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COMPARISON OF DIVERGENCE SCHEMES APPLIED TO THE STATIC CASE OF THE GREAT BELT EAST BRIDGE

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Abstract. The behavior of slender structures to the action of the wind is the object of study in numerous works. With the constant evolution of Computational Fluid Dynamics (CFD), obtaining and analyzing results through numerical methods has been increasingly used by researchers in order to provide reliable parameters for new engineering projects. In this study, simulations are presented with a 2D approach of the static case of the Great Belt East Bridge in turbulent flow with Re 10^5, applying the k-ω SST turbulence model to the simulation with the Reynolds Averaged Navier-Stokes methodology (RANS), which deals with an averaging operation applied to the Navier-Stokes equations to obtain the average fluid flow equations. The simulations were performed via OpenFOAM®, using four divergence schemes: Gauss QUICK, Gauss upwind, Gauss linearUpwind, and Gauss limitedLinear. Then, a comparison was made of the average values of the dimensionless aerodynamic coefficients of drag (Cd), lift (Cl) and moment (Cm), in addition to the Strouhal number (St). There was also an evaluation of the computational performance of the schemes applied in the simulations. In conclusion, the QUICK scheme presented promising results in all analyses, and the others were within an acceptable range, with some values close to those found in the literature. Finally, the results obtained via CFD were validated and considered satisfactory, proving the effectiveness of this methodology.

Keywords: Bridges, Divergence Schemes, Turbulence, Computacional Fluid Dynamics

1 Introduction

Slender structures such as bridges with large spans tend to be more sensitive to the action of the wind, which directly influences the project from its conception. The analysis of the aerodynamic coefficients must be carefully verified and for this there is Computational Fluid Dynamics (CFD), which is constantly evolving, being increasingly used by researchers in order to obtain reliable results via numerical methods.

This study intends to analyze the static case of the Great Belt East Bridge, located in Denmark, in a scaled-down model of $B/H = 7/1$. In the present study, simulations were performed in order to obtain the averages of the aerodynamic coefficients of drag (Cd), lift (Cl) and moment (Cm), in addition to the Strouhal number (St). The turbulence model k-ω SST and four divergence schemes, namely Gauss QUICK, Gauss upwind, Gauss linearUpwind and Gauss limitedLinear, were used in the simulations, comparing them.

2 Theoretical basis

2.1 Governing Equations

The equations that govern the incompressible flow are represented by Navier-Stokes equations that express a physical-mathematical model, based on the conservation of moment and mass for Newtonian, incompressible and viscous fluids, expressed in Equations 1 and 2, respectively:

$$\nabla.\vec{v} = 0 \tag{1}$$

$$\rho\frac{\partial \vec{v}}{\partial t} + \rho\nabla.\vec{v}\vec{v} = -\nabla.p + \nabla.\left(\bar{\bar{\tau}}\right) + \rho\vec{g} + \vec{F} \tag{2}$$

In the equations above, ρ is the density of the fluid; p, the pressure; μ, is viscosity; $\vec{v}$ the velocity field; $\tilde{\tilde{\tau}} = \mu\left[\left(\nabla\vec{v} + \nabla\vec{v}^T\right)\right]$ is the stress tensor; $\rho\vec{g}$ and $\vec{F}$ are gravitational and external forces, respectively.

2.2 Turbulence Model k-ω SST

The turbulence model applied to the simulations of this work was the k-ω SST (Shear Stress Transport), which is a combination of the k-ε and k-ω models. It is a two-equation model that was proposed by Menter [1] to model turbulent flows. This combination made it possible for the models to act at different points, but simultaneously so that there was a better performance of the proposed model. In this way, it is possible to model turbulent regions with adverse pressure gradients and in the vicinity of the boundary layer (Launder and Spalding [2]; Yakhot et al. [3]).

2.3 Aerodynamic Coefficients

The coefficients Cd, Cl, and Cm depend on the geometric characteristics of the cross-section, the wind attack angle and also the Reynolds number of the flow, and are described, respectively, in the Equations 3.

$$Cd = \frac{F_d}{\frac{1}{2}\rho U^2 B}; \qquad Cl = \frac{F_l}{\frac{1}{2}\rho U^2 B}; \qquad Cm = \frac{M}{\frac{1}{2}\rho U^2 B^2} \tag{3}$$

Where F_d and F_l are the average drag and lift forces, M is the average moment, ρ is the fluid density, U is the average velocity, and B is the cross-section width of the bridge deck.

The relationship between the vortex shedding frequency f_v, the flow velocity U, and the characteristic height of the cross section of the bridge deck H, define the St, expressed by the Equation 4.

$$St = \frac{f_v H}{U} \tag{4}$$

3 Divergence Schemes

There is a wide variety of discretization/interpolation schemes for each term of the transport equation. In order to find the fluxes of the surface integrals in this equation, the results stored in the centers of the cells, where they are usually stored in the CFD simulations, are interpolated to the centers of the faces.

Such interpolation may require a flow G through a general face f. Using a variety of schemes, from the values of neighboring cells, one can evaluate the face value (ϕ_f). Obtaining results demands continuity constraints that the flow must satisfy.

Next, some schemes are described, which are applied in the simulations of this study.

3.1 upwind

The upwind scheme, non-TVD (Total Variation Diminishing), has first-order accuracy and is stable, but introduces false diffusion due to its low accuracy, especially in meshes with a lower degree of refinement. According to Versteeg and Malalasekera [4], when determining the value on a cell face, the upwind scheme considers the flow direction. The convection value (ϕ) on a cell face is considered equal to the value on the upstream node.

In Figure 1, the nodal values used to calculate the cell face values, when the flow is in the positive direction, are exemplified.

Analyzing the positive flow, u_w >0, u_e >0 (F_w >0, F_e >0) and defines $\phi_w = \phi_W$ and $\phi_e = \phi_P$.

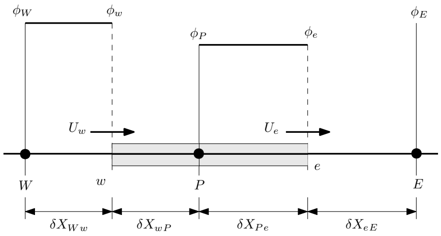

Figure 1. upwind scheme - Adaptation of Versteeg and Malalasekera [4]

3.2 QUICK

Leonard [5] developed the QUICK scheme (Quadratic Upstream Interpolation for Convective Kinematics). It is a non-TVD scheme with third-order precision, and conservative, but it can also present spurious oscillations for high values of the Péclet number. It is based on interpolation of the value of the dependent variable on each face of the element, using a quadratic polynomial polarized in the upstream direction, as shown in Figure 2. The interpolated value is used to calculate the convective term in the governing equations for the dependent variable (Moukalled et al. [6]).

In Figure 2, the calculation of the values of the dependent variable in a face cell is shown.

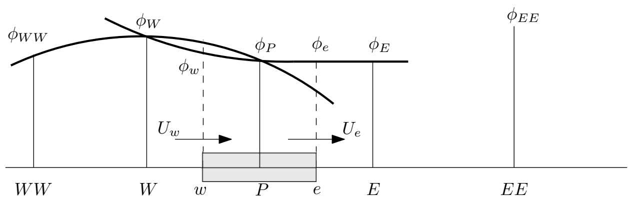

Figure 2. QUICK scheme - Adaptation of Versteeg and Malalasekera [4]

Considering positive flow, when $u_w > 0$, the cluster nodes for the west face w are W and P, and the upstream node is WW. So

$$\phi_w = \frac{6}{8}\phi_W + \frac{3}{8}\phi_P - \frac{1}{8}\phi_{WW} \tag{5}$$

and when $u_e > 0$, the cluster nodes for the east face e are P and E, and the upstream node is W. Then

$$\phi_e = \frac{6}{8}\phi_P + \frac{3}{8}\phi_E - \frac{1}{8}\phi_W \tag{6}$$

3.3 linearUpwind

The linearUpwind scheme is TVD and second order. It was proposed by Warming and Beam [7], who considered the application of explicit second-order techniques, employing a transition operator so that there would be an automatic spatial change between the MacCormack algorithm and the upwind scheme described in 3.1.

MacCormack is said to be uncentered because the space of derivatives in the predictor is approximated by difference quotients forward and corrector backward. The predictor is spatially one-sided and the corrector can be modified in order to obtain a purely upwind scheme since this is also based on the backward difference (Warming and Beam [7]).

Finally, the combination of these techniques, when applied, is highly compatible since the MacCormack algorithm and the upwind scheme share the same predictor (Warming and Beam [7]).

3.4 limitedLinear

The limitedLinear scheme proposed by Sweby [8], is a second-order differentiation scheme, TVD, and with flow limiting. It is written as a sum of the first-order bounded differentiation scheme (upwind) and a bounded higher-order correction, described in Equation 7.

$$\phi_f = (\phi)_{UD} + \Psi[(\phi)_{HO} - (\phi)_{UD}] \tag{7}$$

where ϕ_{HO} represents the nominal value of ϕ for the selected higher-order scheme and Ψ is the bounded flow.

4 Methodology

OpenFOAM® was used in the simulations and, to solve them, the PIMPLE solver, together with divergence schemes in a turbulent flow with Re 10^5 around the cross-section deck of the Great Belt Bridge East in a fixed section, in order to establish a comparison between the schemes. The computational domain and boundary conditions used for this problem are illustrated in Figure 3.

The wind attack angle used in the simulations of this case was $\alpha = 0°$.

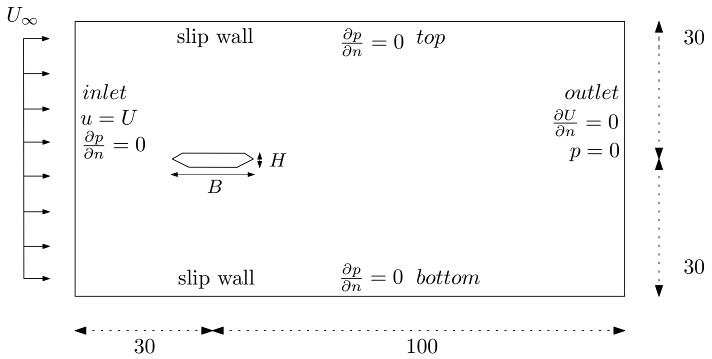

Figure 3. Flow domain definition

5 Results

5.1 Static Aerodynamic Coefficients

Four unstructured meshes were proposed, being M1 88798/259134; M2 100100/287512; M3 118756/342212 and M4 130404/376406 with different node/element characteristics. The meshes are in increasing order of refinement, with M1 being the least refined and M4 being the most refined. Figures 4-11 show the evolution of the averages of Cd, Cl, Cm, in addition to St for the different schemes.

It is noteworthy that to obtain these coefficients, the dimension adopted for the height of the bridge deck was H = 1, and the width was B = 7. Finally, the results were compared with CFD studies by Nieto et al. [9], Larsen and Walther [10], Farsani et al. [11] and Cid Montoya et al. [12].

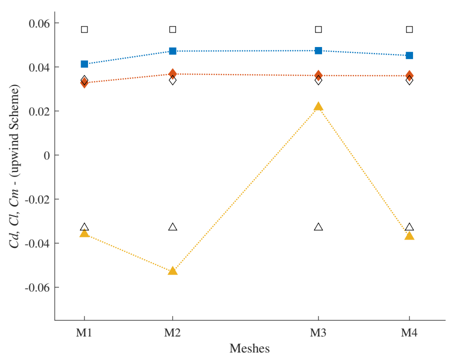

Figure 4. upwind scheme - Cd ■; Cl ▲; Cm ♦; Cd □ CFD Nieto et al. [9]; Cl △ CFD Nieto et al. [9]; Cm ◊ CFD Nieto et al. [9]

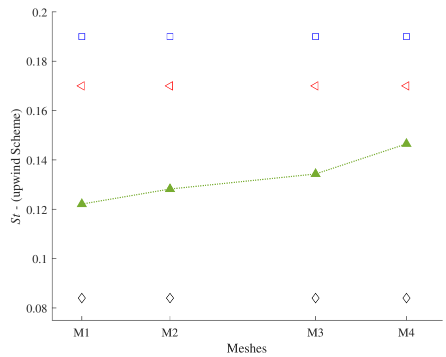

Figure 5. upwind scheme - St ▲; St □ CFD Nieto et al. [9]; St ◊ CFD Farsani et al. [11]; St ◁ CFD Larsen and Walther [10]

It was observed in Figures 4 and 5 that the results obtained for the upwind scheme were satisfactory since they are within an acceptable range and some even with good proximity to literature. Since this is a limited first-order scheme, which may slightly compromise the accuracy of the results, such behavior of the coefficients was expected.

Still in this analysis, it can be observed in Figures 6 and 7 that there is a similarity in the results compared to the previous results, because the linearUpwind scheme, which is derived from the upwind scheme, was capable of reproducing good results since its characteristic of being second-order and not limited contributed to a good agreement between the results.

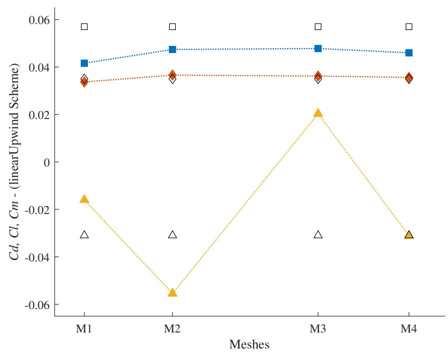

Figure 6. linearUpwind scheme - Cd ■; Cl ▲; Cm ♦; Cd □ CFD Cid Montoya et al. [12]; Cl △ CFD Cid Montoya et al. [12]; Cm ◊ CFD Nieto et al. [9]

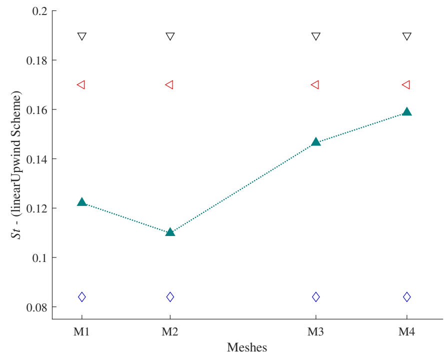

Figure 7. linearUpwind scheme - St ▲; St ▽ CFD Cid Montoya et al. [12]; St ◊ CFD Farsani et al. [11]; St ◁ CFD Larsen and Walther [10]

In Figures 8 and 9, it is observed, by the results obtained with the QUICK scheme, a better performance compared to the previous ones, with emphasis on meshes M1, M2, and M4 that had results very close to the literature.

This behavior of the coefficients can be associated with the refinement of the mesh and also with the scheme's ability to converge the data since it is a differentiation scheme with third-order precision, which considers a weighted quadratic interpolation of three upstream points. for the cell face values.

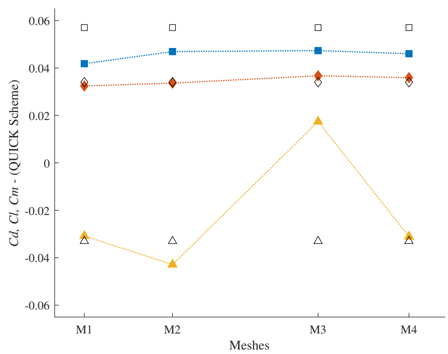

Figure 8. QUICK scheme - Cd ■; Cl ▲; Cm ♦; Cd □ CFD Nieto et al. [9]; Cl △ CFD Nieto et al. [9]; Cm ◊ CFD Nieto et al. [9]

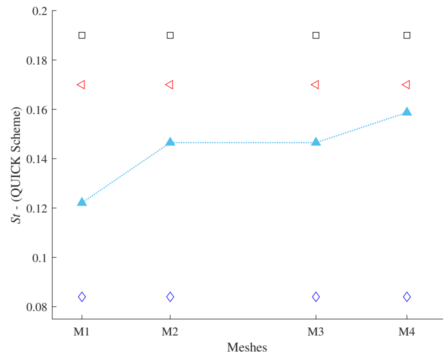

Figure 9. QUICK scheme - St ▲; St □ CFD Nieto et al. [9]; St ◊ CFD Farsani et al. [11]; St ◁ CFD Larsen and Walther [10]

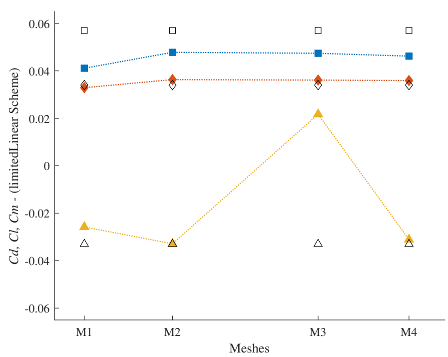

Figure 10. limitedLinear scheme - Cd ■; Cl ▲; Cm ♦; Cd □ CFD Nieto et al. [9]; Cl △ CFD Nieto et al. [9]; Cm ◊ CFD Nieto et al. [9]

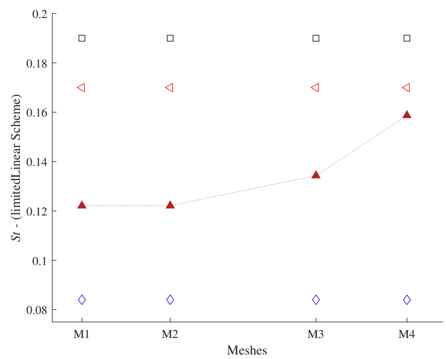

Figure 11. limitedLinear scheme - St ▲; St □ CFD Nieto et al. [9]; St ◊ CFD Farsani et al. [11]; St ◁ CFD Larsen and Walther [10]

Regarding the limitedLinear scheme, it is noted that in Figures 10 and 11, it reached a good consistency of results, as expected, with emphasis on the results of meshes M1, M2, and M4 that achieved a good performance.

As for the M3 mesh, the result is similar to the others presented above, that is, there was an increase of Cl. This difference in the results of Cl in relation to the other meshes can be associated with the refinement or the numerical scheme applied.

5.2 Comparison between Divergence Schemes

In this section, a comparison is made between the schemes in terms of processing time for numerical simulations. For the development of the simulations in this work, a computer with the following configurations was used: Processor Intel® Core™ i7-6700K CPU 4.00 GHz, RAM memory 16 GB and Windows 10 Pro 64-bit operating system.

Table 1, below, describes the number of hours processing for each scheme.

Table 1. Data processing time in numerical simulations

Mesh Nodes/Elements	limitedLinear	linearUpwind	QUICK	upwind
M1 88798/ 259134	37h56min	37h58min	37h11min	37h42min
M2 100100/ 287512	78h16min	72h9min	72h5min	72h23min
M3 118756/ 342212	112h32min	100h48min	98h36min	99h13min
M4 130404/ 376406	140h51min	152h16min	150h11min	152h25min

The table above presents the computational performance of the schemes in three aspects: the mesh refinement, the discretization scheme, and the characteristics of each scheme. Initially, it appears that all schemes proved to be quite competitive.

However, the QUICK scheme outperformed the others in agility in M1, M2, and M3, even though the minimum difference was 4 minutes in M2 and the maximum difference was 37 minutes in M3.

The limitedLinear proved to be faster on the more refined mesh, M4, beating the QUICK by 10h 40min.

On the other hand, the linearUpwind and upwind schemes were surpassed in the four meshes, although the difference in processing time in M1, M2, and M3 was not significant. As these meshes are the least refined and require less processing time, QUICK proved to be the most agile and stable, confirming what was expected due to its characteristics.

6 Conclusions

The case study carried out in this work presented the investigation and verification of the behavior of the aerodynamic coefficients, using four divergent schemes together with the k-ω SST turbulence model, in turbulent flow around the cross-section deck of the Great Bridge Belt East. The objective was to compare the results of these simulations with the CFD references available in the literature and the comparison between the schemes in terms of computational cost.

It was observed that the results presented, in general, had a good consistency, with occasional exceptions, such as some results of meshes M1, M2 and M4, which can be attributed to the type of mesh, its respective refinement or the scheme used in the simulations.

It is known that the upwind scheme is the most computationally expensive, due to errors that can generate a false diffusion in flows with a high Reynolds number, which may be large enough to provide physically incorrect results, being higher for meshes with a lower degree of refinement such as in the M1 mesh. To try to eliminate this false diffusion, one must increase the degree of mesh refinement, which results in an extremely expensive processing, as the computational time increases considerably. This was also verified in this work, when this scheme was only more agile than the others in the more refined mesh, which demands more processing time.

The linearUpwind and limitedLinear schemes are TVD schemes. These schemes are generalizations of existing discretization schemes and, for this reason, must satisfy the necessary requirements that confer transportiveness, conservativeness, and boundedness.

The results obtained when using the TVD schemes show a much smaller false diffusion than the non-TVD schemes. Thus, the results show good consistency, without non-physical overshoots and undershoots. The two TVD schemes were shown to be very close to each other, with the advantage of presenting solutions without oscillations, which is also a recurring feature in comparisons with results available in the literature.

The divergence schemes showed good results when compared to the literature, with emphasis on the QUICK scheme. It is a scheme with third-order precision and not limited, with greater formal precision than the central differentiation or hybrid schemes, which helps to obtain good results, even with a small resulting false diffusion. Solutions achieved with less refined meshes are often considerably more accurate than those of upwind or hybrid schemes. In addition, there was a gain in time, thus resulting in a lower computational cost compared to the other schemes used.

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