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Buckling of piles in soft clay: comparison between analytical and numerical forecasting

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Abstract. Steel piles are deep foundation elements that are mostly used in multi-storey buildings, transmission towers and industrial constructions. It is a structural element that is industrially produced, with laminated or welded steel profiles, single or multiple layer, pipe bending or calendered sheet metal tubes, weld seam or seamless tubes and railway tracks. By the use of reduced cross-sectional area of piles in Brazil, especially thinner steel piles (profile I or simple rails) that cross thick soft clay layers, it is considered the risk of pile buckling, even with fully embedded piles. This work evaluates the critical buckling load of a steel pile in a soft clayed soil using analytical and numerical methods. A comparison between the critical load test result obtained using these methods and the allowable load provided by the pile manufacturer is made. There are differences between the methods mostly due to the limitation on top of the pile. Therefore to verify the critical buckling load on a foundation project, the suitable method should be the one most resembling the real situation acting on the structure.

Keywords: steel pile, buckling, soft soil.

1 Introduction

Steel piles are used for more than 120 years because they have high durability and are considered quite effective on a foundation project.

According to NBR 6122 (ABNT, 2019) [1], steel piles must be dimensioned in accordance with NBR 8800 (ABNT, 2008) [2]. It is also necessary to check if piles will not fail by buckling when the piles have their compressive strength above ground level, considering possible erosion (when there is water flow), or if the soil is too soft.

As a relevant example for bucking analysis, Cabral (2016) [3] mentions the collapse of a residential building in Belém (PA) in 1987 with 39 victims. According to Cabral (2016) [3], after the investigation, the technical report concluded that the cause of the accident was the slow rheological behavior of buckling of the pillar-pile set. The rheological characteristics of buckling were related to a soft organic clay deposit in the site.

Another important aspect is that during pile execution there may be some subtle deviations and lateral displacements on the pile axis. These deviations significantly increase the risk of buckling. Thus, the engineers charged with the responsibility for the execution must be careful, in a detailed executive control, minimizing problems and deviations from pile alignment in order to reduce the risks of foundation failure (Danziger and Lopes, 2021 [4]).

This article evaluates the critical buckling load of a steel H-pile in soft clay using analytical and numerical methods (finite element program SAP2000 – version 22 [5]).

2 Main Analytical Methods Proposed to Estimate the Critical Buckling Load

2.1 Method of Timoshenko and Gere (1961)

Timoshenko and Gere (1961) [6] assume that reaction forces at any cross section of the pile are proportional to the deflection at that point. Therefore, there is a soil reaction coefficient, K_h, expressed by: $K_h = k_h B$, where B is the pile width.

This solution is developed through the energy method. The critical load is given by:

$$Q = \frac{\pi^2 E_p I}{L^2}\left(m^2 + \frac{K_h L^4}{m^2 \pi^4 E_p I}\right) \tag{1}$$

Where m is an integer that represents the amount of sine half-waves into which the pile is subdivided in buckling, $E_p I$ is the pile flexural stiffness and L is the pile length.

For the free pile, $K_h = 0$, and (1) the value m=1 must be assumed. And the equation is reduced:

$$Q_{cr} = \frac{\pi^2 E_p I}{L^2} \tag{2}$$

2.2 Whitaker's Simplified Approach (1957)

Based on Timoshenko's work, Whitaker (1957) [7] assumes that the critical buckling load would be:

$$Q_{cr} = Q_{Euler}\left(n^2 + \frac{L'}{n^2}\right) \tag{3}$$

Where:

Q_{Euler} = Euler's critical load, given by $Q_{Euler} = \frac{\pi^2 \cdot E_p I}{L^2}$.

n = integer number representing the number of sinusoidal half-waves when buckling occurs.

Being:

$$L' = \frac{K_h L^4}{\pi^4 E_p I} \tag{4}$$

Following Whitaker (1957) [7], the buckling load is not determined by the pile length, instead the lateral reaction coefficient of the soil and the flexural stiffness of the pile play this role.

2.3 Bergfelt's Expression (1957)

Bergfelt (1957) [8] developed his theoretical research based on the equation of beams on elastic foundations performing a large number of tests in piles of different materials and dimensions.

Considering the results obtained, the most interesting one points that pile load tests were failed. Specifically in this study there is a linear relationship between the critical pile load and the undrained shear strength of clay (S_u) and, then, the empirical expression was developed:

$$Q_{cr} = 8\ a\ 10\sqrt{S_u EI} \tag{5}$$

Where:

EI = pile flexural stiffness.

2.4 Van Langendonck's Solution (1957)

Based on his studies of buckling of stakes and partially embedded piles, Van Langendonck (1957) [9] obtained an abacus and eq. 6, 7 and 8. The value of ko is used to determine the value of c through the abacus using this method. After the determination of this value, it is possible to calculate the buckling length and the critical load of pile.

$$Q_{fl} = \frac{c^2 E_p I}{L^2} = \frac{\pi^2 E_p I}{{L_{fl}}^2} \tag{6}$$

$$L_{fl} = \frac{\pi}{c} L \tag{7}$$

$$k_0 = \frac{L}{5} \sqrt[4]{\frac{k_h B}{E_p I}} \tag{8}$$

Where:
$L =$ Pile length.
$I =$ Moment of inertia of a pile group.
$B =$ Pile diameter or width.
$k_h =$ Horizontal subgrade reaction coefficient (dimension FL^{-3}).
$E_p =$ Modulus of elasticity in pile material.

2.5 Contribution of Davisson and Robinson (1965)

Davisson and Robinson (1965) [10] proposed a method to calculate the critical load and to verify the buckling of piles. This method compares partially embedded piles and simply-supported beam, presenting the same displacement *yt* or the same critical buckling load. Thus, with the free length of the pile *Lu* added the length *Ls* the result is the length of simply-supported beam, *Le*, as shown in Fig. 1.

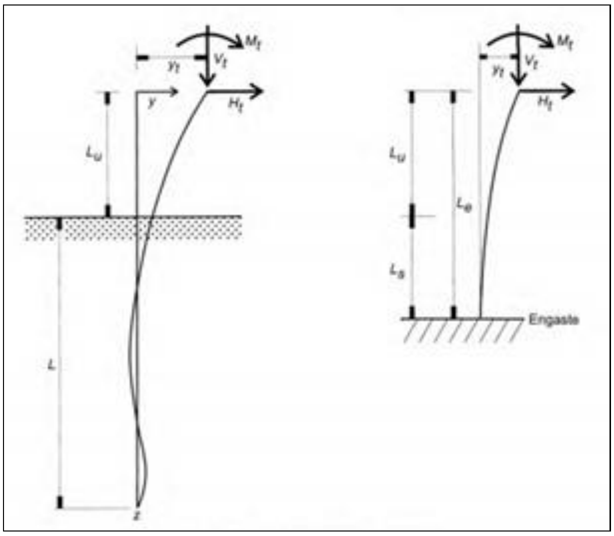

Figure 1. Partially embedded pile

Since the horizontal reaction coefficient is equal to zero from the top pile surface to the ground surface, two cases must be analyzed according to the authors. The first one takes the modulus of horizontal reaction as constant or as growing linearly with depth following the second hypothesis.

For the first case, where K_h = constant, the differential equation of a beam on elastic foundation, represented in eq. 9, is written in 11 considering the quantities in eq. 10.

$$E_p I \frac{d^4 y}{dz^4} + V_1 \frac{d^2 y}{dz^2} + K_h y = 0 \tag{9}$$

$$R = \sqrt[4]{\frac{E_p I}{K_h}},\ L = \frac{z}{R} \quad and \quad U = \frac{V_t R^2}{E_P I} \tag{10}$$

$$\frac{d^4 y}{dL^4} + U\frac{d^2 y}{dL^2} + y = 0 \tag{11}$$

The following dimensionless quantities are introduced as well:

$$L_{máx} = \frac{L}{R},\ S_R = \frac{L_S}{R},\ J_R = \frac{L_u}{R} \tag{12}$$

The equivalent length is given by $L_e = (S_R + J_R)R$.

Buckling and bending coefficients can be presented graphically. The critical buckling load is expressed as:

$$V_{crit} = \frac{\pi^2 E_p I}{4R^2(S_R+J_R)^2} \tag{13}$$

For the second case, $K_h = n_h z$:

$$T = \sqrt[5]{\frac{E_p I}{n_h}},\ Z = \frac{z}{T} \quad and \quad V = \frac{V_t T^2}{E_P I} \tag{14}$$

$$\frac{d^4 y}{dZ^4} + V\frac{d^2 y}{dZ^2} + Zy = 0 \tag{15}$$

The following nondimensional quantities are also introduced:

$$Z_{máx} = \frac{L}{T},\ S_T = \frac{L_S}{T},\ J_t = \frac{L_u}{T} \tag{16}$$

2.6 Solution in Theory of Elasticity (1980)

According to Poulos and Davis' Theory of Elasticity (1980) [11], the critical load value depends on the value of the pile-stiffness factor, given by:

$$K = \frac{E_p}{E_s} R_A \tag{17}$$

Where R_A is the ratio of an area of pile section A_p to the area bounded by the pile outer-circumference:

$$R_A = \frac{A_p}{\frac{\pi d_{ext}^2}{4}} \tag{18}$$

The buckling load Q_{cr} is expressed considering Euler's critical load and is in the abacus which have the factor K_R as entry parameter, given by:

$$K_R = \frac{E_p I_p}{E_s L^4} \tag{19}$$

3 Numeric Method

The discretization by finite elements was performed in this study using the Structural Analysis SAP2000 – version 22. This is the environment for the software SAP2000, a program for linear and non-linear structural and dynamic analysis capable of solving simple static 2D models and high complexity 3D models.

4 Critical Buckling Load Estimation

Aiming to evaluate the critical buckling load through the methods presented above, a hypothetical case is studied trying to show some values found in foundation projects for real constructions. Thus, a steel H-pile 310 x 125 length of 42 meters was considered, fully embedded, surrounded by a thick organic clay layer with undrained shear strength values, *Su*, equal to 20 kPa and subgrade reaction coefficient, *Kh*, equal to 200 kN/m² constant with depth. The cross section shape profile is demonstrated in Fig. 2.

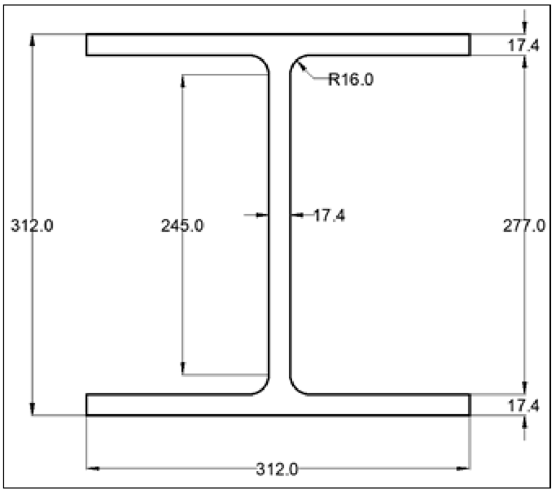

Figure 2. Cross section area HP 310 x 125

Based on Gerdau Steel Piles Guide, A = 159 cm², Ixx =27706 cm^4 and Iyy = 8823 cm^4 are obtained. After deduction of sacrificial steel thickness of 1.5 mm (organic clay), there are A = 132 cm², Ixx = 22452 cm^4 and Iyy = 7091 cm^4 . The smallest moment of inertia is I = 7091 cm^4 . The pile steel, ASTM A572 (Grade 50), has a modulus of elasticity of E = 200,000 MPa. The pile is modeled in SAP2000 using frame elements. A linear buckling analysis is performed for a 3D model. Nonlinear buckling is not investigated in this paper. The properties of A572 - Grade 50 steel are considered, following the references of the program, as shown in Fig. 3.

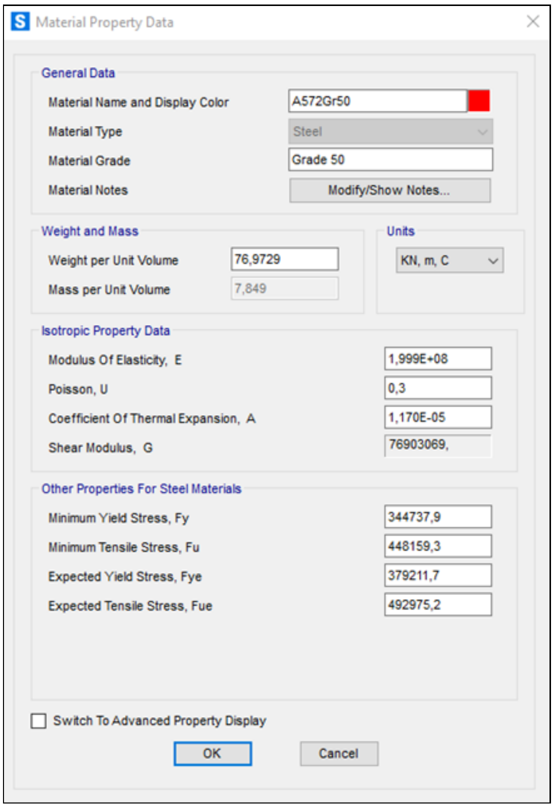

Figure 3. Material properties

The soil is simulated by independent linear elastic springs evenly spaced every 1 m, based on Winkler model (boundary conditions). The coefficients representing the springs can be obtained through the following expression (20):

$$K_i = k \cdot A_i \tag{20}$$

Where:
K_i =Relative stiffness relating to node i;
k = soil reaction coefficient;
A_i = area of influence of node i, pile diameter multiplied by the distance of the nodes; $A_i = Bl$.

Taking into account the equation of the modulus of horizontal soil reaction, $K_H = k_h B$, and the model discretization that occurs every 1 meter, it can be noticed that the value K_i assumes the value of K_H. The springs are applied in both x and y directions and the pile base was considered fixed in a second order support.
Simulation results are expressed in Fig. 4, which presents a critical load value equivalent to Q_{cr} = 1976 kN.

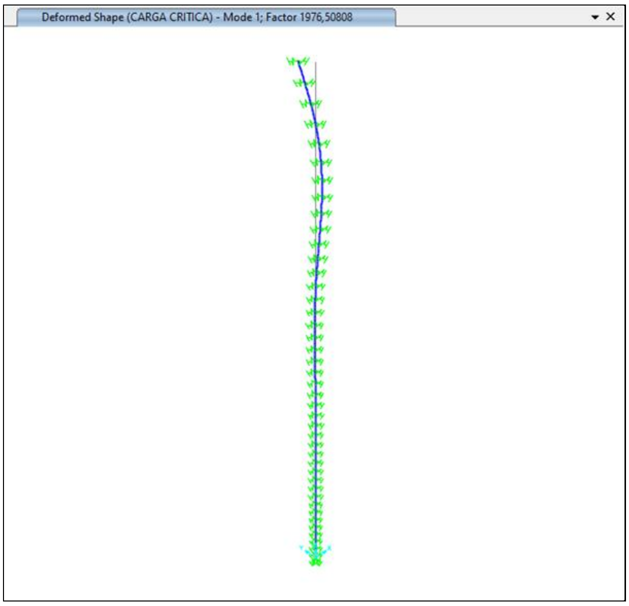

Figure 4. Result for critical buckling load from FEM

The results obtained by some of the methods studied are shown in Fig. 5. It is crucial to observe that critical load values include the global safety factor established by NBR 6122 (2019) [1].

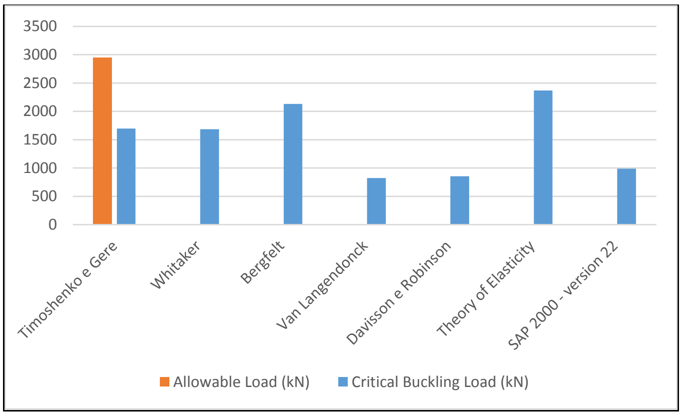

Figure 5. Critical buckling load obtained by each method and allowable load

It is possible to see in Fig. 5 that the methods approached here present critical buckling load value lower than the allowable load given by Gerdau Steel Piles Guide (2946 kN). The solutions proposed by Van Langendonck, Davisson and Robinson and obtained by the Finite Element Method (FEM), provided by the SAP2000 structural calculation program, are different from others. In fact, such solutions were developed based on the assumption that the pile was fully embedded. In these analyses, the free length of pile was considered equal to zero. If there is a limitation on top of the pile, critical buckling load value tends to increase significantly. The variation between the solution using the FEM and the Van Langendonk solution, the one presenting lower results, was approximately 17%. The difference between the Theory of Elasticity solution (the highest critical buckling load obtained) and the solution using the FEM was around 42%. It is important to mention that the results calculated using the methods developed by Timoshenko and Gere and also Whitaker considered the stake labeled in pile caps. The variation between these two methods was less than 1%. Little difference is found between the results of these methods because Whitaker's approach is based on the solution presented by Timoshenko and Gere. Regarding the method employed by Bergfelt and the Theory of Elasticity solution, both presenting greater values of critical buckling load, the variation was approximately 10%.

5 Conclusions

The following conclusions can be drawn: (i) The critical buckling load analysis for fully embedded piles is often neglected, except in cases prescribed in the standard. Nevertheless, cases about steel piles that suffered failure due to buckling are still lacking in the literature. (ii) The methods approached in this study present critical buckling load value lower than the allowable load given by Gerdau. (iii) The solutions proposed by Van Langendonck, Davisson and Robinson and obtained by the Finite Element Method (FEM), provided by the SAP2000 structural calculation program, are different from others. In fact, such solutions were developed based on the assumption that the pile was fully embedded. In these analyses, the free length of pile was considered equal to zero. If there is a limitation on top of the pile, critical buckling load value tends to increase significantly. (iv) The variation between the solution using the FEM and the Van Langendonk solution, the one presenting lower results, was approximately 17%. The difference between the Theory of Elasticity solution (the highest critical buckling load obtained) and the solution using the FEM was around 42%. (v) It is important to mention that the results calculated using the methods developed by Timoshenko and Gere and also Whitaker considered the stake labeled in pile caps. The variation between these two methods was less than 1%. Little difference is found between the results of these methods because Whitaker's approach is based on the solution presented by Timoshenko and Gere. (vi) Regarding the method employed by Bergfelt and the Theory of Elasticity solution, both presenting greater values of critical buckling load, the variation was approximately 10%. (vii) The methods that were proposed here allow the estimation of the critical buckling load of piles in soils with low strength properties. However, considering the different ways to analyse buckling, the suitable method should be the one most resembling the real situation acting on the structure.

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