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Truss chords buckling

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Abstract. The present work proposes reducing coefficients on the buckling length of trusses chords when the normal acting on these chords is not constant throughout the chord. The normal maximum is considered acting on the entire compressed part of the chord, but with a reduced buckling length. In this text, it is obtained an equivalent normal, which, constant in the chord, generates the same deformation energy as the acting variable normal. This equivalent normal has a value lower than the maximum acting normal. It is made, then, the proportion to obtain the reduced buckling length to consider. It is solved, step by step, the case of the double-supported truss under point load in the middle of the span and shown the results for the double-supported truss under uniformly distributed load and the two-span continuous truss, also under uniformly distributed load. The solution of the differential equation with the variable normal in the chord, exact, of difficult solution, is also showed to notice the validity of doing the procedure via energy, approximate. An illustrative numerical example of the application of these buckling length reduction coefficients is shown to easy the practice.

Keywords: buckling lenght factor, truss chord, stability, structural engineering, civil construction.

1 Introduction

On dimensioning trusses, we can assume the truss as a bar and obtain the bending moments that will be acting on it. With the bending moments in hand, these moments can be divided by the height of the truss and obtain the normal acting on the chords. In general these bending moments have a diagram different from the constant, generating diagrams of normal different from the constant in the chords. The Euler buckling load is calculated for a constant normal along the entire member and the buckling length considered is a function of the support conditions of the member. Now, for a normal different from the constant, the buckling length to be considered in Euler's formula changes, being smaller, to the maximum normal acting on the chord.

In this work, it is proposed coefficients to reduce the buckling length due to non-constant normal acting along the bar. We use an energetic method, as the resolution of the differential formulation for the non-constant normal would be difficult to solve, as shown in the text. The buckling length reducing coefficients are shown for double-supported trusses subjected to mid-span point load, single-supported trusses subjected to uniformly distributed load, and continuous two-span trusses under uniformly distributed load.

In the conclusion, the application of the buckling length reducing coefficients for any moment diagram is generalized, showing the reducing coefficients for positive moments, normal with convex diagram and for negative moments, normal with concave diagram, or straight. An illustrative numerical example is made to show the application of the reducing coefficients.

Bibliographic revision. Euler's equations, strain energy and differential formulation can have their development seen in Beer & Johnston [1]. In Timoshenko [2], which is the most complete compendium on stability, addresses a similar issue in the section of buckling under distributed normal load. Grennhill [5] uses Bessel functions to solve analog problems. Dondorff [6] is also a reference cited by Thimoshenko. Another reference is Kato [3], where the topic of bar buckling is widely discussed, there are reference for several articles about bar buckling in this IASSS text. Kongkong [4], Burdzik [7] and Pienaar [8] are more recent articles that also deal with the topic, Pienaar [8] deal with timber chords, Kongkong [4] with steel and Burdzik [7], concrete, precast.

2 Buckling length reduction coefficients

2.1 Bi-supported truss under mid-span point load

Considering a double-supported truss subjected to a point force at midspan, we have a triangular diagram for the moment and the same triangular diagram for the compression normal at the chord, dividing the moment by the height of the truss. It can, then, be calculate what is the strain energy in the bar for this triangular normal diagram and what is the strain energy in the bar for a fictitious constant normal and equate the both. Thus, it is arrived at an equivalent constant normal, smaller than the maximum normal acting on the bar, which generates a deformation energy equal to the triangular normal diagram.

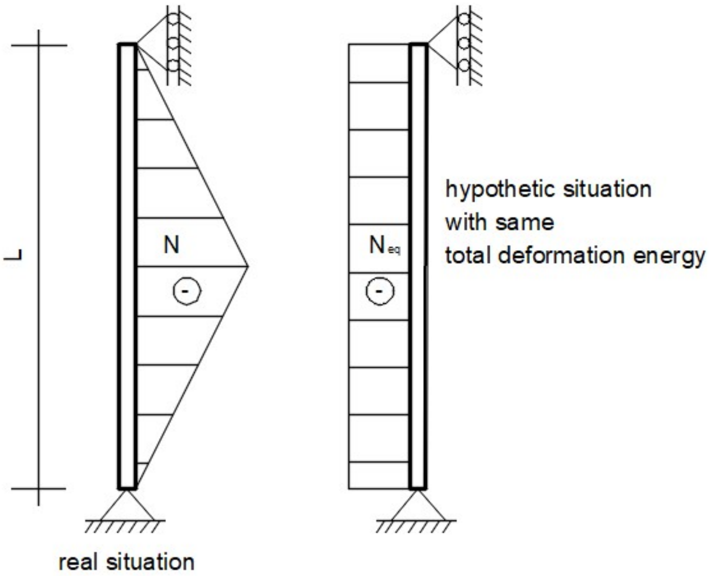

Figure 1 – Equivalent fictitious normal.

The strain energy for normal force acting on the bar is:

$$U = \int_0^L \frac{N(x)^2}{2\,AE}\,dx.$$

Where, N: the normal acting on the chord, L: the chord length, L_b: the buckling length, A: area of the truss chord, E: the modulus of elasticity of the truss chord material, x: distance to the end of the truss. The development of this energy formula can be seen in Beer & Johnston [1]. The strain energy for the triangular normal diagram is:

$$U = 2\int_0^{L/2} \frac{\left(\frac{N_{max}x}{L/2}\right)^2}{2\,AE}\,dx = \frac{{N_{max}}^2\,L}{6\,AE}.$$

Being: N_{max}: the maximum normal acting on the chord. Equating the two energies, we have:

$$\frac{{N_{eq}}^2\,L}{2\,AE} = \frac{{N_{max}}^2\,L}{6\,AE} \rightarrow N_{eq} = N_{max}\sqrt{3}/3.$$

That is, the constant normal along the bar that entails the same energy of deformation of the triangular diagram for the point load is this, N_{eq}, equivalent fictitious normal, and is smaller than the maximum acting normal. Euler's formula for the buckling normal is the one below, and its development can also be seen in Beer & Johnston [1]. Thus, we obtain the buckling length for the triangular diagram, as if the maximum normal were constantly acting along the bar.

$$N_{max} = \pi EI/{L_b}^2 \rightarrow L_b = \sqrt{\frac{\pi EI}{N_{max}}}.$$

The buckling length for the smaller equivalent normal that causes the same strain energy as in the triangular diagram is this:

$$L_{b\,eq} = \sqrt{\frac{\pi EI}{N_{eq}}} = \sqrt{\frac{\pi EI}{N_{max}\sqrt{3}/3}} = \sqrt[4]{3}\sqrt{\frac{\pi EI}{N_{max}}} = \sqrt[4]{3}\,L_b = 1.32\,L_b$$

That is, it is a longer buckling length, due to a shorter normal. Making the proportion, it is arrived at the reducing coefficient of the buckling length:

$$L_{ef} = \frac{1}{1{,}32}L = 0.76\,L$$

2.2 Bi-supported truss under uniform distributed load

For the case of the truss under uniform distributed load, the procedure is similar, the difference is that the normal diagram is parabolic, not triangular. The normal acting on the flange is:

$$N(x) = 4\,N_{max}\left(\frac{x}{L} - \frac{x^2}{L^2}\right)$$

And the strain energy for this normal is:

$$U = \int_0^L \frac{N(x)^2}{2\,AE}dx = \int_0^L \frac{\left[4\,N_{max}\left(\frac{x}{L} - \frac{x^2}{L^2}\right)\right]^2}{2\,AE}dx = \frac{4}{15}\frac{N_{max}^2\,L}{AE}$$

Equating the energy to constant normal and making the proportion, it is arrived at the coefficient for reducing the buckling length of 0.85.

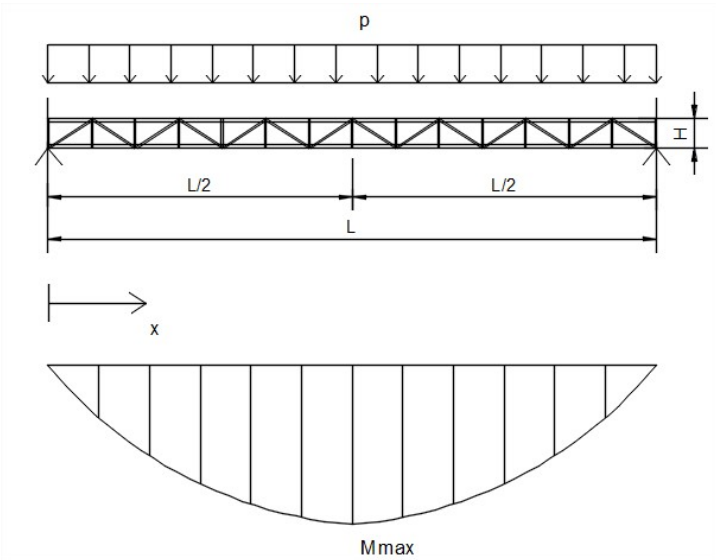

Figure 2 - Bi-supported truss under uniform distributed load.

2.3 Two-span continuous truss under uniform distributed load

In this case, there is a moment inversion, part of the span is compressed and another part is stretched. Thus, the buckling length to consider is, at first, only the compressed part, that is, the bar will present a second balanced deformed configuration, in addition to the straight line, only in the compressed part and, on the lenght of this part, apply the reducing coefficient. This being the positive moment equation:

$$M = M_{max}\left(\frac{16}{3\,L}x - \frac{64}{9\,L^2}x^2\right),$$

And this, the equation for negative moments:

$$M = M_{max}\left(4\frac{x^2}{L^2} - 3\frac{x}{L}\right),$$

The reducing coefficient is 0.85 for the positive moment section, convex diagram and 0.74 for the negative moment section, concave moment and normal diagram. As the section with positive moment is 0.75 L, where L is the span length, the buckling length results in L_{fl} =0.85 x 0.75 = 0.63 L and for the section with negative moment, L_{fl} =0, 74 x 0.25 = 0.18 L.

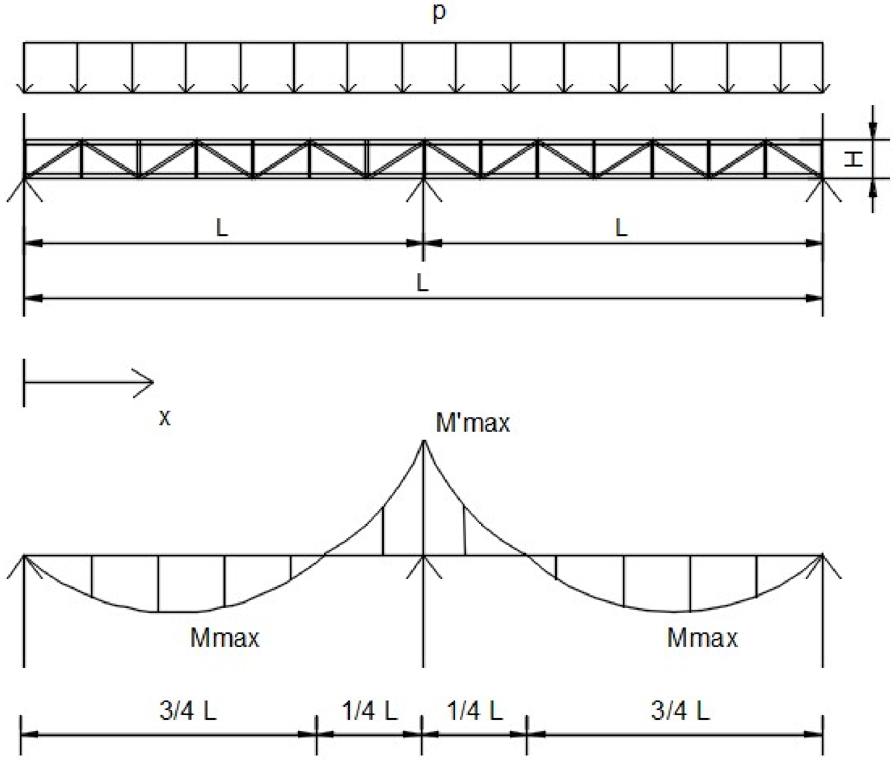

Figure 3 - Continuous truss under uniform distributed load.

3 Differential exact formulation for variable normal along the chord

To solve the problem by the differential formulation, that is, not by the energetic method, for a non-constant normal along the chord, the normal is:

$$N(x) = N_{max}\, w(x)$$

Where N_{max} is the maximum normal acting on the chord and w(x), its diagram for the maximum unit normal. And there must be considered the displacements of the bar for the balanced deformed configuration. The moment acting on the bar is:

$$M(x) = -\,N(x)\, y(x).$$

Where y(x) is the displacement of the bar, transverse to its axis.
According to Beer & Johnston[1] we have:

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI} = -\frac{N(x)}{EI}y.$$

Where results:

$$\frac{d^2y}{dx^2}+\frac{N_{max}w(x)}{EI}\,y=0.$$

It is a second order differential equation with non-constant coefficients, depending on the normal diagram, which is difficult to solve. Then the boundary conditions are applied in order to obtain the value of the maximum buckling normal. Hence the advantage of applying the energy approach, which is simpler and easier to solve.

A more accurate formulation can still be achieved not by considering the moment in the section as the normal element force times the displacement but as the integral of the normal force times the displacement from the beginning of the bar.

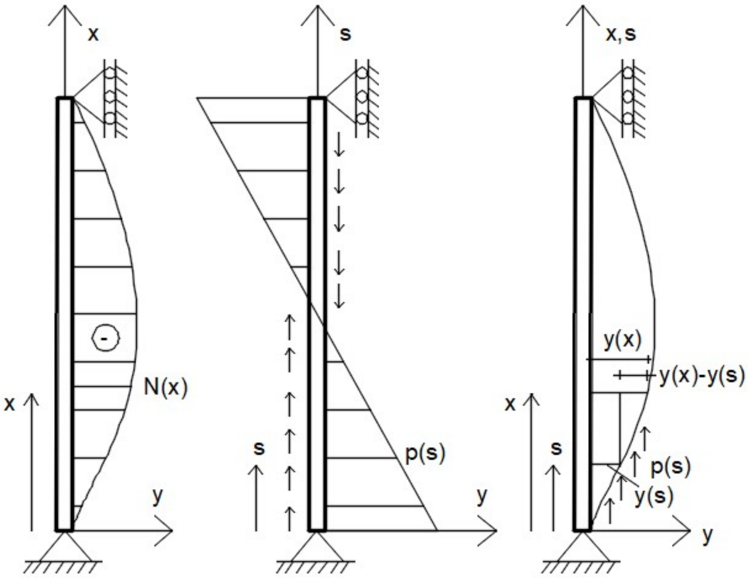

Figura 4 - Forces and displacements in the bar for variable normal.

$$N=N_{max}\,w(x). \qquad\qquad p_b=N'=N_{max}\,w'(x).$$

P_b, the normal force. In this way the normal element force in the bar can be written as:

$$N(x)=\int_0^x p(s)ds=\int_0^x N_{max}\,w'(s)ds.$$

And the moment for the balanced deformed configuration:

$$M(x)=\int_0^x p(s)[y(x)-y(s)]ds=\int_0^x N_{max}\,w'(s)[y(x)-y(s)]ds.$$

Where results:

$$\frac{d^2y}{dx^2}=\frac{M}{EI}=\frac{1}{EI}\int_0^x N_{max}\,w'(s)[y(x)-y(s)]ds$$

And the solution consists of solving the following equation:

$$\frac{d^2y(x)}{dx^2}-\frac{1}{EI}\int_0^x N_{max}\,w'(s)[y(x)-y(s)]ds=0,$$

Applying the boundary conditions, y(x=0) = 0 and y(x=L) = 0. Same procedure used to find the critical normal in Beer & Johnston [1]. Being E, the elasticity modulus and I de inertia moment of the section of the chord being analyzed.

4 Numerical example

Let's see the following practical example, a truss with two spans, 40 cm high, subjected to the following loads:

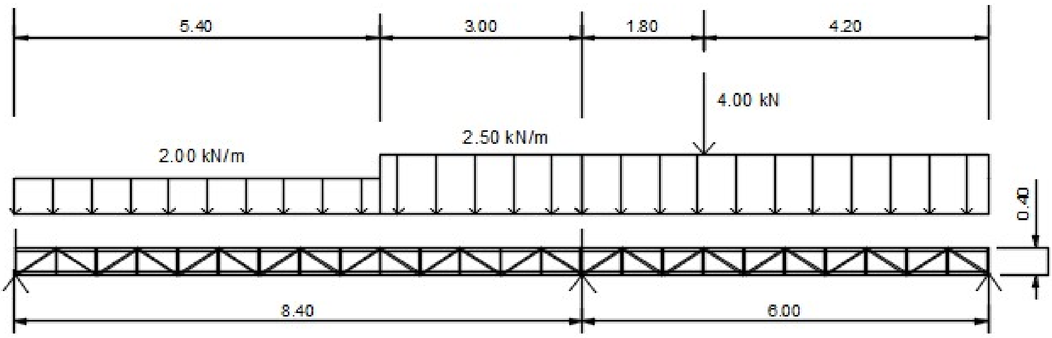

Figure 5 - Practical example of truss.

Measurements in m. Forces in kN. From which the following moment diagram is obtained:

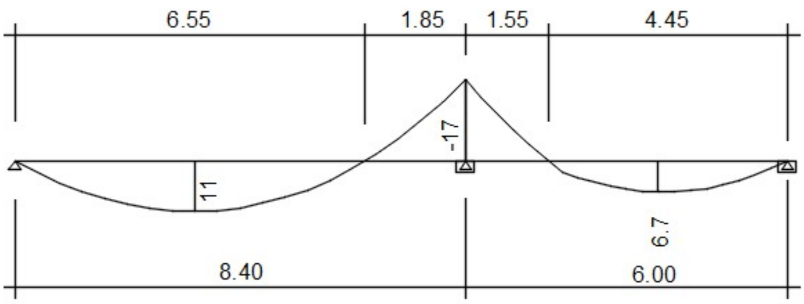

Figure 6 – Moments diagram.

Distances in cm. Moments in kN.m. Dividing the moment by the height of the truss, we obtain the following diagram of normal forces acting on the chords.

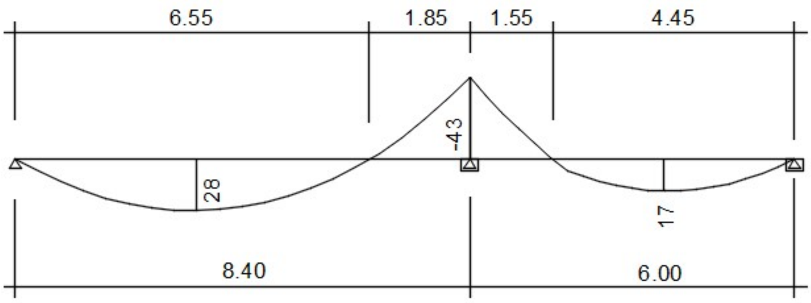

Figure 7 – Compression normal on the chords.

Measurements in m. normal in kN.

In the lower chord the compression appears in the region of negative moments and in the upper chord the compression is in the region of positive moments.

For the lower chord, the maximum normal compression will be 43 kN, acting on a compression length of 1.85 m, so the buckling length to be considered will be: $L_b = 0.75 \times 1.85 = 1.40$ m, with this maximum normal acting over this length.

For the upper chord, the maximum normal compression will be 28 kN, acting on a compression length of 6.55 m, so the buckling length to be considered will be: $L_b = 0.85 \times 6.55 = 5.60$ m, with, also , this maximum normal acting over this entire length.

Lengths much smaller than the 8.40 m span.

5 Conclusion

As we have seen, we can consider a smaller buckling length for normal non constant. The buckling length of a bar enters in the calculation by making the required section smaller or larger. As a generalization we can consider the coefficient of 0.85 for the length of the normal due to positive moments or convex diagram of the normal and 0.75 for the region of the normal due to negative moments, concave or rectilinear moments, applied in the region where the moments are acting.

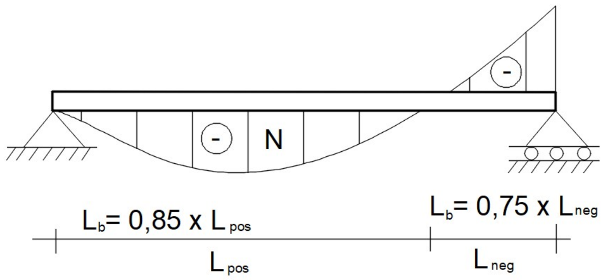

Figure 8 – Generalization.

Being L_{pos}, the chord length with positive moments and L_{neg}, the chord length with negative moments.

A great material economy in the projects is expected with the use of these coefficients, because by reducing the buckling length, the sections to be used in the designs of the trusses chords are also reduced, not generating the use of unnecessary material.

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Authorship statement. The author confirms that he is solely responsible for the authorship of this work and that all material included herein as part of this work is his property (and authorship).

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