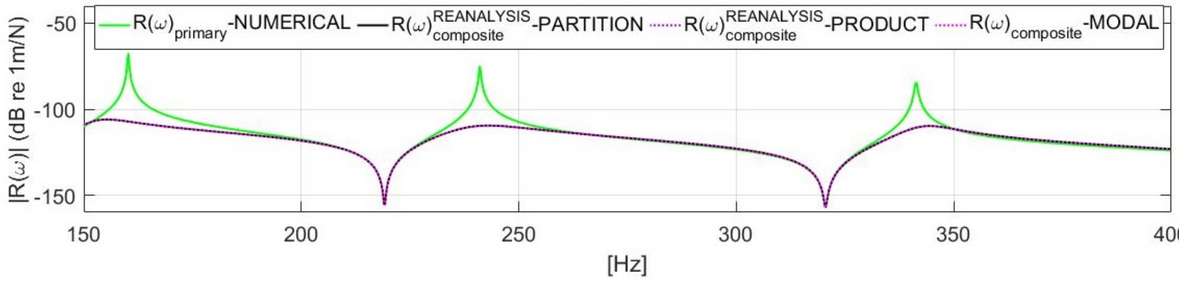

Figure 4. Composite System Receptances via Identical Original Receptance

5 Conclusions

As expected, the investigations showed that there is no difference between the predictions generated by the Matrix Partition Method and the Matrix Product Method. This stems from the fact that both are exact methods, based on concepts from Linear Algebra and Structural Dynamics. It was also found that when the reanalysis methods use the same FRFs generated by the LAVIBS-ND® software through the modal approach, the predictions of one and the other approach, respectively supported by response and modal models, are identical. It is therefore understood that the response reanalysis methods represent alternatives that should also be considered for the optimal design of viscoelastic dynamic neutralizers in the future.

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