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Evaluation of the Nonlinear Behavior of Concrete Structures Using a Flat Shell Finite Element

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Abstract. This work presents a study of the coupling of the geometric and material nonlinearities applied in plain and reinforced concrete structures, using flat shell finite elements. For geometric nonlinearities, the principle of virtual works and a generalized finite element method (GFEM) approximation are explored to directly formulate the nonlinear governing equations in a total Lagrangian approach. The material's behavior will be described by a smeared cracking constitutive model, capable of representing the concrete degradation process, from the phenomenological approach of crack propagation. In order to evaluate variations in stresses, strains, and, consequently, degradation due to cracking along the thickness, the shell element is subdivided into N discrete layers, allowing the measurement of the inelastic behavior of the material. In this proposal, each layer can be represented by different materials, allowing the representation of reinforced concrete structures with the steel described by a classic elastoplastic model. Finally, numerical simulations of shell structures will be presented, considering the coupling of the referred nonlinearities.

Keywords: Nonlinear Shell, Total Lagrangian, Smeared Crack, Von Mises, Concrete Structures

1 Introduction

The modeling of reinforced concrete shell structures has always been an important topic in computational mechanics. Scientific advances arising from numerical methods have allowed the use of these sophisticated structural models such as bridges, shear walls, shell roofs, pressure vessels, water tanks, and a different types of storage structures. So, there is a demand for reliable shell models that are capable greater use of their properties and functionalities taking into acount nonlinear effects associated with the material response, such as concrete cracking or large displacements.

From the structural modeling point of view, the reinforced concrete shells can be understood as a combination of bending and membrane elements. In this scenario, the material behavior plays a crucial role, fitting perfectly from the main approach shell element formulation used by most researchers, the concept of degenerate shell elements [1–6]. Therefore, a proper characterization of the material constitutive behavior is of great importance of the analysis of the reinforced concrete shell structures using the Finite Element Method.

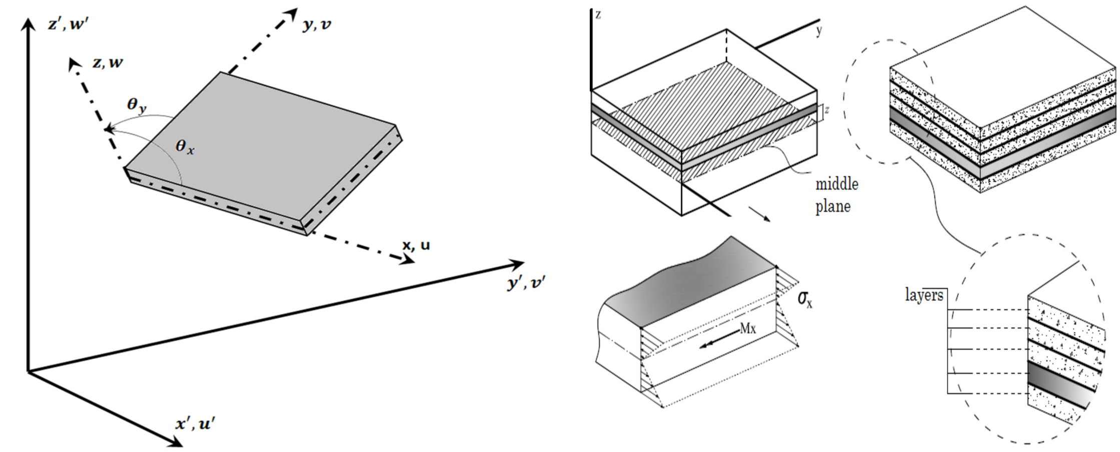
Among the most efficient approaches used in the analysis of concrete structures, there is the concept of smeared crack models, which provides the representations of cracked concrete by a continuous media. So, stresses and strains relations are adopted to follow the cracking process, with the concrete strength limits and fracture mechanics parameters being the bases of the formulation [7].

This paper presents a formulation of finite element shells derived from the degeneration concept on layered shell structures. Problems involving shell finite elements with large displacements and strains, typical from geometrically nonlinear analysis, can induce shear, membrane locking, and the presence of highly distorted elements that deteriorate the quality of the FEM approximation. To overcome these restrictions, the Generalized Finite Element Method (GFEM), which is less susceptible to mesh distortion was adopted. Thus, this work highlights the development and formulation of the reinforced concrete shell element assuming the GFEM.

One of the approaches to shell finite elements is the structural theory based on a reference surface and Kirchhoff's hypothesis. Among a diversity of the shell theories proposed by different authors, this work presents a flat shell finite element based on Marguerre's Shell Theory [8–11] to consider geometric nonlinearity.

2 Shell finite elements

The studied element is an isoparametric degenerated shell based on Marguerre's theory modified by the introduction of Mindlin's hypothesis [12]. The theory consider five degrees of freedom in each node: three displacements $[u_0, v_0, w_0]^T$ in x, y and z directions, like Marguerre's theory, and two rotations $\left[\theta_x, \theta_y\right]^T$ along x and y axes, like Reissner-Mindlin theory. Figure 1 shows the rectangular shell domain defined in a coordinate system (x, y, z) and the discretization of the element thickness in layers.



(a) (b)

Figure 1. (a) Rectangular flat shell domain. [8] (b)Layered decomposition of the shell thickness.

In the layered element formulation, the shell is divided into several layers where it is possible to obtain the variation of the stress and strain state along the thickness of the element, thus allowing to capture the variations of the material properties (each layer can assume as a steel layer or a plain concrete layer, composing the reinforced concrete element) due to damage or plastification. Therefore, to perform a physically nonlinear analysis, each layer must be analyzed separately. From the contribution of each layer, the stiffness of the elements is computed, and the internal stresses are the result of the sum of the parts of the layers.

2.1 Geometric Nonlinearity

A total Lagrangian formulation on Green–Lagrangian strain fields is applied to account the geometrical nonlinearity in the shell element. The formulation adopted follows [13], and it is summarized to ground the discussion about some specific characteristics of the GFEM enrichment strategy.

Based on the method of [14], the enrichment scheme is obtained by multiplying a PU function of C_0 constructed in regions called clouds, ω_j, by the function $L_{ji}(x)$, denominated enrichment function. The use of FEM functions as the PU simplifies the formulation of the method and avoids problems related to the numerical integration and imposition of the boundary conditions. Thus, generic global approximation, denoted by $\tilde{u}(x)$, can be described as a linear combination of the shape functions associated with each node and can be written as:

$$\tilde{u}(x) = \sum_{j=1}^{n} \mathcal{N}_j(x) \left\{ u_j + \sum_{i=2}^{q} L_{ij}(x) b_{ji} \right\} \tag{1}$$

where u_j is a nodal parameter related to standard FE shape function ($\mathcal{N}_j$) and b_{ji} are nodal parameters associated with GFEM shape functions ($\mathcal{N}_j \times L_{ij}$). Considering a shell finite element subjected to a surface load, $\mathbf{t}$ (a vector containing the distributed loads acting on the shell surface and distributed moments contained in the plane xz and yz), and the concentrated loads and moments, $\mathbf{p}_i$. The Principle of Virtual Work for a flat shell in two dimensions, is

$$\int_{A_0} \delta\hat{\varepsilon}^T \hat{\sigma} dA_0 = \int_{A_0} \delta\tilde{u}^T \mathbf{t} dA_0 + \sum_i \delta\tilde{u}_i^T \mathbf{p}_i \tag{2}$$

In Marguerre's theory for flat shells, all the derivatives in the integrals of the eq. (2) are first-order, allowing to use of C_0 continuous elements. Inthe discretization of a surface into C_0 isoparametric flat shell elements with n nodes (Fig. 2). In matrix form, the generalized local strain can be written as

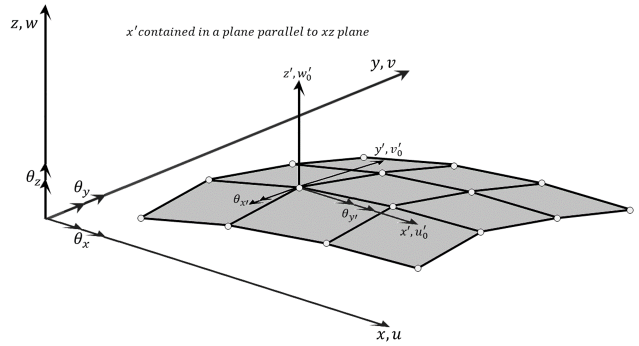


Figure 2. Shell discretized with 4-noded flat rectangles ([8]).

$$\varepsilon = [B_1, B_2, \cdots, B_n] \begin{Bmatrix} d_1^{(e)} \\ d_2^{(e)} \\ \vdots \\ d_n^{(e)} \end{Bmatrix} = \mathbf{B}\mathbf{d}^{(e)} \tag{3}$$

where $\mathbf{B}$ are the local generalized strain matrices for the element, and can be written as

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}^m \\ \mathbf{B}^b \\ \mathbf{B}^s \end{bmatrix} = \begin{bmatrix} \mathbf{B}^m \\ \mathbf{B}^b \\ \mathbf{B}^s \end{bmatrix}_L + \frac{1}{2} \begin{bmatrix} \mathbf{B}^m \\ 0 \\ 0 \end{bmatrix}_{NL} \tag{4}$$

where the subscripts L and NL refers to the linear and nonlinear parts, and $\mathbf{B}^m$, $\mathbf{B}^b$, and $\mathbf{B}^s$ are respectively the membrane, bending, and transverse shear strain matrices of a node respectively. Introducing the finite element discretization and following the standard process from the Principle of Virtual Work in its finite element total Lagrangian formulation (eq. (2)), the equilibrium equations to a single element are obtained as

$$\mathbf{p}_{(e)} = \mathbf{K}_{(e)}\mathbf{d}_{(e)} - \mathbf{f}_{(e)} \tag{5}$$

where the stiffness matrix and the equivalent nodal force vector of the element in local axes are

$$\mathbf{K}_{(e)} = K_L + K_{NL} = \int_{V_0} \mathbf{B}_L^T \mathbf{D} \mathbf{B}_L dV_0 + \int_{V_0} \mathbf{B}_{NL}^T \mathbf{S} \mathbf{B}_{NL} dV_0 \tag{6a}$$

$$\mathbf{f}_{(e)} = \int_{V_0} \mathbf{B}_L^T \mathbf{S} dV_0 \tag{6b}$$

where **S** is the second Piolla-Kirchhoff stress tensor.

The expression of $K_{(e)}$ of eq. 6a is rewritten as the local stiffness matrices corresponding to membrane, bending, transverse shear and membrane-bending coupling effects, respectively, as follows

$$\mathbf{K}^m_{(e)} = \int_A (\mathbf{B}^m)^T \mathbf{D}^m \mathbf{B}^m dA \tag{7a}$$

$$\mathbf{K}^b_{(e)} = \int_A (\mathbf{B}^b)^T \mathbf{D}^b \mathbf{B}^b dA \tag{7b}$$

$$\mathbf{K}^s_{(e)} = \int_A (\mathbf{B}^s)^T \mathbf{D}^s \mathbf{B}^s dA \tag{7c}$$

The local stiffness matrix for the element is obtained, using numerical integration techniques, as the sum of the membrane, bending, and transverse shear contributions. The GFEM stiffness matrix contains the FEM standard stiffness matrix, including the additional degrees of freedom arising from enrichment. Matrix $\mathbf{K}_L$ and $\mathbf{K}_{NL}$ in GFEM problems can be partitioned as follow:

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{FEM} & \mathbf{K}_{FEM,GFEM} \\ \mathbf{K}_{GFEM,FEM} & \mathbf{K}_{GFEM} \end{bmatrix} \tag{8}$$

where $\mathbf{K}_{FEM}$ is the FEM stiffness matrix, $\mathbf{K}_{GFEM}$ represents the additional elements in the matrix associated only with the GFEM enrichments and $\mathbf{K}_{FEM,GFEM}$ is related to the combination of the FEM degrees of freedom and the ones associated with the enrichment functions.

Furthermore, it should be emphasized the formulation simplicity, considering that the construction of the element is made from the existence of the deformation matrices referring to the bending (plate bending element), and membrane (membrane element) states, which can be obtained separately.

3 Material Constitutive Models

The adopted constitutive models to represent the plain concrete and the reinforcement steel were based on the unified framework for constitutive modeling presented by [7], [15], and [16]. For more details, these references should be consulted.

3.1 Smeared Cracking Models for Concrete

The plain concrete was represented by a smeared crack model. Such a model treats the material as continuous, and the crack effects are spread over the element by changing the constitutive equation. Stress-strain relationships are adopted to follow the fracture process, being the concrete limits and fracture energy the characteristic parameters of the material.

This model depends directly on evolution laws (stress-strain relationships or damage laws) capable of computing the degradation of the material physical properties in the principal strain directions. In this work, the smeared crack model was combined with Carreira and Chu's [17] law for compression and Boone and Ingraffea's [18] law for tension (Fig. 4). The necessary material parameters for the stress-strain relations description are: elastic modulus (E_0), compressive strength (f_c), tensile strength (f_t) and fracture energy (G_f).

The concrete compressive model [17] presents polynomial forms based on the stress and strain limits for compression behavior, given by the equation

$$\sigma = f_t \frac{k\left(\frac{\varepsilon}{\varepsilon_c}\right)}{k - 1 + \left(\frac{\varepsilon}{\varepsilon_c}\right)^k} \quad \text{with} \quad k = \frac{1}{1 - \left(\frac{f_c}{\varepsilon_c E_0}\right)} \tag{9}$$

where σ is the stress, ε is the current strain, and ε_c is the strain relative to strength compressive limit.

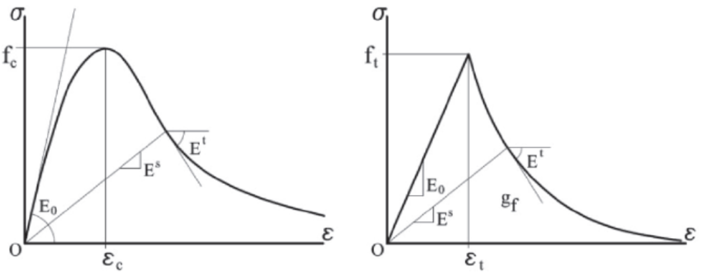


Figure 3. Carreira-Ingraffea Model.

The proposal by [18] witch approximates the tensile behavior of concrete by an exponential law based on the fracture energy and the limits of stress and strain. The model is given by

$$\sigma = f_t e^{-k(\varepsilon - \varepsilon_t)} \quad \text{with} \quad k = \frac{hf_t}{G_t} \quad \text{or} \quad k = \frac{f_t}{g_f} \tag{10}$$

where ε_t is the strain relative to the elastic tensile limit, h is the characteristic length, and g_f is the specific fracture energy.

3.2 Von Mises's Constitutive Model for Steel

In this paper, an elastoplastic constitutive model was adopted to represent the steel reinforcement. The criterion proposed by von Mises in 1913, which can be used to describe plastic yielding in metals, requires that the yielding starts when the critical value of distortion energy per unit volume is reached [19]. This is equivalent to considering that the second principal invariant of the stress deviator tensor, J_2, arrived at a critical value. The mathematical representation of the von Mises criterion and the flow rule for mixed hardening is given by:

$$F(\sigma, \alpha, \beta) = \sqrt{3J_2(\eta(\sigma, \beta))} - (\sigma_y - H\alpha), \tag{11}$$

where H is the linear isotropic hardening modulus, α is the function of the accumulated plastic flow.

For the theoretical basis for a unified formulation for constitutive models implementation in the INSANE system the reader may refer to Penna [7]. For the detailed description of plasticity models, see Oliveira [16].

4 Numerical Simulation of a panel under out-of-plane bending

A numerical simulation of a panel under out-of-plane bending was performed. In this simulation, the combination of the GFEM and layered shell elements is evaluated by a comparison with experimental and numerical results presented in the literature. For numerical simulation, the INSANE (INteractive Structural ANalysis Environment) system is used, a free software developed at the Structural Engineering Department of the Engineering School of the Federal University of Minas Gerais [20]. As showed in Fig. 4, a square reinforced concrete panel tested by Polak and Vecchio[3] under pure bending were analyzed.

The panel thickness was discretized into 10 concrete layers reinforced by 4 steel layers (2 horizontal and 2 vertical) with 1.25% and 0.42% steel per layer in the x- and y-directions. The concrete is characterized by an elastic modulus E = 30000.0 MPa, a Poisson's coeficient ν = 0.20, tensile and compressive uniaxial strengths f_t = 4.70 MPa and f_c = 47.0 MPa, a fracture energy G_f = 0.0001 MN/m, and a characteristic length of the material h = 0.075 m. For steel, the elastoplastic criterion of von Mises was adopted, assuming perfect plasticity, and is characterized by an elastic modulus E = 210000.0 MPa, a Poisson's modulus ν = 0.3 and initial yield stress in the x- and y-directions are 425 MPa and 430 MPa, respectively. It was considered perfect adhesion between steel and concrete.

In this simulation, were adopted a mesh of 16 four-nodes quadrilateral elements for GFEM with polynomial enrichment of P2 type associated with all nodes. For comparison purposes, Polak and Vecchio [3] used 16 eigth-nodes quadrilateral elements. Silva and Horowitz [6] used 64 four-nodes quadrilateral elements.

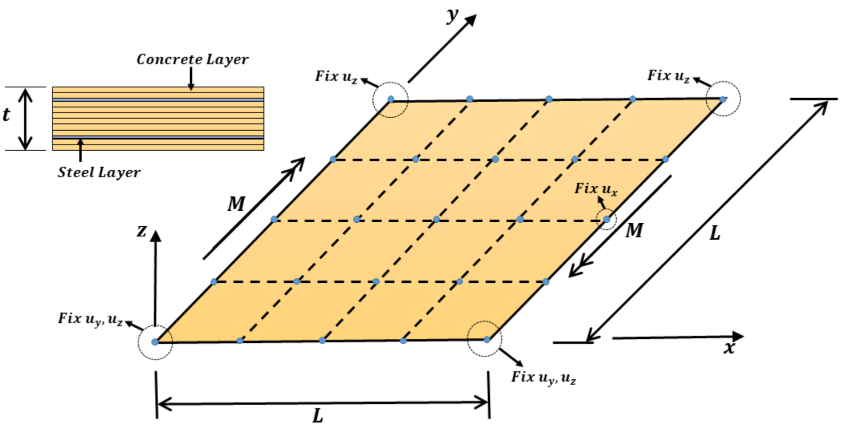


Figure 4. Finite element model of Polak shell

The loading process is driven by the displacement control method [21], assuming increments of $0.001uc$ on the central node, and a tolerance for the convergence in displacement of 1×10^{-2}. The results of moments *versus* curvatures of the panel obtained from the GFEM analysis were compared with the experimental and numerical results presented in the literature, as shown in Fig. 5.

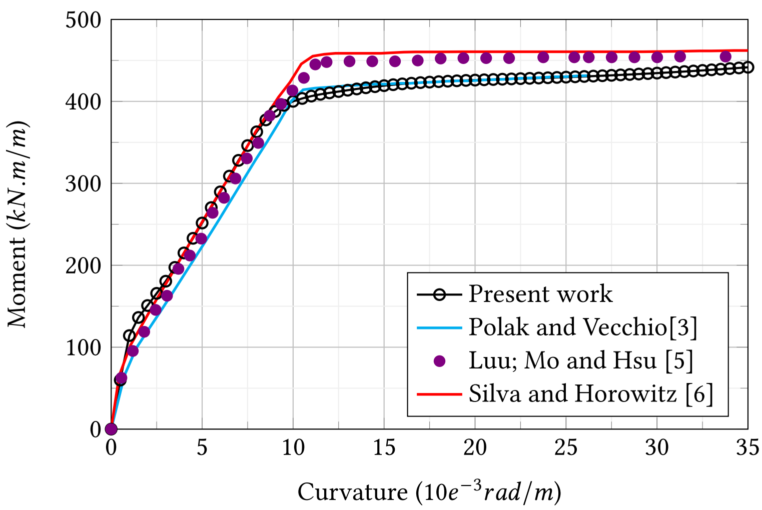


Figure 5. Polak Shell - Moment-curvature response

It was observed a good agreement between the developed model and the experimental and numerical results of the references. It should be noted that with the GFEM enrichment strategy, approximations of different polynomial degrees can be combined, making it possible to represent the structural response very close to the one observed by the referential analysis achieved with a simpler mesh. The applied polynomial enrichment strategy results in an equilibrium path practically coincide with the response described by FEM, however classical FEM requires a more discretized mesh.

5 Conclusions

This paper presented a shell element implemented into the structures analysis program INSANE as a framework to predict the nonlinear behavior of reinforced concrete shell-type structures. The element was developed for coupling a geometrically and physically nonlinear analysis of layered reinforced concrete shells problems using the Generalized Finite Element Method (GFEM).

To validate the results and show the accuracy of the proposed approach, a numerical simulation of a reinforced concrete panel was presented. The agreements were obtained when the results were compared to the numerical and experimental results found in the literature. GFEM was successfully investigated as an alternative to FEM to overcome mesh dependence and shear locking problems.

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