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Numerical analysis of optimum designed models of viscoelastic supports for rotating machines

Bruno F. A. Prado[1], Carlos A. Bavastri[1], Eduardo A. Ribeiro[2]

[1]*Dept. of Mechanical Engineering, Federal University of Paraná*
Av. Cel. Francisco H. dos Santos, 100 – bloco IV,81530-900, Curitiba/PR, Brazil
brunoprado@ufpr.br, bavastri@ufpr.br
[2]*Dyn Technologies*
Lodovico Geronazzo 1847, 82.560-040, Curitiba/PR, Brazil
eduardo@dyntechnologies.com.br

Abstract. Viscoelastic supports (VES) are a simple solution for vibrations in rotating machinery with a low associated cost. The objective of this work is to complement an optimal design methodology for VES, developed by the GVIBS/UFPR group for rotating systems subject to unbalanced excitations, inserting a static stiffness in parallel, whose purpose is to increase the load capacity of the device. The dynamic behavior of the rotating system is represented through the finite element method. For the viscoelastic material, the fractional derivatives model of four parameters is used, which allows considering the effect of temperature and excitation frequency. For the optimal design, the concept of generalized equivalent parameters (GEP) is used, allowing to describe the equation of motion of the composed system with VES, being able to obtain the response of the composed system in a space or subspace of the primary system, efficiently from a computational time perspective. The primary system is modeled considering a simple rotor and the support is introduced via GEP. Nonlinear techniques allow for optimal design of the VES. Numerical simulations on rotors with known dynamic behavior allow showing the results of the proposed methodology and the effectiveness of viscoelastic supports.

Keywords: Rotordynamics, Passive vibration control, Viscoelastic materials, Optimization, Viscoelastic supports.

1 Introduction

Rotating machines such as pumps, compressors, electric motors and turbo-generators are widely used in the most diverse sectors of the production system, including oil and gas and energy. In these sectors, the relentless search for greater energy efficiency and high productivity stands out, since their costs reflect throughout the production chain. Thus, rotating machines capable of providing high power with low energy consumption, volume and mass, but with high operating safety and reliability become essential.

Increasing the energy efficiency of rotating machines is mainly achieved by operating at elevated nominal speeds and through the use of thinner shafts. Such configuration, however, generates operating conditions close to its critical operating conditions from the perspective of vibration level. Systems that operate at high speeds generally operate at supercritical speeds, that is, during starts and stops, the equipment passes through at least one critical speed, which is the speed that excites the system at one of its natural frequencies, causing high radial vibrations.

In order to enable safe operation under conditions that provide high energy efficiency, several control methods are studied for rotating systems. Among them is the use of viscoelastic supports (VESs), which are support elements of the rotating system that have elements with high energy dissipation capacity, in this case, viscoelastic materials (VEMs). These devices have great potential for passive vibration control in rotating machines due to the associated low cost and high performance in vibration control. Several works have been developed by GVIBS research group to consolidate the use of viscoelastic supports as an option for vibration control in rotating machines, among them the works of Bavastri et al [1], Silvério [2] and Ribeiro [3].

Aiming to increase the load capacity of the VESs, both to support the own weight of the rotating system and to support external loads, the present work proposes the insertion of a static stiffness in parallel with the viscoelastic layers in the device. The optimal design is carried out to provide the maximum reduction in system vibration response over the systems operating range.

2 Mathematical model

2.1 Viscoelastic support model

Viscoelastic materials are those whose behavior combine elastic and viscous properties, i.e. store and dissipate mechanical energy. Many mathematical models were developed to represent the dynamic behavior of these materials, among them, the four-parameter fractional derivative model, as described in Bagley and Torvik [4]. This model adequately represents a linear VEM on the frequency domain. The complex shear modulus is given by

$$\bar{G}(\Omega,T)=\frac{G_0+G_\infty b_1(\mathrm{i}\Omega)^\beta}{1+b_1(\mathrm{i}\Omega)^\beta}, \tag{1}$$

where G_0 represents its lowest asymptotic value of the modulus and G_∞its upper asymptotic value, b_1 is a constant related to the material relaxation time and β is the fractional order of the derivative.

The VEM's dependency of temperature is considered as a shift in the frequency domain for simple thermoreologic materials. According to Ferry [5], this displacement is written as

$$\alpha(T)=10^{-\varphi_1\frac{(T-T_0)}{(\varphi_2+T-T_0)}}, \tag{2}$$

where φ_1 and φ_2 are parameters of the material experimentally determined and T_0 is the reference temperature.

Considering the inverse proportionality of thermorheologically simple materials to variations in dynamic behavior with temperature and frequency, up to a scale factor, it is possible to define a single parameter that allows describing the dynamic behavior as a function of temperature and frequency. This parameter is called reduced frequency and is defined as

$$\Omega_r(T)=\alpha(T)\Omega. \tag{3}$$

Thus, Eq. 1 can be rewritten, giving the complex shear modulus as function of frequency and temperature as

$$\bar{G}(\Omega,T)=\frac{G_0+G_\infty b_1(\mathrm{i}\Omega_r)^\beta}{1+b_1(\mathrm{i}\Omega_r)^\beta}. \tag{4}$$

The complex stiffness of a VEM sheet is associated with its geometric factor l_g, as

$$\bar{k}(\Omega)=\mathrm{l_g}\bar{G}_a(\Omega), \tag{5}$$

where $\bar{G}_a(\Omega)$ is the apparent shear modulus, obtained multiplying $\bar{G}(\Omega)$ by the form factor l_f, according to Nashif et al [6]. The influence of l_f, although, is soft, because in this work a pure shear strain state in the VEM sheet is considered.

The VES proposed in this work is composed by two parallel stifness, the equivalent stiffness of the bearing with viscoelastic sheets and a static stiffness – called static because it does not vary with frequency. Fig. 1 presents the proposed VES representation. Because of its parallel configuration, the total equivalent stiffness of the support is the sum of the equivalent stiffness of the viscoelastic part and the static stiffness.

In the present paper, the viscoelastic part model used is the one with one additional DOF. According to Ribeiro [3], this model is dynamically equivalent to the traditional model, which adds DOFs to the system. The equivalent stiffness of it is obtained through the GEP approach, as presented in Ribeiro [3]. The translational and angular equivalent stiffness coefficients are given, respectively, by

$$\bar{k}_{t\,VE}(\Omega)=k_{tb}-\frac{k_{tb}^2}{k_{tb}+\bar{k}_{t1}(\Omega)-\Omega^2 m_b-\frac{(\bar{k}_{t1}(\Omega))^2}{\bar{k}_{t1}(\Omega)+\bar{k}_{t2}(\Omega)-\Omega^2 m_{f1}}} \tag{6}$$

and

$$\bar{k}_{r\,VE}(\Omega) = k_{rb} - \frac{k_{rb}^2}{k_{rb}+\bar{k}_{r1}(\Omega)-\Omega^2 I_b - \frac{(\bar{k}_{r1}(\Omega))^2}{\bar{k}_{r1}(\Omega)+\bar{k}_{r2}(\Omega)-\Omega^2 I_{f1}}}. \tag{7}$$

The subscripts t and r are related to the DOF which the control is provided, translational or rotative, while the subscript b is related to the bearing. k is the stiffness, m is mass and I is the moment of inertia.

Then, the VES equivalent stiffness, for translational and angular cases, is given by

$$\bar{k}_{t\,eq}(\Omega) = \bar{k}_{t\,VE}(\Omega) + k_{t\,st} \tag{8}$$

and

$$\bar{k}_{r\,eq}(\Omega) = \bar{k}_{r\,VE}(\Omega) + k_{r\,st}. \tag{9}$$

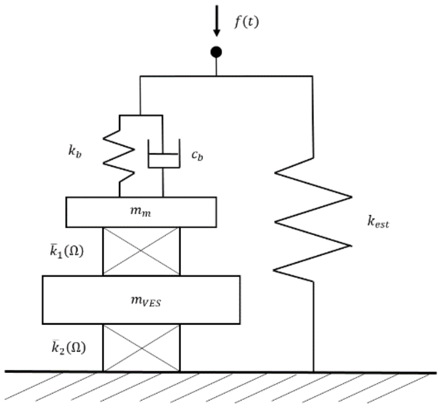

Figure 1. VES with parallel static stiffness representation.

2.2 Unbalance response

Through the generalized equivalent parameters (GEP) methodology, developed by Espíndola and Silva [7] it is possible to describe the dynamic of the composed system (rotor + VES) in terms of the generalized coordinates of the primary system, the system to be controlled, a constrained rotor with known elastic bounds. The modal parameters of the primary system are used as a basis in the modal space to describe the response of the composed system. This is possible because the utilization of the concept of simplified Campbell, proposed by Espíndola and Bavastri [8], which allows to describe the modal parameters of the primary system independently from the frequency for the case of synchronous excitation, as unbalance. The influence of the supports in the system are considered using the VES equivalent stiffness. According to Ribeiro [3], the equations of motion of the composed system in the frequency domain, in the space state are

$$\begin{bmatrix}\mathbf{C} & \widehat{\mathbf{M}}\\ \widehat{\mathbf{M}} & \mathbf{0}\end{bmatrix}\begin{Bmatrix}\mathrm{i}\Omega\mathbf{Q}(\Omega)\\ -\Omega^2\mathbf{Q}(\Omega)\end{Bmatrix} + \left(\begin{bmatrix}\mathbf{K} & \mathbf{0}\\ \mathbf{0} & -\widehat{\mathbf{M}}\end{bmatrix} + \begin{bmatrix}\bar{\mathbf{K}}_{\mathrm{eq}}(\Omega) & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\right)\begin{Bmatrix}\mathbf{Q}(\Omega)\\ \mathrm{i}\Omega\mathbf{Q}(\Omega)\end{Bmatrix} = \begin{Bmatrix}\mathbf{F}(\Omega)\\ \mathbf{0}\end{Bmatrix}, \tag{10}$$

or simply

$$\left(\mathrm{i}\Omega\mathbf{A} + \mathbf{B} + \mathbf{B}_{\mathbf{eq}}\right)\mathbf{Y}(\Omega) = \mathbf{N}(\Omega). \tag{11}$$

In Eq. 10, $\widehat{\mathbf{M}}$ results from the use of the simplified Campbell concept and is $\widehat{\mathbf{M}} = \mathbf{M} - \mathrm{i}\mathbf{G}_0$. $\bar{\mathbf{K}}_{\mathrm{eq}}(\Omega)$ is the sparse matrix in which the VES equivalent stiffness are inserted, in the respective positions related to the node of link between the VES and the rotor, as

$$\bar{\boldsymbol{K}}_{\boldsymbol{eq}}(\Omega) = \begin{array}{c} \begin{array}{cccccc} \cdots & u & v & \theta & \psi & \cdots \end{array} \\ \begin{bmatrix} \ddots & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{k}_{t\,eq}(\Omega) & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{k}_{t\,eq}(\Omega) & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{k}_{r\,eq}(\Omega) & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{k}_{r\,eq}(\Omega) & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddots \end{bmatrix} \end{array} \begin{array}{c} \vdots \\ u \\ v \\ \theta \\ \psi \\ \vdots \end{array}. \tag{12}$$

The unbalance response of the composed system in terms of the generalized coordinates of the rotor is given by

$$\mathbf{Q}(\Omega) = \boldsymbol{\Theta}\left(\mathrm{i}\Omega\mathbf{I} + \boldsymbol{\Lambda} + \boldsymbol{\Psi}^{\mathrm{T}}\mathbf{B}_{\mathrm{eq}}\boldsymbol{\Theta}\right)^{-1}\boldsymbol{\Psi}^{\mathrm{T}}\mathbf{F}(\Omega), \tag{13}$$

where $\boldsymbol{\Theta}$ and $\boldsymbol{\Psi}$ are the orthonormalized eigenvectors, $\mathbf{I}$ is the identity matrix and $\boldsymbol{\Lambda}$ is the spectral matrix, which contain the eigenvalues on its main diagonal. $\mathbf{F}(\Omega)$ is the unbalance excitation vector.

2.3 Optimization

A hybrid optimization technique is used to obtain the VES optimum parameters in terms of the unbalance response of the compound system. Combining genetic algorithms (GA) and the Nelder-Mead method, it is possible to obtain the global minimum with relative low computational time. The optimization begins with GA to get points close to global minimum. The Nelder-Mead technique start from this to obtain the global minimum point, located at the region.

The optimization problem is defined as

$$\min f(\boldsymbol{x}) \colon \mathbf{D}^{\mathbf{n}} \to \mathbf{D}, \tag{14}$$

where $\boldsymbol{x}$ is the design vector, composed by the parameters of the VES. This depends on the type of VES designed, in the most general case, the combined VES, the design vector is given by

$$\boldsymbol{x} = \left\{l_{\mathrm{gt1}}, l_{\mathrm{gt2}}, l_{\mathrm{gr1}}, l_{\mathrm{gr2}}, m_{\mathrm{f}}, I_{\mathrm{f}}\right\}. \tag{15}$$

In the case of a translational VES, for instance, the terms related to the angular DOFs ($l_{\mathrm{gr1}}, l_{\mathrm{gr2}}$ and I_{f}) would not appear in Eq. 15, the same happens to the angular VES with terms related to translational DOFs. Preliminary studies shown that the optimization of the parallel stiffness results in extreme values for this parameter. When the optimization is done for a point in the middle of the shaft, the optimum stiffness obtained is very small. On the other hand, when the optimization is done for the support point, the stiffness results extremely high. These results are coherent with the physical phenomena but are not interesting from the design perspective of the device. Thus, it was decided to define the static parallel stiffness from the maximum static deflection allowed to the system and optimize the VES parameters which combined with this stiffness results in the minimum unbalance response.

The cost function to be minimized is the maximum value of the unbalance response vector in one or more selected generalized coordinates of the rotor, i.e.

$$f(\mathbf{x}) = \max|\mathbf{q}(\boldsymbol{x}, \boldsymbol{\Omega})|, \tag{16}$$

with

$$\mathbf{f}(\boldsymbol{x}, \boldsymbol{\Omega}) = \mathbf{Q}_j(\boldsymbol{\Omega}). \tag{17}$$

3 Numerical simulations

The analysis of the control provided by the optimal VESs designed is conducted comparing the unbalance responses of the system supported in rolling bearings and supported in VES in the frequency domain via numerical simulations. The finite element model of the rotor supported in rolling bearings is presented in Fig. 2. It consists in a shaft divided in 8 elements, one disc and two rolling bearings. The data present in the model is shown in Table

1.

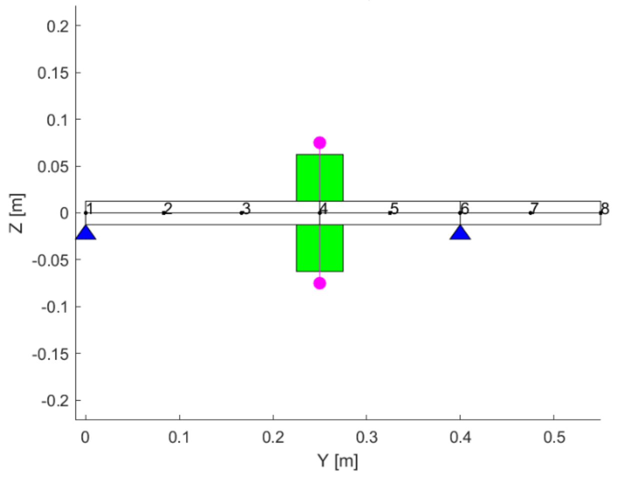

Figure 2. Rotor finite element model.

The frequency unbalance response of the rotor supported in rolling bearing is presented in Fig. 3, in log scale.

For comparison, four VESs were optimally designed, without and with the static parallel stiffness for purely translational VES and purely angular VES, in order to evaluate the unbalance response control and the influence of the parallel stiffness in each case. For the purely angular VES, there are angular stiffness on the bearings, besides the properties shown in Table 1. It is necessary to consider it to transfer the shaft tilt to the VES sheets. In this case, isotropic angular stiffness is considered as 1 kN/m.

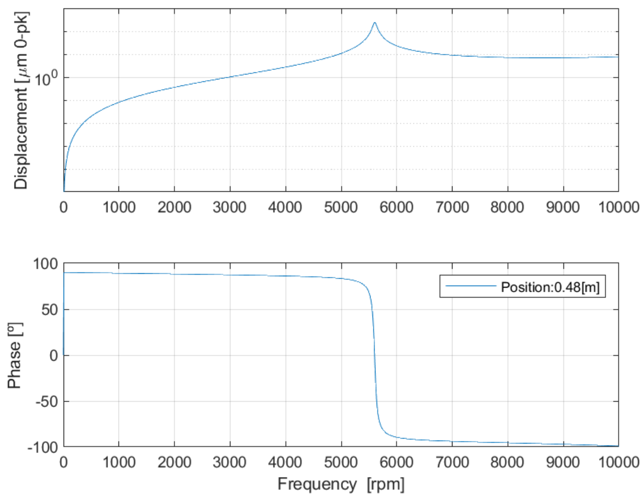

Figure 3. Primary system's frequency unbalance response.

Table 1. Rotor-bearing data

Shaft				
Length [mm]	Diameter [mm]	Density [kg/m³]	Young Mod. [GPa]	Poisson ratio
550	50	7800	200	0.3

Disc			
Position [mm]	Diameter [mm]	Thickness [mm]	Density [kg/m³]
250	125	50	7386

Rolling bearings					
k_{xx}[kN/m]	k_{zz}[kN/m]	c_{xx}[Ns/m]	c_{zz}[Ns/m]	Mass [kg]	Inertia [kgm²]
2300	2300	500	500	0.15	$1.8\ 10^{-5}$
Unbalance					
Position [mm]		Magnitude [mm]		Phase [°]	
250		75		0	

The VEM used in the simulations is a butyl rubber, its parameters are presented in Table 2.

Table 2. VEM parameters

G_0[MPa]	G_∞[MPa]	b_1	β	φ_1	φ_2	T_0[K]	T[K]
3.57	179	0.00246	0.435	6.57	68	273	295

4 Results

Table 3 presents the optimization results for all VES models studied. The parallel stiffness was obtained dividing the rotors weight by the maximum deflection allowed, defined as 0.5 mm. Fig. 4 presents the frequency unbalance response of the composed system for the four optimum VES in linear scale.

Table 3. Optimization results

Translational VES			
l_{gt1}[m]	l_{gt2}[m]	Mass [kg]	Parallel Stiffness [kN/m]
0.0249	0.0027	0.476	-
0.3804	0.0684	4.932	84.7
Angular VES			
l_{gr1}[m]	l_{gr2}[m]	Inertia [kg.m²]	Parallel Stiffness [kN/m]
0.0849	$8.4742\ 10^{-5}$	0.0036	-
0.0108	$7.9092\ 10^{-5}$	0.0034	84.7

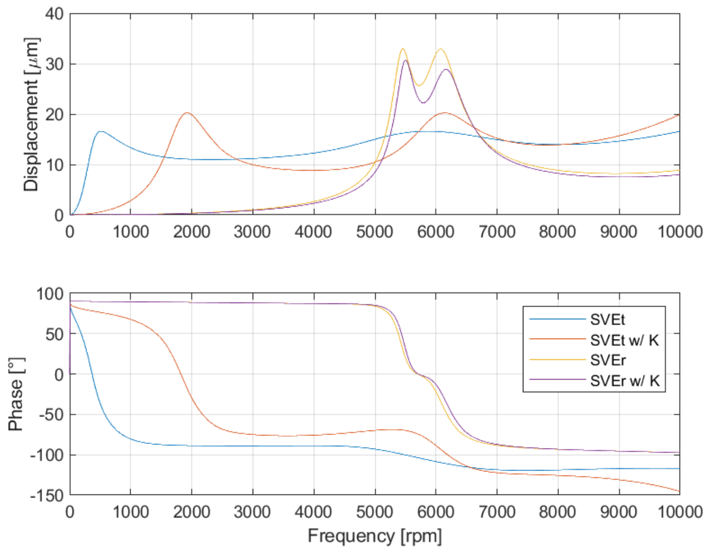

Figure 4. Frequency unbalance response of purely translational and purely rotational, without and with static parallel stiffness.

Comparing the frequency unbalance response of the primary system (Fig. 3) with the response of the compound system (Fig. 4) for any type of VES it is possible to claim that all designs have great capacity for control. When comparing the composed system response for the different types of VESs, it is noticed that the control provided by translative DOFs is more effective than by rotational DOFs. On the other hand, it is possible to claim that the influence of parallel static stiffness on the response for the purely rotational VES impact less the control provided than the purely translational device.

Trough the analysis of the optimum parameters obtained for the devices, shown in Table 3, it is noticed that the results of the optimization vary more between the case of the device without and with parallel stiffness for the translational VES.

5 Conclusions

This paper presented a methodology for optimal design of VES with static parallel stiffness for unbalance response control. Insertion of the stiffness in the device improves it load capacity, enabling its application in heavier and more robust rotating equipment. Four types of VES were numerically simulated to comprehend the influence of the stiffness in the system's behavior. Simulations showed that the VESs with static parallel stiffness are able to control the unbalance response.

All devices result in great reduction of unbalance response, the greater is obtained with purely translational VES. The rotational VES is less sensitive to the static parallel stiffness than the translational.

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