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Acoustic cavity theoretical and numerical analyses for noise attenuation by applying Helmholtz resonators

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Abstract. From the passive methods of acoustic cavity sound control, the use of reactive silencers is widely applied in many engineering sectors. These devices consist of segments of interconnected cavities that do not use materials with dissipative properties. There is a variety of reactive silencers and the present work has highlighted the Helmholtz resonator. Methodologies for the evaluation of acoustic cavities using these resonators for sound control purposes are not extensive, so a theoretical method for the analysis of noise attenuation performance in this acoustic system is presented. In addition, in order to evaluate and discuss the theoretical results obtained, analysis was performed using the finite element method with the aid of ANSYS® software with acoustic extension. In this methodology, the pressure-formulated element has been used. The theoretical and numerical results demonstrated good agreement and evidenced the sensitivity of the acoustic parameters and sound control efficiency to the resonator neck geometry which exerted a strong influence on the modal parameters and on the transmission loss in theoretical results. Lastly, an experimental study of acoustic cavity sound control using Helmholtz resonators developed at the University of Brasilia is evaluated by the methodology presented, in order to obtain a numerical-experimental comparison of the results obtain.

Keywords: Noise control, acoustic cavity, Helmholtz resonator, transmission loss.

1 Introduction

There are several mechanisms for controlling noise in acoustic cavities (such as ducts and enclosed rooms) that can use passive and/or active methods, depending on the characteristics of the sound waves and the desired efficiency. Among the passive methods, the Helmholtz resonator, which consists of segments of interconnected cavities, as show in the Fig. 1, is often used to reduce noise in low frequency bands. This type of resonator reduces noise by an impedance mismatch that causes reflection of the incident acoustic energy and attenuation in the resonator's neck (Seo and Kim [1]). The Helmholtz resonator shows high efficiency in sound transmission loss (parameter that quantifies noise attenuation) when the frequency band of the noise is close to the acoustic resonance frequency of the resonator.

However, these devices must be designed with specific geometric properties, since their acoustic effects are strongly associated with their dimensions, show in the Fig. 1, and is restricted to narrow bands of incident frequency (Santos [2]). Therefore, it is essential to have tools capable of estimating the noise attenuation characteristics of this devices for good acoustical design.

Thus, based on the works of Selamet [3], Chanaud [4], Panton [5] and Esandiari [6] about the acoustic performance of Helmholtz resonators, this paper describes a theoretical methodology for the determination of sound transmission loss in acoustic cavities (ducts), which considers the geometrical properties of the resonator-cavity system, and a Finite Element-based numerical methodology for the same purpose. Both methods are then used to analyze the noise attenuation efficiency in the acoustic systems of interest and to study sensitivity to changing resonator neck dimensions. Finally, a numerical-experimental comparison is given between experimental data from a paper taken from the public domain (Queiroz [7]) on the TL of an acoustic cavity with a single-neck resonator and the results obtained reproducing, by the described numerical methodology, the same experimental case.

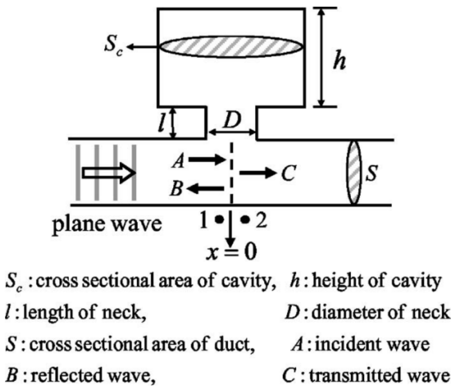

Figure 1. Helmholtz resonator with a acoustic cavity (duct) and main dimensions of this acoustic system. Figure taken from Seo and Kim [1].

2 Background

2.1 Transmission loss of a branch resonator in a acoustic cavity

Considering only the planar propagation of sound waves one, no internal flow and anechoic terminations in the acoustic cavity we can obtain the sound transmission loss in a duct by inserting an acoustic element, in this case a Helmholtz resonator, as (Selamet [3])

$$TL = 10\log_{10}\left|\frac{A}{C}\right|^2 \tag{1}$$

Where $\mathbf{A}$ and $\mathbf{C}$ are complex constants representing magnitudes of incident harmonics and transmitted wave pressure, respectively. Assuming constant pressure, conservation of volume flow at the tube intersection, neglecting viscous effects, and incorporating wave motion in the volume and neck of the resonator, according to the classical one-dimensional acoustic field theory of Helmholtz resonators, the transmission loss is given by (Esandiari [6])

$$TL = 10\log_{10}\left|1 + \frac{S_n}{2S}\left[\frac{1+\varphi+(\varphi+1)e^{-2ikl'}}{1+\varphi-(\varphi+1)e^{-2ikl'}}\right]\right|^2 \varphi = \frac{S_c}{S_n}\left(\frac{e^{2kih-1}}{e^{2kih+1}}\right) \tag{2}$$

Where $\mathbf{S}$ is the cross-sectional area of the main duct, $\mathbf{k}$ is the wavenumber, $\mathbf{l}$ is the neck length, $\mathbf{S_n}$ is the cross-sectional area of the neck, $\mathbf{h}$ is the cavity length, $\mathbf{S_c}$ is the cross sectional area of the cavity and $\mathbf{k = 2}\pi/\lambda$ the wavenumber.

This equation requires correction in the neck length to account for the effect of mass transport that occurs at the inner and outer exits of the geometry. For this purpose, Panton [5] propose the following equation for adjusting the effective neck length of the resonator ($\mathbf{l'}$).

$$l' = l + \frac{8r_n}{3\pi}\left(1 - 1.24\frac{r_n}{r_c}\right) + \frac{8r_n}{3\pi} \tag{3}$$

Then, using eq. (3) and rearranged the eq. (2) to an equivalent trigonometric form we can obtain the following expression for the trasmission loss (Esandiari [6])

$$TL = 10\log_{10}\left[1 + \left(\frac{S_n}{2S}\frac{tan(kl') + \left(\frac{S_c}{S_n}\right)tan(kh)}{1 - \left(\frac{S_c}{S_n}\right)tan(kl')tan(kh)}\right)^2\right] \tag{4}$$

It can be observed that the resonator transmission loss becomes infinite with the denominator in eq. (4) close to zero, this occurs when the incident acoustic frequency becomes close to the resonator resonance frequency.

2.2 Finite Elements Analysis (FEA)

Finite Element Analysis (FEA) is a numerical method that can be used to calculate the response of a complicated structure due to the application of forcing functions, which could be an acoustic source or a distribution of mechanical forces. FEA can also be used to estimate the sound power radiated by a structure or the distribution of the sound field in an enclosed space. The most commonly used finite element to analyze acoustic problems is the pressure-formulated elements. In this method, the acoustic pressure $\mathbf{p}$ in a finite element can be expressed as (Howard and Cazzolat [8])

$$p = \sum_{i=1}^{m} N_i p_i \tag{5}$$

where $\mathbf{N_i}$ is a set of linear shape functions, $\mathbf{p_i}$ are acoustic nodal pressures at node $\mathbf{i}$, and $\mathbf{m}$ is the number of nodes in element. For pressure-formulated acoustic elements, the lossless finite element equation for the fluid in matrix form is (Howard and Cazzolat [8])

$$[M_f]\{\ddot{p}\} + [K_f]\{p\} = \{F_f\} \tag{6}$$

where $[\mathbf{K_f}]$ is the equivalent fluid stiffness matrix, $[\mathbf{M_f}]$ is the equivalent fluid mass matrix, $[\mathbf{F_f}]$ is a vector of applied fluid loads, $\{\mathbf{p}\}$ is a vector of unknown nodal acoustic pressures, and $\{\ddot{\mathbf{p}}\}$ is a vector of the second derivative of acoustic pressure with respect to time.

3 Methodology

Based on the literature review and the available references, three cases of noise attenuation in a duct using cylindrical Helmholtz resonator have been analyzed by theoretical method (using the eq. (4) implemented in a MATLAB® script) and numerical method (Finite Elements using the ANSYS software with acoustic extension). In these three cases the $\mathbf{l/D}$ (ratio between length and characteristic diameter) df the resonator neck was varied, keeping the other characteristic dimensions, show in Fig. 1, fixed. The objectives of these analyses was to study the variation in noise reduction efficiency by quantifying the sound transmission loss of the system due to changes in resonator neck dimensions, and to relate the theoretical and numerical results for each case analyzed.

Subsequently, the analysis domain of an experimental study of noise attenuation by Helmholtz resonators, conducted by Queiroz [7] in his graduation conclusion work, was modeled and numerically analyzed, Fig. 2. This process was performed to obtain a relationship between the noise attenuation results of the numerical solution, obtained by the aforementioned methodology, with empirical data.

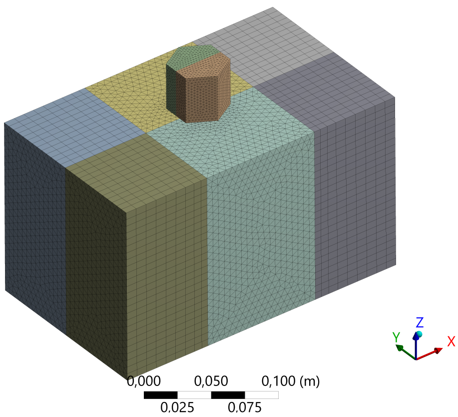

Figure 2. Three-dimensional model of the resonator-acoustic cavity assembly that represents the experimentally analyzed acoustic system.

All finite elements analyses performed were full harmonic type with pressure-formulated elements and no flow in the main duct (acoustic cavity). The cases studied had their three-dimensional acoustic domains discretized with linear acoustic elements of Hexahedral and pyramidal geometries, as show in Fig. 3 (a), with attention to the necessary mesh refinement. "Acoustic Radiation Boundary" was used on the faces of the duct exit and entrance sections, as show in Fig.3 (b), in order to guarantee anechoic terminations (Howard and Cazzolat [8]). Lastly, the duct inlet was excited by a "Normal Acoustic Surface Velocity" to create a volume velocity source needed to perform the harmonic analysis.

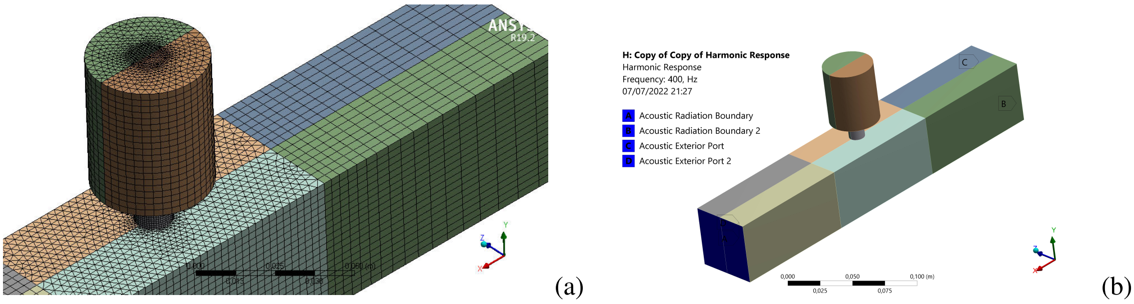

Figure 3. (a) Three-dimensional model of the resonator-acoustic cavity assembly of one of the proposed cases for numerical analysis of sound transmission loss. (b) Mesh of the same three-dimensional model.

4 Analysis and results

4.1 Theorycal and numerical noise attenuation analysis

The results of the theoretical and numerical solutions are illustrated in the graph in Fig. 4. This graph correlates the frequency of the incident sound wave with the resulting transmission loss for each case analyzed and compares the results of both solutions used.

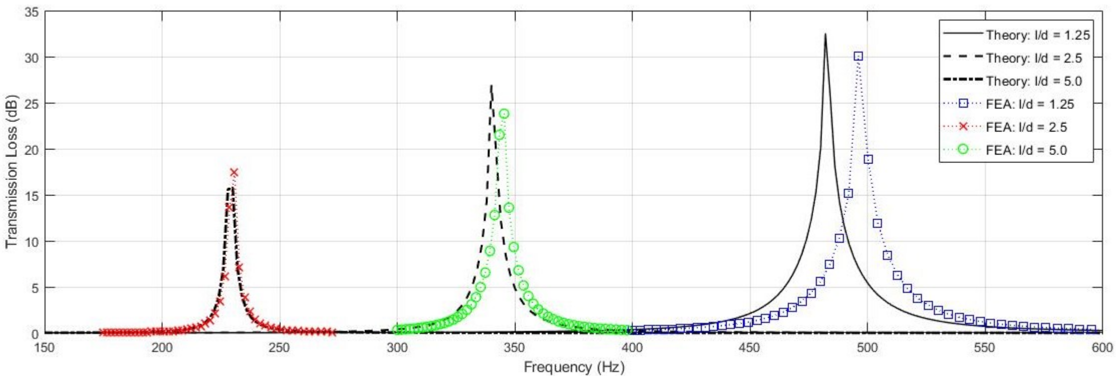

Figure 4. Influence on the l/D (length/diameter ratio) of helmholtz resonator neck dimensions in TL study of the resonator-cavity system and comparison between numerical and theoretical solutions ($\mathbf{h}$: 4.85e-2 m, $\mathbf{r_c}$: 2.0e-2 m, $\mathbf{S_n}$: 7.854e-5 m² and $\mathbf{S}$: 2.5e-3 m²).

For the cases with higher l/D ratios the graph illustrates good agreement between the values obtained by both solutions, both with respect to the peak TL and the frequency band in which the resonator shows some effectiveness. However, for the case where the l/D ratio is equal to 1.25, the comparison between the theoretical and numerical results obtained considerable discrepancy. This discrepancy is related to the correction of the resonator neck length applied in theoretical solution that becomes more sensitive as the neck radius increases compared to the cavity radius (Panton and Miler [5]). In these cases, it is necessary to develop specific corrections for the resonator neck length based on experimental analysis.

4.2 Numerical and experimental noise attenuation analysis

The Table 1 demonstrates the peak transmission loss and the frequency at which it occurs in both solutions.The peak transmission loss reported in experiment was considerably lower than that obtained by the numerical solution, but it is possible to see that both maximum amplitudes happened at frequencies of magnitudes close to each other,

Table 1. Resonance frequency and maximum transmission loss obtained by experimental and numerical (FEA) analysis.

Analysis method	Resonance frequency [Hz]	Maximum transmission loss [dB]
Experimental [7]	600.0	6.07e-3
Numerical	632.5	0.18

5 Conclusion

In view of the above, the smaller the ratio between the neck and cavity radius of the resonator, the more effective it becomes to predict the behavior of the duct sound transmission loss with a Helmholtz resonator by the theoretical methodology. For larger ratios it is necessary to implement other models of resonator neck length corrections which must be based on experimental data. In this regard, the papers of Ingard [9] presents some solutions for neck length corrections for certain resonator geometries and characteristics.

It is also observed that the results of the finite element solution showed a good approximation to both the theoretical solution results and the experimental data, demonstrating the capabilities of the method to obtain satisfactory results in calculations related to noise attenuation in duct cavities.

Future prospects of this research include formulating theoretical methods for analyzing acoustic resonance and sound transmission loss in noise reduction projects in ducts with resonator combinations. Also, we will evolve into a numerical study of acoustic performance using the finite volume method.

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