XLIII CILAMCE
21-25, November, 2022
FOZ DO IGUAÇU - PR - BRAZIL

Sensitivity analysis of dry friction damper position in stay cables

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Abstract. The aim of this study is to perform a comparative analysis regarding position influence of a damper with the dry friction mechanism to control the cable vibrations in cable-stayed bridges. The finite element method is used to implement the cable model and its validation by analytical theory. Lumped mass matrix and stiffness matrix considering the applied stress were generated considering the cable discretization in one hundred elements. The proportional damping matrix is obtained by the Rayleigh method with a damping rate of 0.13%. Damping rate evaluation take into account the damper position varying unitarily in the first 10% of the cable total length. The excitation force simulates the cable shape in its first mode of the vibration. To numerically solve the second-order nonlinear ordinary differential equation, Newmark's method with average acceleration is used. The damping factors obtained made it possible to compare with other studies for viscous dampers. Time domain was used to analysis the amplitude variation in the middle of the cable and at damper position, where it was possible to assess mobility limitations and the feasibility of choosing the position. It can be concluded that changing the damper position towards the cable center caused a total damping increase.

Keywords: finite element method, stay cables, dynamic analysis, dry friction damper.

1 Introduction

Since the popularization of microcomputers, vibration analysis has greatly increased due to the possibility of analyzing systems by numerical methods. The structures were extensively optimized requiring increasingly refined simulations.

Stay cables are severely affected by externally loads due to their low flexural strength and internal damping. Aiming reduce the displacements external dampers are installed close to its end, usually in the first 10% of the length. However, choosing their right position has always been a challenge, because it can be effectiveness varies for each position and mode vibration. Aiming to solve this problem Pacheco, Fujino and Sulekh [1] proposed a universal curve to evaluate the effect of the viscous damper position on the first vibration modes, where larger displacements are achieved. Noting that the behavior between viscous and dry friction dampers is different, this work analyzes the dry friction behavior from position sensitivity for the first vibration mode using this damper.

The kinematic dry friction force is considered with its classical formulation by Coulomb. Due to the difficulties in keeping a rigid system of ordinary differential equations stable, Newmark's method is used to evaluate displacements, velocity and acceleration. This is particularly useful to avoid model reduction and also because it is a method that does not add numerical damping to the system (Soriano [2]). After a theoretical background, a numerical application is developed to analyze the cable dynamic behavior considering a dry friction damper mechanism applied in different positions.

2 Problem formulation

The finite element method combined with Newmark method is used to obtain the dynamic responses. The cable is simply supported in both ends and the dry friction damper varies its position over the first 10% of the cable length. It is also excited by a sinusoidal force. Svensson [3] illustrated the single damper arrangement shown in Fig. 1.

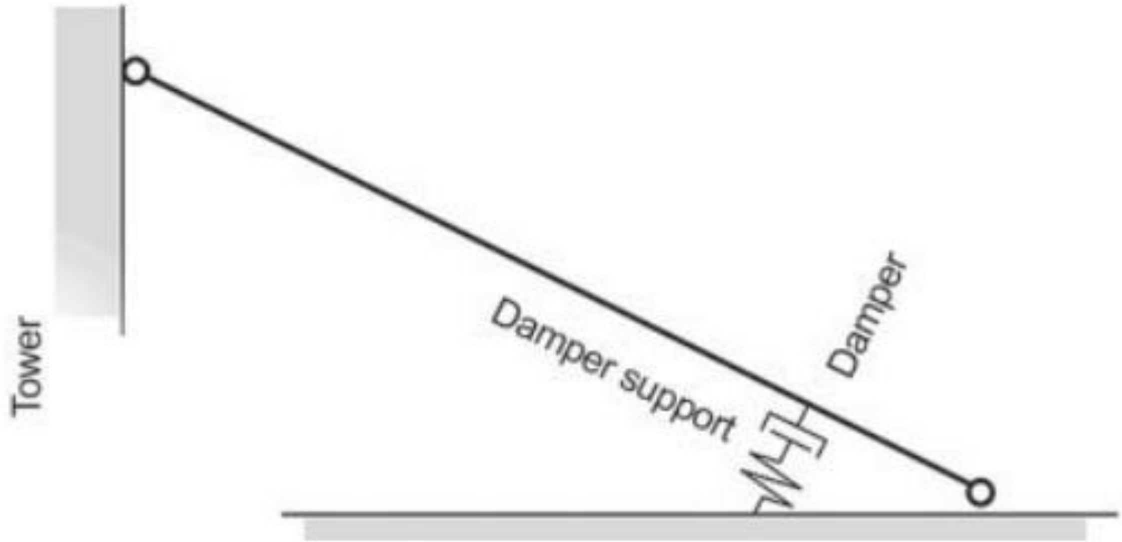

Figure 1. Single damper arrangement, Svensson [3]

2.1 Finite element method

The cable structure is discretized into one hundred bar elements with four degrees of freedom each. To assess the cable dynamic behavior, the system requires a mass $[M]$, stiffness $[K]$ and proportional damping $[C]$ (by Rayleigh) matrices beyond external influences, as the excitation force and dry friction vectors. The stiffness matrix must consider the tension force effects and damping matrix uses a damping factor of 0.13%, as Caetano [4] proposed. The assumed frictional force is 25 Newtons.

Physical cable properties from Sutong Yangtze River Bridge, in China, was used to sensitivity analysis of dry friction damper position (Tab. 1).

Table 1. Cable properties, Gao et al. [5]

Item	Value
Cable length	253.34 m
Mass per unit length	62.09 kg/m
Modulus of elasticity	1.9972×10^{11} N/m²
Diameter	0.127 m
Inclination angle	43.1º
Tension force	4227 kN
Sag parameter	0.4249

2.2 Equivalent viscous damper

Equalizing the energy dissipation per cycle of the viscous damper (ΔW_v) and dry friction damper (ΔW_f), the equivalent viscous damping constant c can be obtained from eq. (1) to (3).

$$\Delta W_f = 4F_f Y; \quad (1)$$

$$\Delta W_v = \pi c \omega Y^2; \quad (2)$$

$$\mathrm{c} = \frac{9.838}{Y}. \quad (3)$$

where F_f, Y and ω are friction force, maximum amplitude and natural angular frequency, respectively. Possible comparisons with this work can be made through the application of eq. (3), which contains the system parameters.

2.3 Excitation force

Weber, Krenk and Hogsberg [6], in eq. (4), proposed a sinusoidal force that excite the cable in first shape mode.

$$F_{exc}(t,x) = \begin{cases} sin(\pi x/L)\, sin(\omega_{01} t) & for\ \ t < t_{off} \\ 0 & for\ t \geq\ \ t_{off} \end{cases}. \quad (4)$$

where L, ω_{01}, x and t are, respectively, cable length, first angular natural frequency, variable length and time. The time when withdrawn the force (t_{off}) is 9.71 seconds, equivalent to five periods.

2.4 Equation of motion

The system's behavior has the matrices with boundary conditions applied in eq. (5):

$$[M]\ddot{y}(t) + [C]\dot{y}(t) + [K]y(t) = \{F_{exc}\} - \{a\}F_f sign(\dot{y}), \quad (5)$$

where F_{exc}, a, F_f and y are the excitation force, damper position, dry friction force and position, respectively; $sign(\dot{y})$ is the signal function as follows:

$$sign(\dot{y}) = \begin{cases} 1 & \dot{y} > 0 \\ 0 & \dot{y} = 0. \\ -1 & \dot{y} < 0 \end{cases} \quad (6)$$

2.5 Newmark method

Dynamic systems with dry friction dampers can be presented an instability due to the signal function. To avoid this problem, it was convenient use the Newmark method with average acceleration for its uncondicionally stability. The general equations for velocity and acceleration are formulated as follows:

$$\dot{y}_{i+1} = \frac{2}{\Delta t}\Delta y - \dot{y}_i; \quad (7)$$

$$\ddot{y}_{i+1} = \frac{4}{\Delta t^2}\Delta y - \frac{4}{\Delta t}\dot{y}_i - \ddot{y}_i; \quad (8)$$

in which Δt is the time step and the displacement is represented by Δy .

Substituting the velocity and acceleration values in the eq. (5) obtains:

$$\left(\frac{4}{\Delta t^2}M + \frac{2}{\Delta t}C + K\right)\Delta y = \left\{F_{exc_{i+1}}\right\} - \{x_c\}F_{at} sign(\dot{y}_i) + \left(\frac{4}{\Delta t}M + C\right)\dot{y}_i + M\ddot{y}_i - Ky_i. \quad (9)$$

The method stability depends on the following condition:

$$\frac{\Delta t}{T_n} \leq \frac{1}{\pi\sqrt{2\gamma - 4\beta}}, \tag{10}$$

where T_n is the period. For average acceleration condition, the parameters $\beta = 1/4$ and $\gamma = 1/2$. Therefore:

$$\frac{\Delta t}{T_n} < \infty. \tag{11}$$

2.6 Numerical application

The numerical application described herein was performed on a cable structure with dry friction damper. In order to illustrate the effect of external damping, the internal damping was disregarded. Firstly, the displacements amplitude was compared considering the center of the cable and the damper position. Analyzing Fig. 2, it shows that after a certain time the displacements on the center of the cable remains constant. This happens because the damper locks the cable and there is no more energy dissipation in its position. However, there are still displacements along the cable that are not damped by the frictional force. The undamped portion varies depending on the damper position, the closer to the center, the residual amplitude is smaller. This part not influenced by dry friction must only be internally damped.

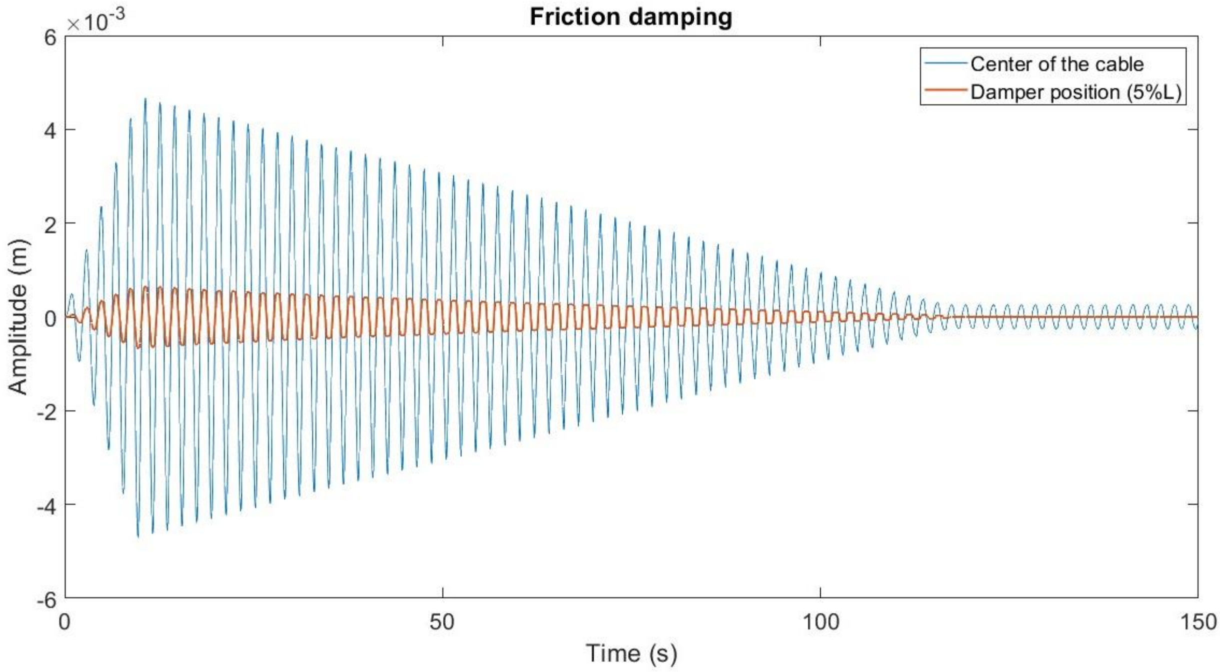

Figure 2. Amplitude comparison with and without external damping

After analyzing the isolated behavior of dry friction, it is interesting to observe its influence on the cable containing internal damping. The addition of external damper considerably reduces the displacement amplitude when compared to internal damping only, as shown in Fig. 3. It is observed that the displacement envelope is altered by dry friction, which tends to linearize the decay and makes square waves at damper position.

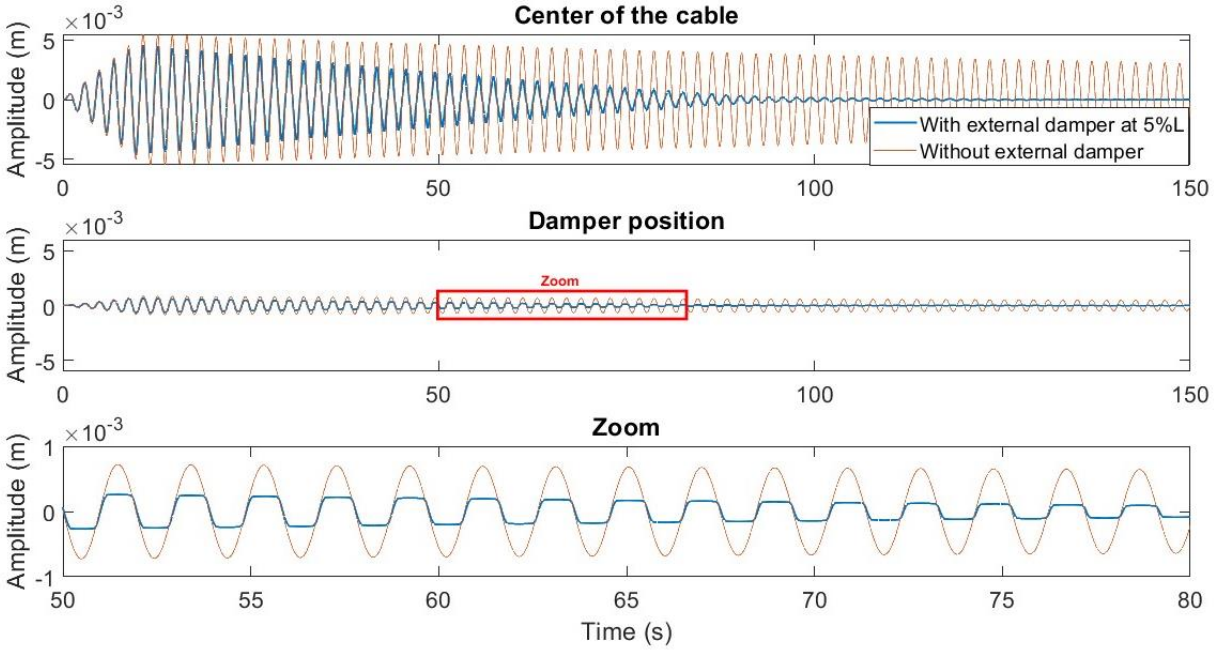

Figure 3. Amplitude comparison with and without external damping

By varying the damper position in first 10% of the cable length, the amplitude difference is observed at the center position in Fig. 4. The maximum displacement achieved by excitation is different for each position, as is the decay. To facilitate exposure, only the first and last result is shown.

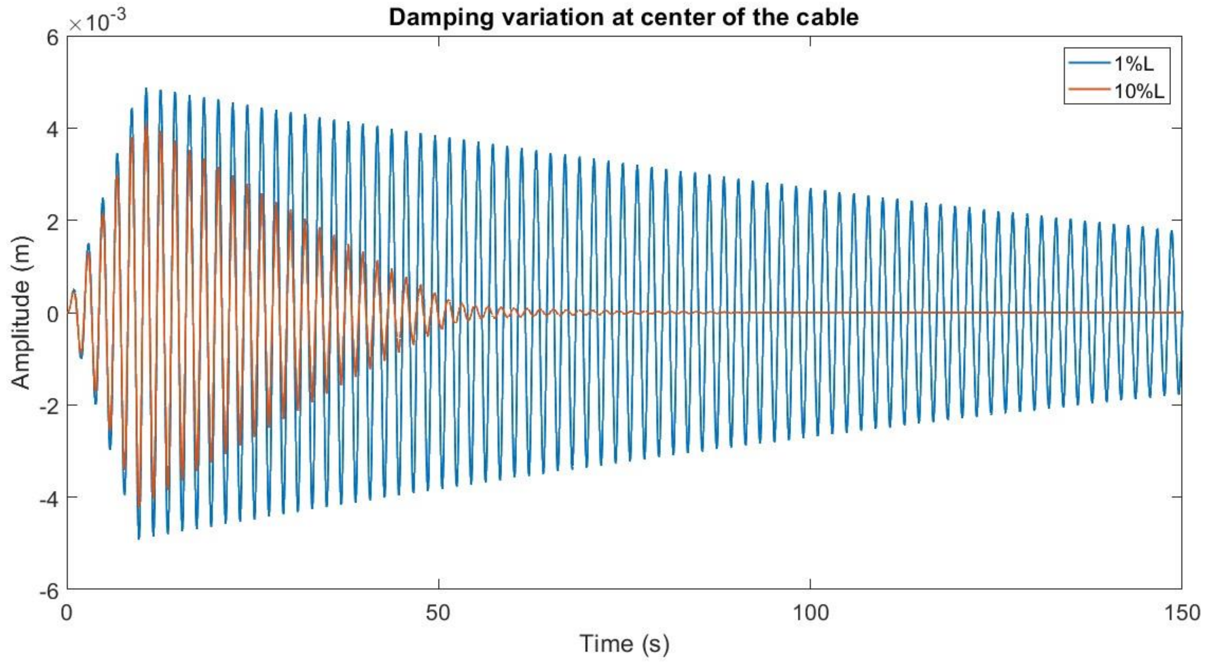

Figure 4. Sensitivity analysis of damper position

To compare the damping effect, the upper envelope at center of the cable for each damper position is shown in Fig. 5. A parameterization by tangents and dissipation energy are used to compare the results at each position

(Fig. 6). Through this, it is possible to quantify the displacements sensitivity by the variation of damper position, identifying a linear gain in damper efficiency over the first position at 1% of the length. This gain (G) can be found approximately as a function of the relative damper position (a_p), in percentage, by eq. (12). For the energy dissipation gain (E_d), eq. (13) is used.

$$G = 0.2764a_p + 0.7276; \tag{12}$$

$$E_d = -0.0256a_p^2 + 0.9855a_p + 0.0626. \tag{13}$$

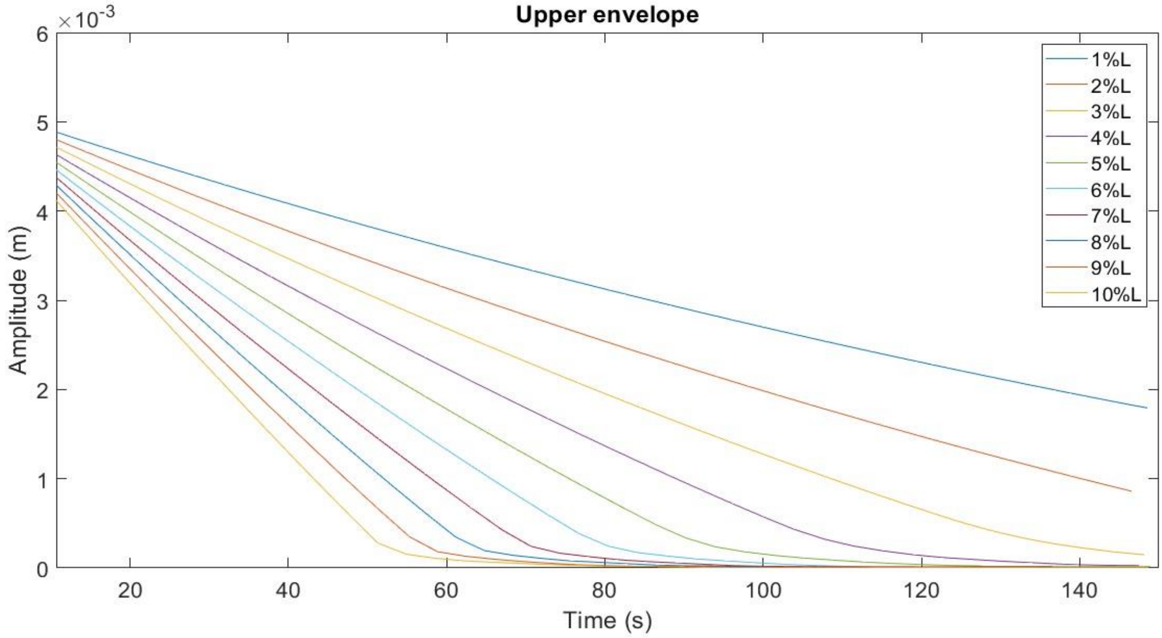

Figure 5. Amplitude comparison with and without external damping

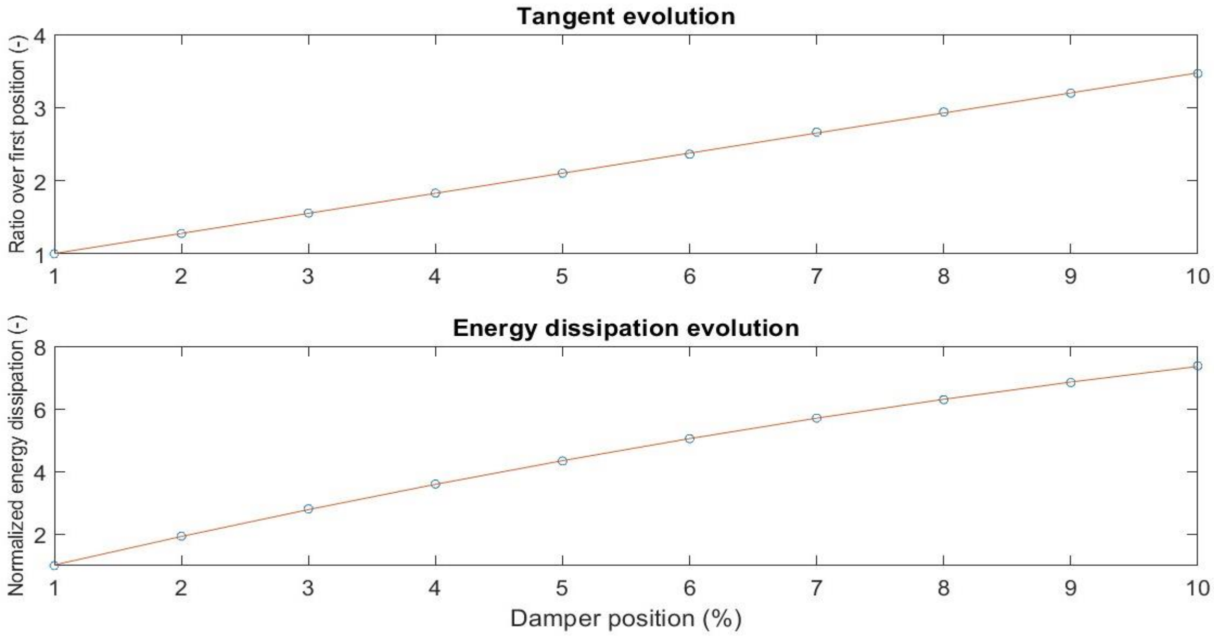

Figure 6. Damping evolution by energy and tangent parameters

It was observed that the greater dry friction damper participation in the damping also tends to intensify the linearization of the decay as expected. Furthermore, the energy dissipation at 10% of the length is 7.37 greater than at the start position indicating a considerable gain with just changing position. At the center of the cable, the tangents progression is linearized while at the damper position the energy dissipation follows a quadratic function. This happens due to the internal damper influence which has different characteristics from the damping by dry friction and possible frequency changes due to cable clamping at low speeds at damper position.

3 Conclusions

It is observed that when the damper is placed closer to the center of the cable, the damping in the middle is increased at first vibration mode and, as the dissipation energy per cycle of dry friction is linear, the variation tends to be linearized. However, the cable geometric limitations directly influence the maximum displacement obtained, making the function slightly quadratic. This variation also increases the participation of the damper on the total displacements of the cable, decreasing the residual portion internally damped only.

When making decisions regarding the dry friction damper position, it is important that this analysis is taken into account the best position that meets the critical design criteria, avoiding the loss of damping capacity by being too close to the support or making the execution difficult by being unnecessarily too far.

Acknowledgements. The authors thank the Department of Civil Engineering of the Federal University of Uberlândia for their support.

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