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Recent Advances in a Multiscale Flux-Based Method for Simulating Flow in Fractured Porous Media

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Abstract. Computational simulation of reservoir flow is an important tool that provides valuable insight into the decision process in oil extraction. Several types of commercial software have been developed over the years for this application, the majority using low-order schemes, which can become prohibitive for very large models. This issue becomes more apparent since, nowadays, the accuracy of a simulator is dominated by the accurate simulation of the multiscale characteristics of a reservoir such as permeability heterogeneity. To capture these multiscale features in low-order schemes, very refined models are required. Therefore, developing a high-order scheme able to simulate fractured reservoir flow that is accurate and can efficiently capture the multiscale features of the reservoir is of great value for the field. With this motivation, this presentation reports on recent advances in a methodology to simulate flow in highly heterogeneous fractured porous media using the Multiscale Hybrid-Mixed (MHM) method with H(div)-confirming flux approximations. This method is particularly appealing because of its inherent properties such as local mass conservation, multiscale features, and strong divergence-free enforcement for incompressible flows. Flow in the porous media is modeled with traditional Darcy's equations and the coupling between flow in the porous media and fractures is based on the conceptual Discrete-Fracture-Matrix representation, where the fractures are idealized as lower-dimensional elements at the interface of matrix elements. The methodology is compared with benchmark examples to demonstrate its robustness, accuracy, and efficiency.

Keywords: Discrete Fracture Networks, Porous Media Flow, Multiscale Method, Mixed Finite Elements

1 Introduction

One of the main objectives of the numerical simulation of oil reservoirs is to computationally reproduce and predict the real behavior observed in explorations fields. The main characteristics that should be taken into account are the geometry of the reservoir, rock and fluid material parameters, the pressure field, flux patterns, and phenomena associated with the interaction between these properties.

According to Singhal and Gupta [1], more than half of the surface of the continents is covered with low-permeability rocks. However, the permeability of these rocks can be enhanced through the process of fracturing. Additionally, around 65% of oil reserves are in naturally fractured reservoirs (Shah et al. [2]). These facts motivate the development of a reservoir simulator that can account for the secondary permeability introduced by the presence of fractures in the porous media, which can significantly affect the flux patterns. Some of the previous work towards simulating this process can be found in [3–5].

The approximation error of current reservoir simulators is mostly dominated by the multiscale features of the reservoir, such as permeability heterogeneity. The Multiscale Hybrid-Mixed Method (MHM) with H(div)-conforming flux approximations has been shown to accurately and robustly be able to simulate flow in porous media (Durán et al. [6]). The method possesses inherent properties that are considered key in the simulation of porous media flow such as its multiscale properties, local mass conservation, and strong divergence-free enforcement for incompressible flows.

With these two main motivations, this article presents the recent advances in the development of a multiscale, locally conservative method to simulate flow in fractured porous media that is robust, accurate, and efficient. In this work, porous media flow is modeled with the traditional Darcy's model, and coupling between flow in the porous media and flow in the fractures is based on the conceptual Discrete-Fracture-Matrix representation, where the fractures are idealized as lower-dimensional elements at the interface of matrix finite elements (see Section 2). A novel methodology to generate meshes tailored for the method is briefly presented in Section 2.3. Results are compared with the use of benchmark problems in Section 3 where great agreement is verified between the proposed method and other methods available in the literature.

2 Methodology

In this section, the methodology to approximate Darcy's flow in fractured porous media with H(div)-conforming spaces is briefly explained. The multiscale method MHM and the mesh generator DFNMesh are also briefly presented. More details can be found in Durán et al. [6], Berre et al. [7] and Lima [8].

2.1 Strong form of the governing equations

For the porous media flow, the strong form of the governing equation can be stated as finding the fluid pressure in the matrix p_3 and the fluid flux in the matrix $\boldsymbol{\sigma}_3$ such that

$$\begin{cases} K_3^{-1}\boldsymbol{\sigma}_3 + \nabla p_3 = 0 & \text{in } \Omega_3 \\ \text{div}(\boldsymbol{\sigma}_3) = f & \text{in } \Omega_3 \\ p_3 = p_{D_2} & \text{on } \partial\Omega_{D_2} \\ \boldsymbol{\sigma}_3 \cdot \mathbf{n} = \boldsymbol{\sigma}_{N_2} & \text{on } \partial\Omega_{N_2} \\ p_3 = p_2 & \text{on } \Omega_2 \end{cases}, \tag{1}$$

where K_3 is the isotropic permeability of the matrix, f is a source term, Ω_3 is the matrix domain, p_{D_2} is the imposed pressure at the boundary $\partial\Omega_{D_2}$, $\boldsymbol{\sigma}_{N_2}$ is the imposed flux at the boundary $\partial\Omega_{N_2}$, and p_2 is the fluid pressure inside of a fracture.

The strong form of the governing equation for fluid flow inside of the fractures is stated as finding the fluid pressure inside the fracture p_2 and the fluid flux inside the fracture $\boldsymbol{\sigma}_2$ such that

$$\begin{cases} K_2^{-1}\boldsymbol{\sigma}_2 + \nabla p_2 = 0 & \text{in } \Omega_2 \\ \text{div}(\boldsymbol{\sigma}_2) = [\![\boldsymbol{\sigma}_3 \cdot \mathbf{n}]\!] & \text{in } \Omega_2 \\ p_2 = p_{D_1} & \text{on } \partial\Omega_{D_1} \\ \boldsymbol{\sigma}_2 \cdot \mathbf{n} = \boldsymbol{\sigma}_{N_1} & \text{on } \partial\Omega_{N_1} \end{cases}, \tag{2}$$

where K_2 is the isotropic permeability of the fracture, Ω_2 is the fracture domain, $\mathbf{n}$ is the normal to the fracture, p_{D_1} is the imposed pressure at the boundary $\partial\Omega_{D_1}$, and $\boldsymbol{\sigma}_{N_1}$ is the imposed flux at the boundary $\partial\Omega_{N_2}$.

2.2 The Multiscale Hybrid-Mixed Method applied to Discrete Fracture Networks

The Multiscale Hybrid-Mixed Method (MHM) is a numerical technique tailored to approximate solutions with multiscale features such as strong permeability heterogeneity (Durán et al. [6]). In this method, the problem is divided into two steps: First, the solution for the normal flux (Lagrange multiplier) is sought at the interfaces of macro elements (red lines in Figure 1) together with the constant pressure for each macro element. This phase is denoted as *upscaling*. The second step is denoted *downscaling* where the solution of the normal fluxes between macro elements is used as Neumann boundary conditions for each macro element. The procedure is illustrated in figures 1 and 2. More details of the MHM applied to porous media flow problems are found in Durán et al. [6]. It is emphasized that, with the MHM, the degrees of freedom of the fine discretization can be condensed into the macro fluxes at the interface, therefore decreasing the size of the final system of equations.

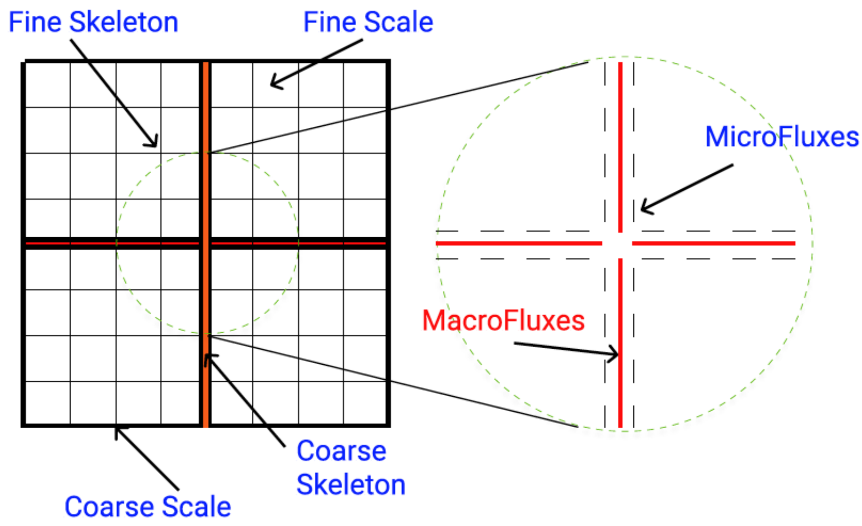


Figure 1. Example of MHM coarse and fine meshes. The coarse mesh is composed of 4 quadrilateral elements and each macro element has a 4x4 uniform refinement leading to 16 quadrilateral microelements. The global system is only composed of normal macro fluxes at the interfaces and constant pressure at each macro element.

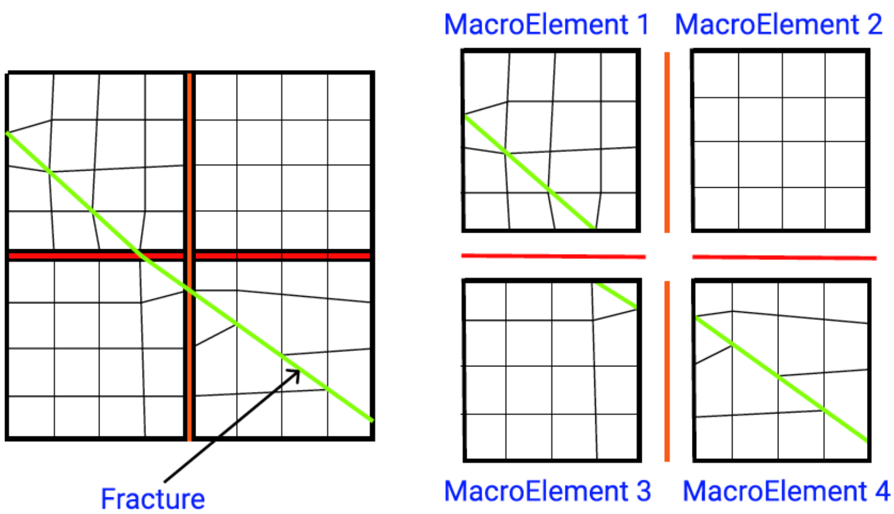


Figure 2. Example of MHM meshes for the case of a diagonal fracture crossing the domain.

2.3 DFNMesh: A mesh generator tailored for the MHM applied to fractured porous media problems

The meshes used in this article are generated using DFNMesh (GitHub) developed by Lima [8]. This mesh generator is tailored for the proposed multiscale methodology where it is sought that the macro elements are as coarse as possible with a fine refinement that accurately models the fractures at the interface of the fine mesh discretization. The algorithm consists of progressively refining an initial coarse mesh based on polygons that represent the fracture location. The obtained fine mesh for the example analyzed in Section 3 starting with a coarse discretization of $4 \times 6 \times 9$ hexahedra is illustrated in Figure 3 where a 2D fracture mesh (blue) is seen at the interfaces of a 3D matrix mesh (gray). More details on the procedure adopted in DFNMesh are found in Lima [8].

3 Comparisons with benchmark problems

In this section, the proposed methodology is compared with other methods based on the benchmark problems proposed by Berre et al. [7]. Cases 2 and 3 of the benchmark problems are analyzed here. These cases show

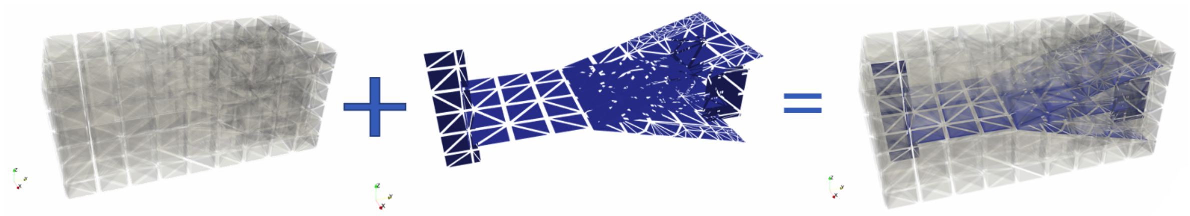


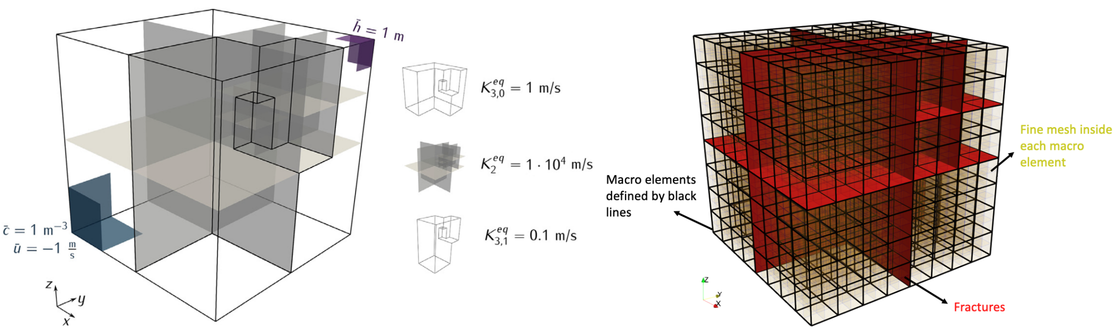
Figure 3. Final fine mesh obtained by DFNMesh if starting with a coarse discretization of 4x6x9 hexahedra. The blue 2D elements represent the fractures at the interface of matrix gray 3D elements.

complex fracture networks and therefore are good tests to evaluate the accuracy and robustness of the method. It is noted that in the benchmark problem, solutions of both pressure and saturation are analyzed. However, in this manuscript, only pressure comparisons are shown.

3.1 Benchmark problem case 2

In this problem, a unit cube $\Omega_3 = [0\,\mathrm{m}, 1\,\mathrm{m}]^3$ with nine fractures is analyzed. The domain with the fractures is illustrated in Figure 4. Two different permeabilities are assigned to different parts of the domain ($\Omega_{3,0}$ and $\Omega_{3,1}$) as defined next:

$$\begin{cases} \Omega_{3,0} & \Omega_3 \backslash \Omega_{3,1} \\ \Omega_{3,1} & \{(x,y,z) \in \Omega_3 : x > 0.5\,\mathrm{m} \;\cap\; y < 0.5\,\mathrm{m}\} \\ & \cup \{(x,y,z) \in \Omega_3 : x > 0.5\,\mathrm{m} \;\cap\; 0.5\,\mathrm{m} < y < 0.75\,\mathrm{m} \;\cap\; z > 0.5\,\mathrm{m}\} \\ & \cup \{(x,y,z) \in \Omega_3 : 0.625\,\mathrm{m} < x < 0.75\,\mathrm{m} \;\cap\; 0.5\,\mathrm{m} < y < 0.625\,\mathrm{m} \;\cap\; 0.5\,\mathrm{m} < z < 0.75\,\mathrm{m}\} \end{cases} \tag{3}$$



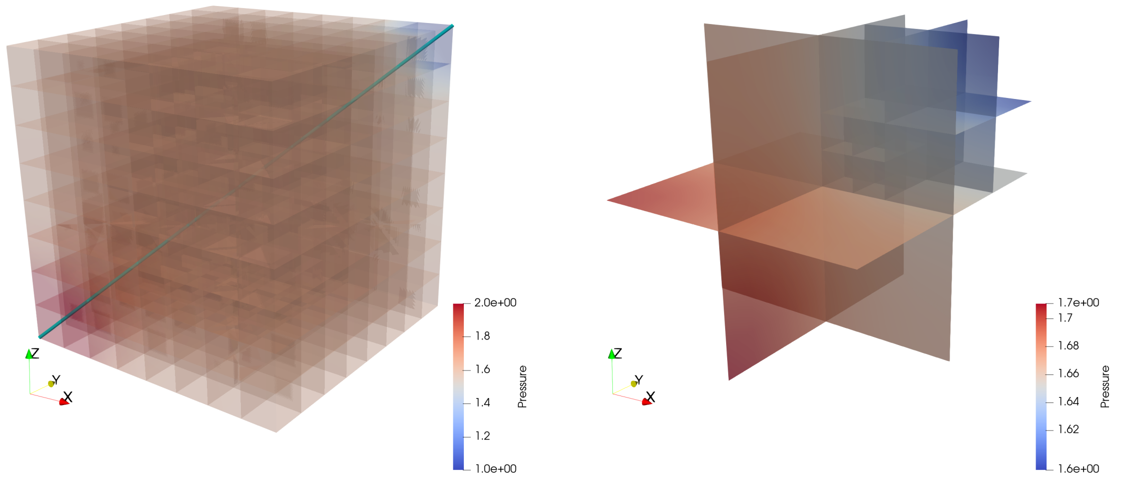
(a) Problem domain and boundary conditions (b) Multiscale FEM mesh

Figure 4. Illustration of problem domain, fractures, and multiscale mesh for benchmark case 2.

A constant entering flux of $\sigma_{N_2} = -1.0$ is imposed at $\partial\Omega_{N_2} = \{(x,y,z) \in \partial\Omega : x,y,z < 0.25\,\mathrm{m}\}$ and a condition of $p_3 = 1\,\mathrm{m}$ is imposed at $\partial\Omega_{D_2} = \{(x,y,z) \in \partial\Omega : x,y,z < 0.0875\,\mathrm{m}\}$. Zero flux is applied to the remaining part of the boundary of the 3D domain. The adopted material properties are omitted here and can be verified in Berre et al. [7].

Results

The total number of degrees of freedom of the simulation is 162 728. However, as explained in Section 2.2, condensation is applied towards the skeleton mesh leading to a final system of 23 528 equations. The matrix pressure is sampled along a line defined between the points $(0\,\mathrm{m}, 0\,\mathrm{m}, 0\,\mathrm{m})$ and $(1\,\mathrm{m}, 1\,\mathrm{m}, 1\,\mathrm{m})$ as shown in Figure 5. This pressure is compared against the results by other methods in the benchmark in Figure 6. It can be seen that the solution obtained is within the range of the other methods.



(a) Matrix pressure and line for sampling points (cyan) (b) Fluid pressure on fractures

Figure 5. Pressure solution for case 2 in both matrix and fractures.

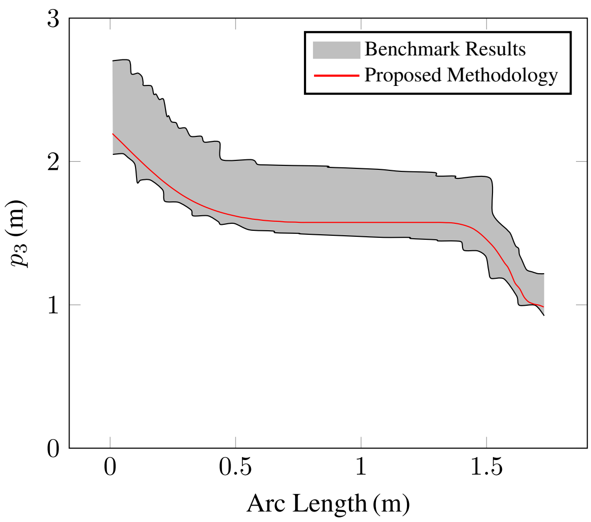


Figure 6. Comparison of matrix pressure over line through domain between proposed methodology and other methods available for the benchmark case 2 (Berre et al. [7])

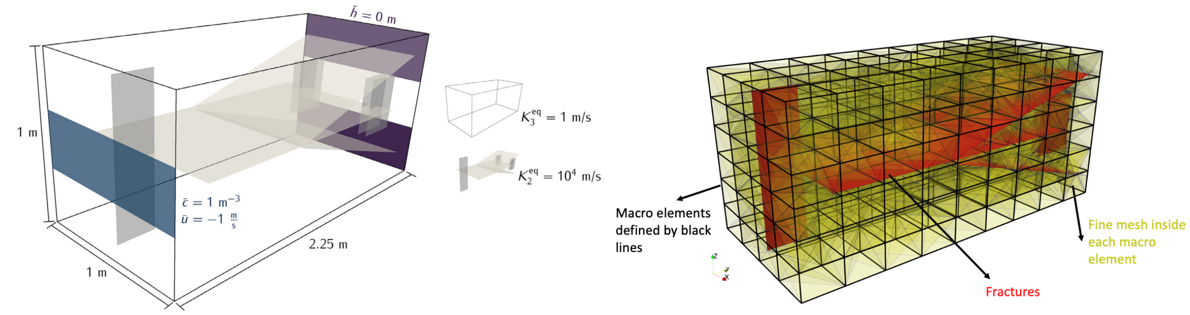
3.2 Benchmark problem case 3

This test is designed to test the method's precision in the presence of small fractures. These can cause problems for strategies that use uniform conformal meshes. The problem domain is defined at $\Omega = [0\,\text{m}, 1\,\text{m}] \times [0\,\text{m}, 2.25\,\text{m}] \times [0\,\text{m}, 1\,\text{m}]$ and 8 fractures are present as illustrated in Figure 7.

A constant entering flux of $\sigma_{N_2} = -1.0$ is imposed at $\partial\Omega_{N_2} = \{(x, y, z) \in \partial\Omega : (0\,\text{m}, 1\,\text{m}) \times \{0\,\text{m}\} \times (1/3\,\text{m}, 3\,\text{m})\}$ and a condition of $p_3 = 1\,\text{m}$ is imposed at $\partial\Omega_{D_{2,1}} = \{(x, y, z) \in \partial\Omega : (0\,\text{m}, 1\,\text{m}) \times \{2.25\,\text{m}\} \times (0\,\text{m}, 1/3\,\text{m})\}$ and $\partial\Omega_{D_{2,2}} = \{(x, y, z) \in \partial\Omega : (0\,\text{m}, 1\,\text{m}) \times \{2.25\,\text{m}\} \times (2/3\,\text{m}, 1\,\text{m})\}$. Zero flux is applied to the remaining part of the boundary of the 3D domain. The adopted material properties are omitted here and can be verified in Berre et al. [7].

Results

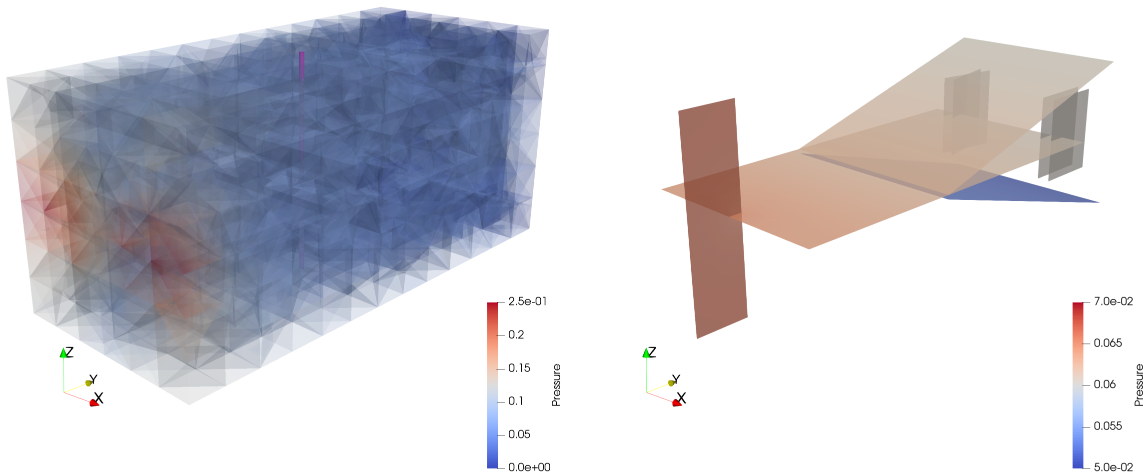
The total number of degrees of freedom of the simulation is 96 951. By using condensation of the fine discretization towards the skeleton mesh, a final system of 6 394 equations is obtained. The matrix pressure is sampled along a line defined between the points $(0.5\,\text{m}, 1.1\,\text{m}, 0\,\text{m})$ and $(0.5\,\text{m}, 1.1\,\text{m}, 1\,\text{m})$ as shown in Figure 8.



(a) Problem domain and boundary conditions (b) Multiscale FEM mesh

Figure 7. Illustration of problem domain, fractures, and multiscale mesh for benchmark case 3.

This pressure is compared against the results by other methods in the benchmark in Figure 9. It can be seen that the solution obtained is within the range of the other methods.



(a) Matrix pressure and line for sampling points (pink) (b) Fluid pressure on fractures

Figure 8. Pressure solution for case 3 in both matrix and fractures.

4 Conclusions

The presented methodology is able to solve complex fracture networks and leads to a great matching of pressure for the benchmark cases 2 and 3 proposed in Berre et al. [7]. By using the MHM and condensing the degrees of freedom of each macro element into the skeleton mesh, the size of the final system of equations in both examples is considerably reduced. This is a very attractive feature when simulating more realistic reservoirs that involve time-dependency and non-linearities, where the final system of equations needs to be solved several times during a simulation cycle.

Assessing the performance of the method when analyzing real reservoir cases with a high degree of permeability heterogeneity is the next step in this research. Also, since the MHM is an inherently highly parallelizable method, focus will be given to studying optimal parallel paradigms to accelerate this methodology.

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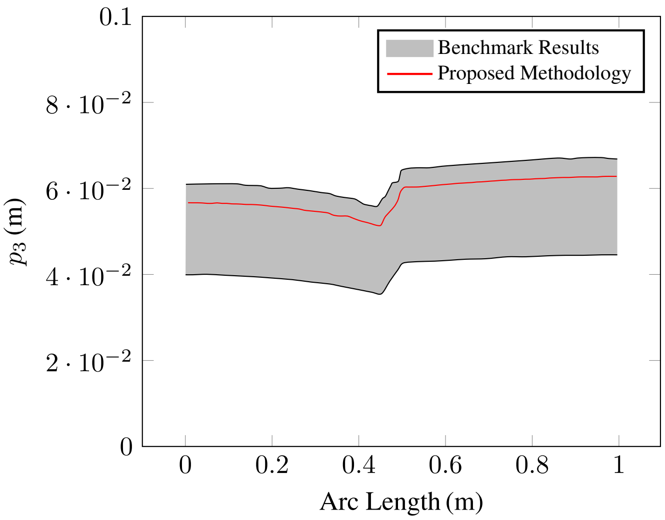

Figure 9. Comparison of matrix pressure over line through domain between proposed methodology and other methods available for the benchmark case 3 (Berre et al. [7])

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