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Evaluation of Numerical Parameters of a global-local GFEM approach simulating damage propagation in a L-shaped concrete panel

Anelize B. Monteiro[1], Felício B. Barros[2], Roque L. S. Pitangueira[2], Samuel S. Penna[2]

[1]*Federal University of Santa Catarina*
8300 Dona Francisca St., CEP 89219-600, Joinville, SC, Brazil
anelize.m@ufsc.br
[2]*Federal University of Minas Gerais*
Federal University of Minas Gerais, 6627 Antonio Carlos Ave.,CEP 31270-901, Belo Horizonte, MG, Brazil
felicio@dees.ufmg.br, roque@dees.ufmg.br, spenna@dees.ufmg.br

Abstract. The nonlinear modeling of concrete structures requires strain-softening models that properly represent the nucleation and propagation of damage. The description of those phenomena, by Finite Element Method (FEM), is highly dependent on the quality of the mesh and the type of the approximation function adopted. The Generalized Finite Element Method (GFEM) has been developed in order to overcome some limitations inherent to the FEM aiming to use some knowledgment about the expected solution behavior to improve the analysis. The GFEM enriches the space of the polynomial FEM solution with a priori known information based on the concept of Partition of Unit. In this context, the global-local approach to the GFEM (GFEM global-local) is investigated here as an alternative to the standard GFEM to describe the deterioration process of concrete media in the context of Continuous Damage Mechanics. Succinctly, the global-local function used to enrich the global problem is obtained through physically nonlinear analysis performed only in the local domain, represented by constitutive models and discretized by a refined mesh, where in fact damage propagation occurs. In the global domain discretized by a coarse mesh, it is performed a linear analysis considering the incorporation of local damage through the global-local enrichment functions and damage state mapped from local problem. In this paper, the Smeared Crack Model is the constitutive model used in the local domain to simulate the damage propagation experimentally obtained in an L-shaped concrete panel. The numerical simulations aim to evaluate the influence of the following numerical parameters of the GFEM global-local approach on the equilibrium paths: number of local steps added to each block of global-local analysis and the size of the global step. The obtained results are compared with the experimental ones, the most suitable sets of parameters can be found and then applied in other simulations that involve the expansion of the local domain and the variation of the nodes enriched with the global-local function.

Keywords: GFEM global-local, nonlinear analysis, Continuous Damage Mechanics.

1 Introduction

The application of the GFEM to the nonlinear analysis and representation of the damage and plasticity fronts is well consolidated. In this context, the GFEM global-local was proposed by Duarte and Babuška (2005)[1] and widely studied by authors such as Kim *et al*. (2012)[2], Freitas *et al*. (2015)[3] and Kim and Duarte (2015)[4]. The latter applied the global-local GFEM in the three-dimensional analysis of the propagation of cohesive cracks in concrete structures. Evangelista Jr *et al*. (2020)[5] proposed the formulation and development of GFEM global-local strategy which incorporates a Continuum Damage Model that uses scalar damage variable for quasi-brittle materials to simulate failure in mode I and mixed-mode crack propagation.

In this paper, a global-local approach to the GFEM based on Kim and Duarte (2015)[4] is applied to describe the deterioration process of quasi-brittle media within the context of Continuous Damage Mechanics, by assuming a constitutive models to represent the concrete in the local region: smeared crack model of fixed direction with the Carreira and Chu (1985, 1986)[6] stress-strain laws. The numerical solution used to enrich the global problem is obtained through physically nonlinear analysis performed only in the local region. With the damage of the local region incorporated into the global problem, through the global-local enrichment functions, the linear analysis is performed in the global region. This process is carried out in blocks of global-local analysis able to capture the evolution of the deterioration process and their influence on the global behavior of structures.

This GFEM global-local approach was implemented by Monteiro (2019)[7] in the computational system INSANE (*INteractive Structural ANalysis Environment*) ((Gori *et al.*, 2017)[8]), and in the Section 2 the implemented formulation is summarized. The Section 3 presents the numerical examples.

2 Formulation

Each block of global-local analysis has three stages:

Stage 1. Initial and estimated linear global problem

For the initial linear global problem, step $k = 0$, the global domain is defined by $\bar{\Omega}_G = \Omega_G \cup \partial\Omega_G$ in R^n. The vector field $\mathbf{u}^0_{G,0}$ is the approximate solution of the weak form of the initial global problem:

$$\int_{\Omega_G} \sigma(\mathbf{u}^0_{G,0}) : \varepsilon(\mathbf{v}^0_{G,0})\, d\mathbf{x} + \int_{\partial\Omega^u_G} \mathbf{u}^0_{G,0} \cdot \mathbf{v}^0_{G,0}\, d\mathbf{s} = \int_{\partial\Omega^\sigma_G} \bar{\mathbf{t}} \cdot \mathbf{v}^0_{G,0}\, d\mathbf{s} + \int_{\partial\Omega^u_G} \bar{\mathbf{u}} \cdot \mathbf{v}^0_{G,0}\, d\mathbf{s}, \tag{1}$$

where $\mathbf{v}^0_{G,0}$ are the test functions of the initial global problem, σ is the stress tensor, ε is the strain tensor, $\bar{\mathbf{t}}$ is the prescribed stress vector, and $\bar{\mathbf{u}}$ is the prescribed displacement vector.

The solution $\mathbf{u}^0_{G,0}$ is obtained for the entire load (load factor $\lambda = 1$) and then it is adjusted according to the size of the displacement step P_{DG} (predefined for the global problem in the control node). The load factor is obtained by:

$$\lambda^0 = \frac{P_{DG}}{u^0_{G,0,DC}}, \tag{2}$$

where $u^0_{G,0,DC}$ is a displacement component of the control node, obtained from eq. (1).

To $k \geq 1$, in the estimated linear global problem, $\mathbf{u}^k_{G,0}$ is estimated by the following expression, adapted from Kim and Duarte (2015):

$$\mathbf{u}^k_{G,0} = \frac{(k+1)}{k}\mathbf{u}^{k-1}_G. \tag{3}$$

Stage 2. Nonlinear local problem

The local problem is solved incrementally-iteratively in the local domain Ω_L.The local displacement vector $\mathbf{u}^k_L$ is calculated by the following equation, which has boundary conditions from the initial global solution of the Stage 1.

$$\int_{\Omega_L} \sigma(\mathbf{u}^k_L) : \varepsilon(\mathbf{v}^k_L)\, d\mathbf{x} + \eta \int_{\partial\Omega_L \cap \partial\Omega^u_G} \mathbf{u}^k_L \cdot \mathbf{v}_L\, d\mathbf{s} = \int_{\partial\Omega_L \cap \Omega^\sigma_G} \bar{\mathbf{t}} \cdot \mathbf{v}^k_L\, d\mathbf{s} + \int_{\partial\Omega_L \backslash (\partial\Omega_L \cap \partial\Omega_G)} [\mathbf{t}(\mathbf{u}^k_G) + \eta\mathbf{u}^k_G] \cdot \mathbf{v}^k_L\, d\mathbf{s}, \tag{4}$$

where η is the penalty parameter, $\mathbf{v}^k_L$ are the test functions of the local problem, and $\mathbf{t}(\mathbf{u}^k_G)$ is the stress vector

In this stage of each block of global-local analysis, it is necessary to solve the problem from the beginning of the loading, up to the level of loading of the block. In order to adequately represent the problem, the number of local steps resolved at each block is increased. The number of total local steps (N_{LS}) solved in each block k is given by:

$$N_{LS} = N_{IL} + [(k+1)N_{AL}], \tag{5}$$

where N_{IL} is the number of initial local steps, and N_{AL} is the number of local steps added to each global-local block.

Stage 3. Enriched linear global problem

The constitutive relation is given by $\sigma = \mathbf{C}^s : \varepsilon$, where ε is the strain tensor and $\mathbf{C}^s$ is the secant approximation of the constitutive tensor adopted in the balance of the global model and obtained considering the damage occurred in the local problem. In this stage, local solution $\mathbf{u}^k_L$ is applied as extrinsic basis for enriching the global problem:

$$\{\phi_J\}(x) = \mathcal{N}_J(x) \times \mathbf{u}^k_L, \tag{6}$$

where J is referred to nodal points, $\mathcal{N}_J$ is the PoU function of the initial global problem and $\mathbf{u}^k_L$ is the local solution, named global-local enrichment function. The global enriched problem is defined by:

$$\int_{\Omega_G} \sigma(\mathbf{u}^k_G) : \varepsilon(\mathbf{v}^k_G)\, d\mathbf{x} + \int_{\partial\Omega^u_G} \mathbf{u}^k_G \cdot \mathbf{v}^k_G\, d\mathbf{s} = \int_{\partial\Omega^\sigma_G} \bar{\mathbf{t}} \cdot \mathbf{v}^k_G\, d\mathbf{s} + \int_{\partial\Omega^u_G} \bar{\mathbf{u}} \cdot \mathbf{v}^k_G\, d\mathbf{s}, \tag{7}$$

The solution $\mathbf{u}_G^k$ is obtained for the entire load (load factor $\lambda = 1$). $\mathbf{u}_G^k$ is adjusted according to the size of the displacement step S_{GD} predefined for the global problem. The load factor λ_E^k is defined as:

$$\lambda_E^k = \frac{(k+1)S_{GD}}{u_{G,DC}^k}, \tag{8}$$

where $u_{G,DC}^k$ is a displacement component of the control node, obtained from eq.(7).

Figure 1 presents the solution algorithm of the proposed approach. k is the global-local analysis block, i is the local step and j is the local iteration.

begin
- execute();
- **foreach** *block* k **do**
  - Solve Stage 1:
  - **if** k*=0* **then**
    - Solve linear equation system and get $\mathbf{u}_{G,0}^0$
  - **else**
    - Get the estimated solution $\mathbf{u}_{G,0}^k = \frac{(k+1)}{k}\mathbf{u}_G^{k-1}$
  - **end**
  - Transfer boundary condition from Stage 1 to Stage 2;
  - Solve Stage 2:
  - **foreach** *local step i=i+1* **do**
    - **repeat**
      - Assemble stiffness matrix $[K]_{j-1}^i$;
      - Get the incremental displacement $\left\{\Delta U^P\right\}_j^i$ e $\left\{\Delta U^Q\right\}_j^i$;
      - Get the load factor increment $\Delta\lambda_j^i$;
      - Update the nodal displacement vector $\{U\}_j^i = \{U\}_{j-1}^i + \Delta\lambda_j^i \ \left\{\Delta U^P\right\}_j^i + \left\{\Delta U^Q\right\}_j^i$;
      - Update the load factor $\lambda_j^i = \lambda_{j-1}^i + \Delta\lambda_j^i$;
      - Get the vector of equivalent nodal internal forces $\{F\}_j^i$;
      - Update the residual forces vector $\{Q\}_j^i = \lambda_j^i \ \{P\} - \{F\}_j^i$;
    - **until** *convergence*;
  - **end**
  - Solve Stage 3:
  - Enrich global problem with $\mathbf{u}_L^k$ obtained in 2;
  - Solve the linear equation system and calculate $\mathbf{u}_G^k$;
- **end**

end

Figure 1. Solution algorithm to the nonlinear global-local approach.

3 Numerical Simulations

The smeared crack model of fixed direction with the Carreira and Chu (1985, 1986) [6] stress-strain laws, available in the library of constitutive models of INSANE, it is used in the numerical simulations of a L-shaped concrete panel. To evaluate the performance of the proposed global-local approach to the GFEM in the nonlinear analysis of quasi-brittle media, the numerical results are compared to the experimental ones of Winkler *et al*. (2004) [9].

Figure 2 shows the geometry, loading and boundary conditions of these tests. The loading is distributed with the value of $q = 28.0$ N/mm and the panel thickness is 100 mm. In the global meshes the point A corresponds to the node whose vertical displacement is considered in the composition of the equilibrium paths. In the local mesh point B is adopted as a control node in the nonlinear analysis by the displacement control method.

The smeared crack model is based on monitoring the deterioration of the material's physical properties, and the crack evolution process is described by the gradual decay of stress with an increase in strain. In relation to the adopted materials, in the global problem the material is initially linear elastic and its parameters are defined in the Tab. 1. In the local problem, as well as in the conventional GFEM analysis, it is adopted a smeared crack model of fixed direction and plane stress state. The concrete properties obtained by Winkler *et al*. (2004) [9] and the parameters adopted in thesmeared crack model are also presented in the Tab. 1.

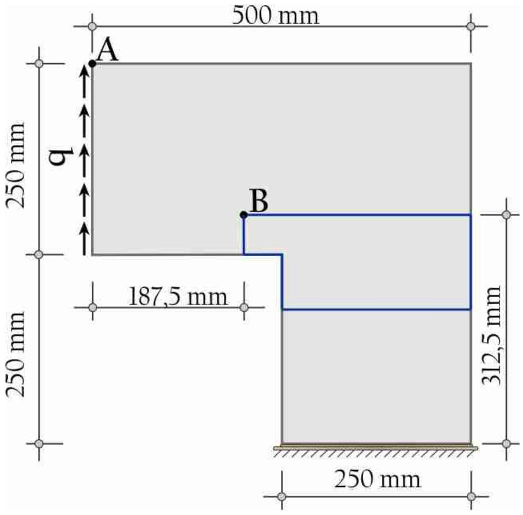

Figure 2. L-shaped concrete panel: geometry, boundary and loading conditions.

Table 1. Material parameters of L panel

L-shapep concrete panel: material properties (Winkler *et al*., 2004) [9]	
Poisson ratio	$\nu = 0.18$
Young's modulus	$E = 25850.0$ MPa
Compressive strength	$F_c = 31.0$ MPa
Tensile strength	$F_t = 2.70$ MPa
Carreira and Chu (1985, 1986) parameters	
Compressive strength	$F_c = 31.0$ MPa
Tensile strength	$F_t = 2.70$ MPa
Compressive strain	$\varepsilon_c = 0.002$
Tensile strain	$\varepsilon_t = 0.0001925$
Shear retention factor	$\beta_r = 0.0$

The nonlinear analysis of the local problem is performed with the displacement control method (Batoz and Dhatt, 1979) [10], secant approximation to the constitutive tensor and tolerance to convergence equals to $1 \times 10^{-5}(\times 100\%) = 0.0010\%$ in relation to the norm of incremental displacements vector. In local mesh, penalty parameter is $\eta = 1 \times 10^{10}$ and there are 4×4 Gauss points per element (same number in the global mesh).

The numerical analysis evaluated the influence of the following numeric parameters on the obtained equilibrium paths: number of local steps, variables or not, added steps and global steps. The global and local meshes are shown in the Fig. 3. The global mesh has 192 four-noded quadrilateral elements and 29 global nodes enriched with only the local numerical solution. The local mesh is composed of 36 four-noded quadrilateral elements. As these analysis are preliminary in order to understand the influence of numerical parameters of the proposed approach, there was no refinement of the local mesh. The aim here is to isolate the effect of the process of solving the nonlinear problem at the local scale, disregarding the best description of the behavior that the refinement of the local model would provide.

The first investigation is about the influence of the number of local steps added to each block of analysis. This parameter ranged from 0, 5, 10, 15 and 20, while the following parameters were fixed:

- 25 global steps;
- 10 initial local steps;
- displacement global step of 0.04 mm; and
- displacement control method: point B in the vertical direction.

The equilibrium paths of the Fig. 4 refer to the variation in the number of steps added to each local step of the global-local analysis block, together with the experimental results of Winkler *et al*. (2004) [9].

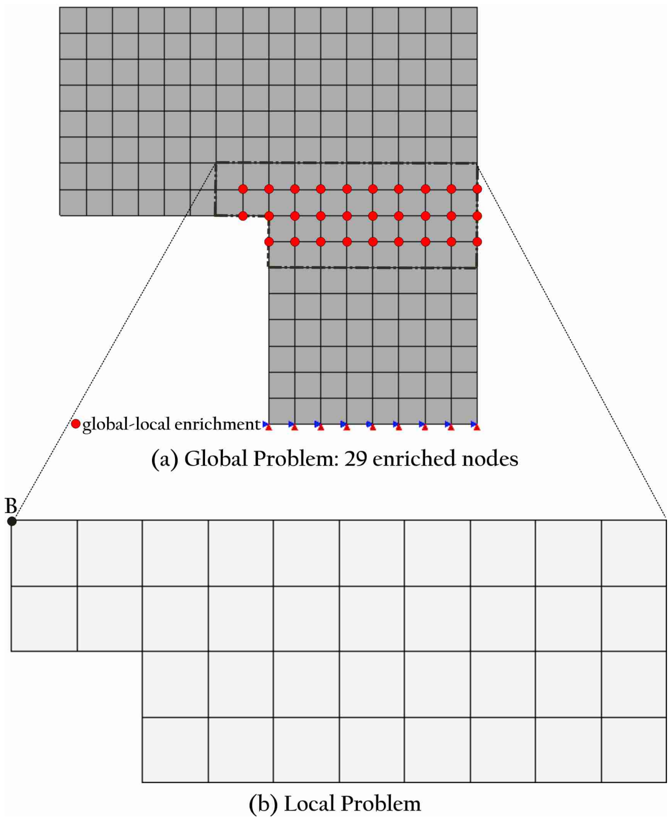

Figure 3. L-shaped concrete panel: global and local problems.

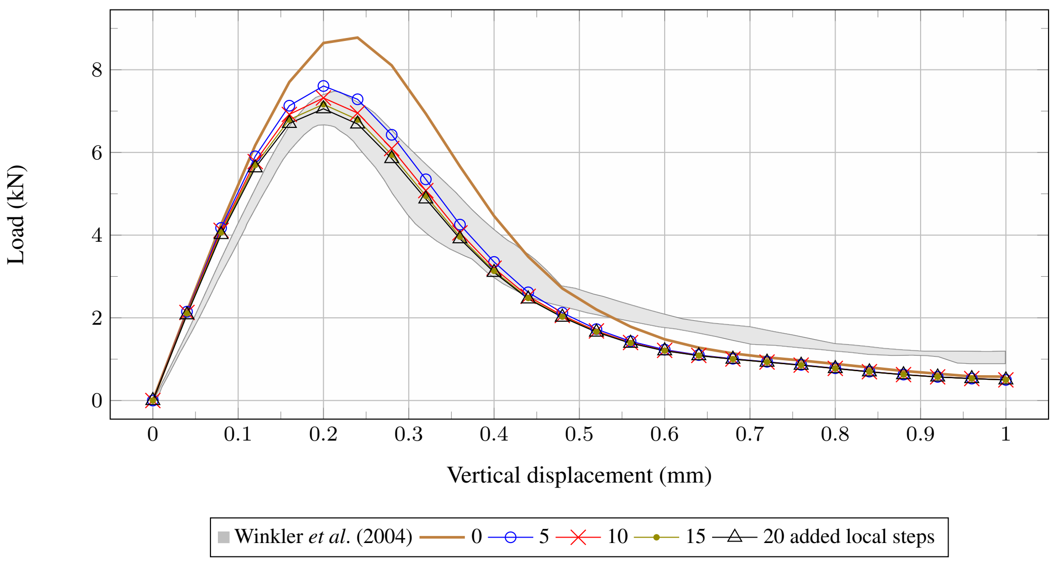

Figure 4. Variation of the number of steps added to each local step of the global-local analysis block for the global displacement step of $0,04$ mm.

In the Fig. 4, except for the analysis without additional steps in each analysis block, the other values added produced very close equilibrium paths, especially those of 10, 15 and 20 local steps added. As the number of local steps makes the analysis more expensive, it is possible to indicate the use of 10 additional local steps, without prejudice to the representation of the global behavior of the problem. In the equilibrium path named 0, it is observed that not adding local steps led to a peak load $14,48\%$ higher than the experimental load limit of Winkler *et al*. (2004) [9].

In the second set of analysis, the added local steps are again varied, but now with displacement step of 0.02 mm. The following parameters are used:

- 50 global steps;

- 10 initial local steps;
- displacement global step of 0.02 mm; and
- displacement control method: point B in the vertical direction.

The equilibrium paths of the Fig. 5 refer to the variation in the number of steps added to each local step of the global-local analysis block, together with the experimental results of Winkler *et al*. (2004) [9].

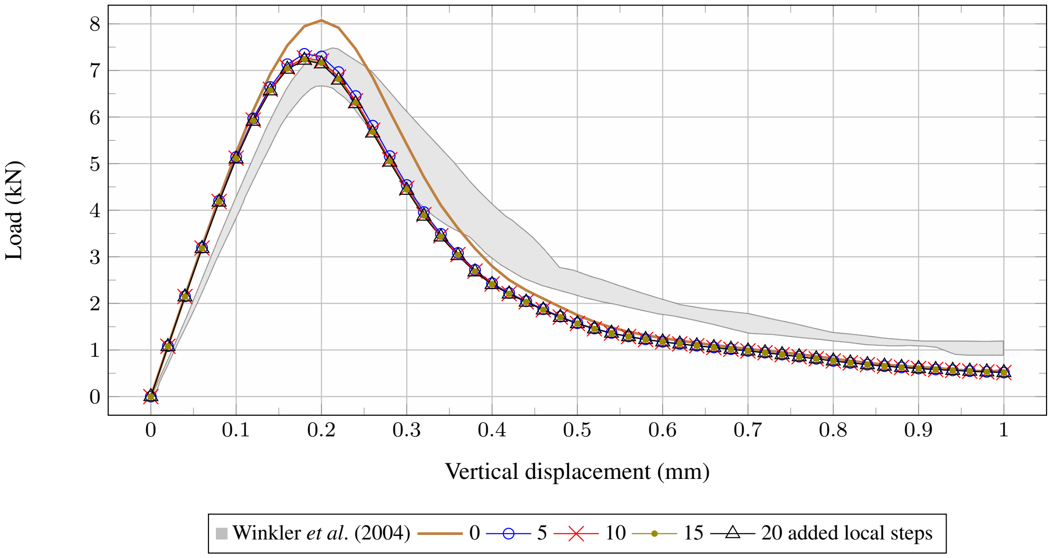

Figure 5. Variation of the number of steps added to each local step of the global-local analysis block for the global displacement step of 0.02 mm.

In the equilibrium path named 0, it is observed that not adding local steps, keeping 10 local steps in each global-local analysis block, led to a peak load approximately $7,0\%$ higher than the experimental load limit of Winkler *et al*. (2004) [9]. It is observed, again, the convergence of the equilibrium paths with 5, 10, 15 and 20 steps added.

4 Conclusions

Observing the effect of the global displacement step on the equilibrium path behavior, it is verified that for 0.04 mm, it was more unstable in relation to the local steps used than to 0.02 mm. In fact, when no local step is added, there is a greater distance between the curves when the largest global displacement step (0.04 mm) is used. This study of the numerical parameters of the nonlinear analysis allowed to conclude that an optimal number of added local steps is necessary to adequately represent the problem. Additionally, the increase in the size of the global displacement step may interfer in the structural response if the number of added local steps is not appropriately chosen. The increment of 0.02 mm proved to be more suitable for describing the equilibrium paths. A more complete investigation will be presented in future works, in which the size and the refinement level of the local problem, and the number of enriched nodes in the global problem are considered.

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