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Tri-objective optimization of steel frames with the bracing system configuration as a design variable

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Abstract. In steel structural design, especially concerning tall buildings, it may be desired to minimize its cost and improve its performance concerning horizontal displacements, dynamic behavior, and structural stability. Also, the predefinition of which bracing system geometric configuration is more suitable for each objective is not evident, and usually, it is made according to the designer's experience. Thus, solving this problem considering different cases of three simultaneous objectives is not a trivial task. Therefore, this paper deals with the tri-objective optimization of spatial steel frames, considering the bracing system configuration as a design variable. The third evolution step of generalized differential evolution (GDE3), the success history-based adaptive multi-objective differential evolution (SHAMODE), and the multi-objective meta-heuristic with iterative parameter distribution estimation (MM-IPDE) are the differential evolution algorithms adopted in this paper. In addition, a multi-criteria tournament method is used to extract desired solutions from the Pareto front according to the decision-maker preferences.

Keywords: 3D-steel frames; Multi-objective optimization; Maximum displacement; Global stability; Bracing systems.

1 Introduction

A general structural optimization problem regarding steel frames has the sole purpose of minimizing the total cost of the structure, or in a simplified way, its weight. However, in real engineering problems, the goal may be to minimize the weight of a structure and improve its performance, such as minimizing horizontal displacement due to wind or improving its dynamic behavior or its global stability. Moreover, it is not possible to know in advance which configuration of the bracing system leads to the best results for a particular objective and is generally determined by the experience of the engineer. Bracing systems are required for tall buildings to stiffen the structure, making it function as a vertical truss, which redistributes internal forces in a more balanced way and improves overall structural performance in horizontal displacements and vibrations.

This work consists of tri-objective optimizations of 3D steel frames considering different configurations of bracing systems as a design variable. The objectives concern the total structural weight, the horizontal displacements, and the critical load factor for the first global buckling mode. Some bracing system configurations commonly used in practice such as diagonal, "Z", "V", and "X" is encoded by an integer index in the candidate solution.

Besides, in a tri-objective problem, the result is a 3D-Pareto curve of several solutions in the objective functions space, from where the engineer must extract the most suitable ones according to the "importance" of each objective. For that, a multi-criteria decision-making developed by Parreiras and Vasconcelos [1] is applied. The search methodology employed in this paper is four differential evolution-based algorithms mentioned in Section 3.

Other studies that consider different bracing systems configuration in multi-objective problems can be found on the literature, such as: Kicinger and Arciszewski [2], Kicinger et al. [3], Richardson et al. [4] and Babaei and Sanaei [5]. However, there is no proven better way of considering different configurations of bracing systems, and the present work presents an alternative to what is done in the cited works. However, there is no proven best way to consider different bracing systems configurations and this paper presents an alternative to what is done in the cited works.

The remainder of this paper is organized as follows: Section 2 describes the formulation of the optimization problem. Section 3 briefly presents the four search methods employed. The multi-criteria decision-maker is described in Section 4. Numerical experiment and its analysis are detailed in Sections 5 and 6, respectively. Finally, conclusions and future works are reported in Section 7.

2 Formulation of the optimization problem

The structural optimization problem presented in this paper consists in finding a bracing system configuration and a set of commercial steel profiles, designated by an integer index vector $\mathbf{x} = \{I_1, I_2, ..., I_i\}$ (design variables), in which the first index indicates which configuration of bracing elements will be applied and the others point to commercial profiles. This vector is a candidate solution, and have to minimize the first objective function $W(\mathbf{x})$, the second objective function $\delta_{max}(\mathbf{x})$, and maximize the third objective function $\lambda_{cr}(\mathbf{x})$ subjected to structural design constraints (eq. (1)). Where $W(\mathbf{x})$ is the total weight of the structure, $\delta_{max}(\mathbf{x})$ is the maximum horizontal displacement and $\lambda_{cr}(\mathbf{x})$ is the critical load factor concerning the global stability, which it is obtained by solving an eigenvalue problem concerning the elastic and geometric stiffness matrices (McGuire et al. [6]).

$$\begin{gathered} \min \quad W(\mathbf{x}) \quad \text{and} \quad \min \quad \delta_{max}(\mathbf{x}) \quad \text{and} \quad \max \quad \lambda_{cr}(\mathbf{x}) \\ \text{s.t.} \quad \text{structural constraints} \\ \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U \end{gathered} \tag{1}$$

The problem's constraints are the inter-story drift, the LRFD (Load and Resistance Factor Design) interaction equations for combined axial force and bending moments, the LRDF shearing equation, and geometric constraints referring to column-column connection. The value the maximum inter-story drift is taken as $\bar{d} = h/500$, where h is the height between two consecutive floors (eq.(2)), according with both Brazilian ABNT [7] and American ANSI [8] codes.

$$\frac{d_{max}(\mathbf{x})}{\bar{d}} - 1 \leq 0 \tag{2}$$

The frame elements must satisfy the LRFD equations for unsymmetrical bending (eq.(3)), and shearing (eq. (4)) effects. P_r, M_{rx}, and M_{ry} are the required axial strength, required flexural strength about the major axis and the minor axis, respectively. The available axial and flexural members strength are named as P_c, M_{cx}, and M_{cy}. For the allowable shearing strength equation, V_r is the required shearing strength, and V_c is the available shearing strength. The methodology of determining the allowable strengths are similar in both ABNT [7] and ANSI [8] and adopted in this paper.

$$\begin{cases} \dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) - 1 \leq 0 \quad if \quad \dfrac{P_r}{P_c} \geq 0.2 \\ \dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) - 1 \leq 0 \quad if \quad \dfrac{P_r}{P_c} < 0.2 \end{cases} \tag{3}$$

$$\frac{V_r}{V_c} - 1 \leq 0 \tag{4}$$

The geometric constraints refer to the column-column connection, in order to establish that the upper column must not have, neither the profile depth nor the mass, higher than the lower column. Equations (5) and (6) show the geometric constraints, where $dp_i(\mathbf{x})$ and $dp_{i-1}(\mathbf{x})$ are the depth of the W section selected for the group of columns i and $i-1$, respectively. $ms_i(\mathbf{x})$ and $ms_{i-1}(\mathbf{x})$ are the unit weight of W section selected for the group of columns i and $i-1$, respectively. NG_c is the number of groups of columns.

$$\frac{dp_i(\mathbf{x})}{dp_{i-1}(\mathbf{x})} - 1 \leq 0 \quad i = 1, NG_c \tag{5}$$

$$\frac{ms_i(\mathbf{x})}{ms_{i-1}(\mathbf{x})} - 1 \leq 0 \quad i = 1, NG_c \tag{6}$$

3 Search algorithms

This work adopts four differential evolution based algorithms for multi-objective problems: (i) The Third Evolution Step of Generalized Differential Evolution (GDE3) proposed by Kukkonen and Lampinen [9]; (ii) The Success History Based Adaptive Multi-objective Differential Evolution (SHAMODE) introduced by Panagant et al. [10]; (iii) The Success History Based Adaptive Multi-objective Differential Evolution with Whale Optimization (SHAMODE-WO) also developed by Panagant et al. [10] with the insertion of the spiral movement presented on the Whale Optimization Algorithm (WOA) created by [11]; and (iv) The Multi-objective Meta-heuristic with Iterative Parameter Distribution Estimation (MM-IPDE) proposed by Wansasueb et al. [12]. The selection of the candidate solutions for other generations is made applying the concepts of dominance and crowding distante, described by Deb et al. [13].

4 Multi-criteria decision maker

A multi-objective optimization problem results in a Pareto front with multiple non-dominated solutions, making extracting the best solution a non-trivial task. One way to get around this problem is to choose a solution based on a pre-defined methodology in which the weighting coefficients can be determined by the importance of each objective. The solution extraction in this paper was supported by a multi-criteria tournament introduced by Parreiras and Vasconcelos [1]. According to the objective functions and their respective importance weights (w_i), established by the Decision-Maker, a Multi Tournament Decision Method (MTD) ranks the best and the worst possible solutions on the Pareto front. The complete and detailed description of the MTD method can be found in Parreiras and Vasconcelos [1] and examples in multi-objective structural optimization in Carvalho et al. [14].

5 Numerical examples

The numerical experiment conducted in this paper is about a tri-objective optimization of a six-story and two-bay spatial steel frame where the stories are three meters high and the bays are six meters wide. The objective of the problem is to minimize both the structure's total weight and its maximum horizontal displacement on the top story and maximize its critical load factor concerning global stability. The bracing system configuration is a variable of this problem, and the structure can assume four different configurations: (i) a 90 bars diagonally braced frame; (ii) a 90 bars "Z" braced frame; (iii) a 114 bars "V" braced frame and (iv) a 126 bars "X" braced frame. The first index guides the configuration in the candidate vector, assuming values one to four. The other variables concern the profile employed on columns and beams. The search spaces for members are composed of 29 "H" profiles for the columns and 56 "I" for beams, all of them part of the AISC profile tables. Figure 1 illustrates the candidate vector and the corresponding phenotype of the frame according to the first index. It is important to note that the profile variables are grouped in nine according to symmetry and solicitation, as follows: (i) corner columns; (ii) outer columns; (iii) outer beams; (iv) inner beams; and (v) bracers. The elements of three consecutive floors must have the same cross-sectional area, for instance, corner columns of stories one, two, and three will have the same profile employed, as well as the corner columns of stories four, five, and six.

The structure is subjected to gravity loads of 10 kN/m on the outer beams and 20 kN/m on the inner beams, also a wind load is considered acting on the larger façade is considered based on a basic velocity of 37 m/s (ABNT [15]). The wind load applied is detailed in Table 1, where the middle node has twice the corner node contribution area. It is considered that the pavement plane work as a rigid diaphragm due to the slab stiffness, it is modeled with a multi-freedom constraint master and slave method described in Felippa [16].

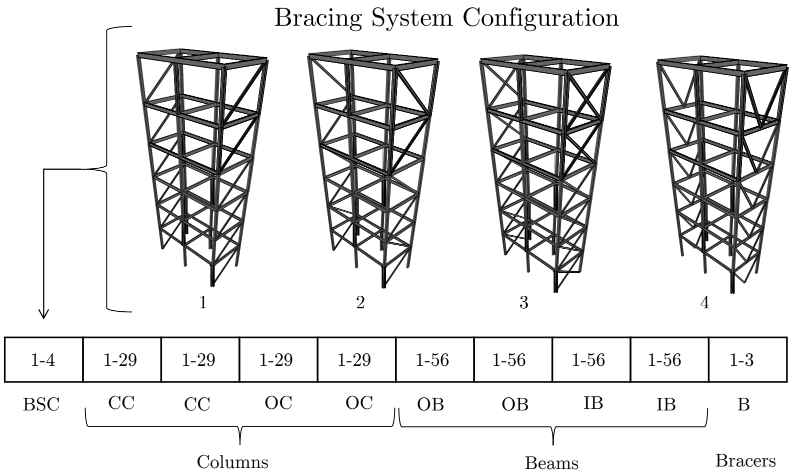


Figure 1. Candidate vector and bracing systems configurations.

Table 1. Nodal wind loads acting on each floor (kN)

Floor	Corner Nodes	Middle Nodes	Floor	Corner Nodes	Middle Nodes
1	6.39	12.78	4	7.88	15.76
2	6.67	13.34	5	8.31	16.62
3	7.35	14.70	6	8.68	17.36

Ten independent runs of 200 generations with 50 candidate vectors are set for the four algorithms applied. In the problem treated in this paper, the solutions are extracted according to three different scenarios: (i) scenario 1: the extracted solution has the structure's weight w_1 = 0.6 of importance and both maximum displacement and critical load factor $w_2 = w_3 = 0.2$; (ii) scenario 2: the extracted solution has the structure's maximum displacement w_2 = 0.6 of importance and both total weight and critical load factor $w_1 = w_3 = 0.2$; (iii) scenario 3: the extracted solution has the structure's critical load factor w_3 = 0.6 of importance and both total weight and critical load factor $w_1 = w_2 = 0.2$. The best results found are displayed on Table 2 detailing the bracing system, the profiles for each group and the values for constraints and objective functions, it is also highlighted from which algorithm the extracted solution came, where $LRFD_{max}(\mathbf{x})$ is the maximum value obtained by the interaction equation for combined flexural and axial effects and $V_{max}(\mathbf{x})$ is the maximum value found for the shearing strength equation. Figure 2 depicts the 3D Pareto front trade-off curve, from where are extracted solutions for each of the three scenarios described before, also the preferential planes for each two out of three objectives are shown in Figure 3.

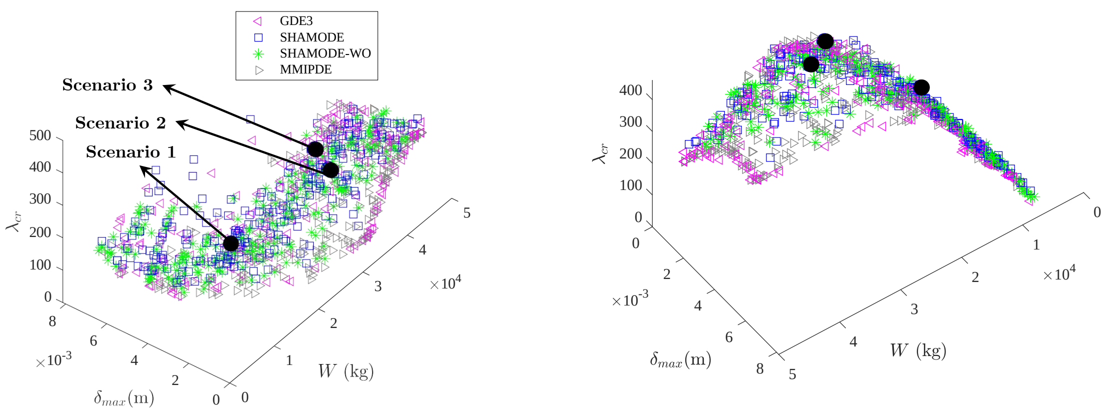
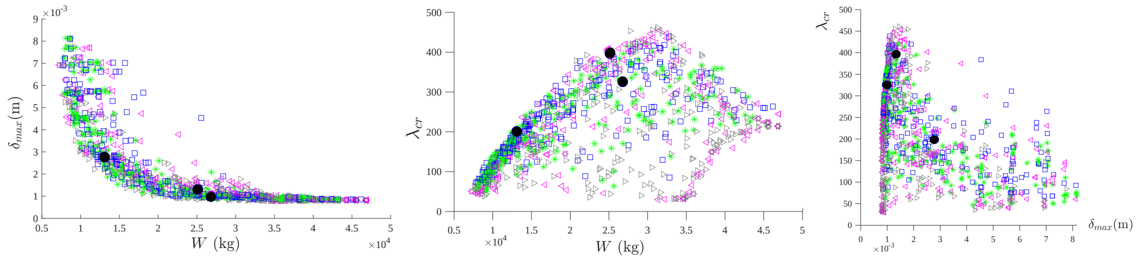


Figure 2. 3D Pareto fronts and extracted solutions.



(a) (b) (c)

Figure 3. 2D Pareto fronts and extracted solutions. (a) Total weight ($W(\mathbf{x})$) x maximum displacement ($\delta_{max}(\mathbf{x})$); (b) Total weight ($W(\mathbf{x})$) x critical load factor ($\lambda_{cr}(\mathbf{x})$); (c) Maximum displacement ($\delta_{max}(\mathbf{x})$) x critical load factor ($\lambda_{cr}(\mathbf{x})$).

Table 2. Best results found for the three scenarios of the multi-objective problem presenting details of the profiles assigned to each member group, constraints, and objective function values.

Scenario	1	2	3
Bracing System	D	Z	Z
Group (Stories)		W Profiles	
CC (1-3)	310x79	310x125	310x117
CC (4-6)	200x35.9	250x85	310x79
OC (1-3)	310x79	310x125	310x117
OC (4-6)	200x46.1	310x79	250x73
OB (1-3)	310x28	460x60	460x82
OB (4-6)	200x15	360x32.9	310x23.8
IB (1-3)	310x22.7	610x113	530x72
IB (4-6)	250x17.9	460x82	310x21
BC (1-6)	150x13	150x24	150x18
	Constraints values		
$LRFD_{max}(\mathbf{x})$	0.98	0.66	0.73
$V_{max}(\mathbf{x})$	0.39	0.23	0.28
$d_{max}(\mathbf{x})$ (mm)	0.5	0.2	0.3
	Objective functions values		
$W(\mathbf{x})$ (kg)	13068	26772	25127
$\delta_{max}(\mathbf{x})$ (mm)	2.8	1	1.3
$\lambda_{cr}(\mathbf{x})$	198	325	395
	Search method		
Algorithm	MMIPDE	SHAMODE	SHAMODE-WO

6 Results Analysis

It is necessary to consider that the extracted solutions are just examples of what a decision-maker can choose, but the entire universe composed of the Pareto frontier is available and the designer can extract as many solutions with different weights of importance for each objective as is most convenient.

It is possible to make interesting observations when analyzing Table 1. First, as expected, as the importance of a given objective grows to the detriment of others, the opposite occurs with the value of its function in case of minimization and the same in case of maximization. This can be seen by analyzing the first objective function W(x), which presents values W1(x) = 13068 kg, W2(x)=26772 kg and W3(x)=25127 kg for weights of importance $w_1 = 0.6$, $w_2 = 0.2$ and $w_3 = 0.2$, respectively. The same can be seen in the other objectives. When analyzing the results of the extracted solutions simultaneously, another point to be noted is the fact that objective 1 conflicts with objectives 2 and 3, but objectives 2 and 3 do not conflict with each other, since when increasing the critical load factor importance weight in scenario 3 for $w_3 = 0.6$, the maximum displacement also reduces. In a problem with multiple objectives, there can be the most diverse conflicting relationships between multiple objectives, hence the usefulness of a method that considers their weighted importance to extract solutions. In terms of bracing systems, it is interesting to note that the system adopted in the solution of scenario 1 was the diagonal bracing, while the "Z" bracing was chosen for scenarios 2 and 3.

As for the constraints, it is important to note that only in scenario 1 the constraint of combined axial and bending effects was close to being active ($LRFD_{max} = 0.98$), since weight was the most important function and, therefore, presented the lightest structure. On Pareto fronts, not necessarily all solutions must present active constraints since some constraints may be conditioned by objectives, as is the case of the inter-story drift by the maximum horizontal displacement. Another point to note are the algorithms that were responsible for the existence of the extracted solutions, which in scenario 1 was MMIPDE, in scenario 2, SHAMODE and in scenario 3, SHAMODE-WO.

7 Conclusions and extensions

This work consisted of a preliminary numerical experiment for a steel frame optimization problem with three objectives and considering different bracing systems as variables of the problem. The objective of such a problem is to generate a 3D Pareto front so that the decision-maker can extract the most attractive solution given the weighting of importance for each objective. Three scenarios of different extracted solutions are studied and their results are analyzed, in which it is possible to notice the use of diagonal bracing for the first and the "Z" bracing for the others. As extensions of this work, we can mention the application of metrics and comparative studies of the meta-heuristics used, optimization problems with more than three objectives, the application to larger structures with multiple load combinations and more varieties of bracing systems, the orientation of the pillars as variables and the consideration of second-order effects.

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