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DYNAMIC ANALYSIS AND OPTIMIZATION OF A 6X6 VEHICLE'S ACTIVE SUSPENSION SYSTEM

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Abstract. In a big dimensions vehicle, the suspension system performs an essential role in stability, driveability and comfort, being responsible to reduce vibrations induced by ground irregularities, which provides the increasement of suspension and vehicle components life cycle. In this context, the objective of this work is to analyze the dynamic time response of an active front suspension model in a space-state formulation, obtaining through multibody modeling the optimized response for the system, taking into consideration the variables of interest: control force and reduction of the speed of the suspended mass. The closed loop control systems are designed and compared using different strategies: Linear Quadratic Regulator (LQR) and Genetic Algorithm (GA) associated with LQR in order to check the optimal model. The plant parameters are, at first, equivalent to a ¼ car model of a 6X6 Military Vehicle, and the results obtained are simulated through MATLAB/Simulink® for a ½ car model to understand the vertical dynamics phenomenon including variables as pitch and center of gravity speed.

Keywords: Vertical Dynamics, Active Suspension, Optimization, Genetic Algorithm, Linear Quadratic Regulator.

1 Introduction

Vehicle suspension systems are essential for the management of the driveability and vertical dynamics of a vehicle. In terms of vertical dynamics, the suspension systems must be able to support the vehicle's chassis, guarantee the contact between the tires and the ground, and mainly, attenuate the vibrations induced by road irregularities.

In the Dynamics field, mechanical vibrations are described as oscillations capable to deviate a body from its static state, which can be characterized as free or forced. Forced vibrations, in turn, are classified as physical phenomena caused by disturbing external forces, leading the mechanical system to forced oscillations (MCCALLION, 1973).

In this context, the design of a suspension system must consider parameters such as stiffness and damping coefficients, in order to obtain the behavior of the vehicle's suspended mass within acceptable and desirable levels.

Suspension systems can be characterized as passive, semi-active and active. Passive systems do not rely on external energy sources, while active systems may have sensors and actuators, in order to optimize the suspension behavior (SILVEIRA, 2014). Examples of active systems are electrohydraulic and electropneumatic suspensions. Active control is widely used in engineering, and several techniques can be applied in the design of an active suspension. The use of the Linear Quadratic Regulator (LQR) technique has been frequently discussed in the literature, as it is a method that seeks to identify weighting parameters (Q and R) associated with the system states, in order to model an optimal control force. Thus, the objective of this work is to propose an active front suspension mechanism by the action of a control force in a closed loop electrohydraulic system. The control systems are designed and compared in a ¼ vehicle model, whose physical parameters are equivalent to those of a SCANIA 6x6 truck, using two different techniques: LQR and GA-LQR. Finally, a ½ model is developed and analyzed in order to compare the response for both suspesion systems against the passive system for variables such as the vehicle pitch and the center of gravity speed during a sequence of bumps (sinusoidal input).

2 Dynamic System

For a comparative study of the active and passive systems, the front suspension can be represented in a simplified way in a ¼ vehicle model. This model can be expressed in a multibody mechanism that consists of a coupled dual mass-spring-damper system (ELMADANY AND ABDULJABBAR, 1999).

2.1 Multibody Modelling

For the preliminary study of the vehicular dynamics, the truck suspension system can be represented by a ¼ vehicle model as showed in figure 1.

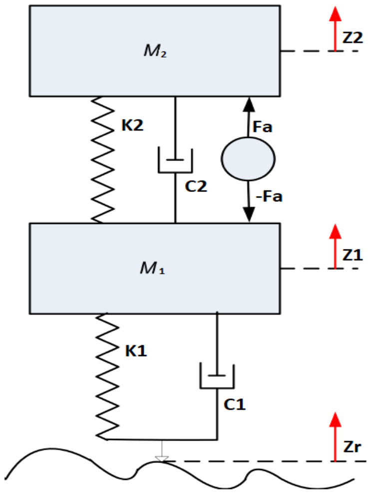

Figure 1. ¼ Car suspension model. Available from: QUANSER MANUAL, 2010

$$M1\ddot{\boldsymbol{Z1}}(t) = C2\left[\dot{\boldsymbol{Z2}}(t) - \dot{\boldsymbol{Z1}}(t)\right] - C1\left[\dot{\boldsymbol{Z1}}(t) - \dot{\boldsymbol{Zr}}(t)\right] + K2\,[\boldsymbol{Z2}(t) - \boldsymbol{Z1}(t)] - K1\,[\boldsymbol{Z1}(t) - \boldsymbol{Zr}(t)] - \boldsymbol{fa}(t) \quad (1)$$

$$M2\ddot{\boldsymbol{Z2}}(t) = -C2\left[\dot{\boldsymbol{Z2}}(t) - \dot{\boldsymbol{Z1}}(t)\right] - K2\,[\boldsymbol{Z2}(t) - \boldsymbol{Z1}(t)] + \boldsymbol{fa}(t) \quad (2)$$

Where the spring stiffness and suspension damping coefficient are $\boldsymbol{K2}$ and $\boldsymbol{C2}$, respectively. The tire is modeled as a spring-damp, also characterized by $\boldsymbol{K1}$ and $\boldsymbol{C1}$ coefficients. The displacement of mass $\boldsymbol{M1}$ is represented by $\boldsymbol{Z1}$ while the displacement of mass $\boldsymbol{M2}$ is represented by $\boldsymbol{Z2}$, and $\boldsymbol{fa}(t)$ represents an active control force.

2.2 State-Space Formulation

The front passive suspension is considered a continuous, linear and time invariant dynamic system, so it is convenient for the study to describe it in terms of it's state variables:

$$x1(t) = \mathrm{Z2(t)}; \quad x2(t) = \dot{Z}2(t); \quad x3(t) = \mathrm{Z2(t) - Z1(t)}; \quad x4(t) = \dot{Z}2(t) - \dot{Z}1(t) \quad (3)$$

Considering a unit step test input, the system above can be written in the state space form as follows, where $\begin{matrix} fa(t) \\ Zr(t) \end{matrix} = u(t)$:

$$\dot{x} = \mathrm{A}x(t) + Bu(t), \quad (4)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{C2}{M2}\frac{C1}{M1} & 0 & \frac{C2}{M2}\left(\frac{C2}{M2} + \frac{C2}{M1} + \frac{C1}{M1}\right) - \frac{K2}{M2} & \frac{C2}{M2} \\ \frac{C1}{M1} & 0 & -\left(\frac{C2}{M2} + \frac{C2}{M1} + \frac{C1}{M1}\right) & 1 \\ \frac{K1}{M1} & 0 & -\left(\frac{K2}{M2} + \frac{K2}{M1} + \frac{K1}{M1}\right) & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ \frac{1}{M2} & \frac{C2}{M2}\frac{C1}{M1} \\ 0 & -\frac{C1}{M1} \\ \frac{1}{M1} + \frac{1}{M2} & -\frac{K1}{M1} \end{bmatrix} \quad (5)$$

Before the action of any road input or active control force, the system is in stationary state, so zero initial conditions can be assumed:

$$\text{x1} = 0, \ \ \text{x2} = 0, \ \ \text{x3} = 0, \ \ \text{x4} = 0. \tag{6}$$

In vehicle active suspension optimization projects, the following indices are generally considered: the passenger comfort and suspension deformation (driveability). The use of this criteria imposes some restrictions to $x2 = \dot{Z}2(t)$ and $x1 = Z2(t)$, respectively, and for sure, to $fa(t)$ which is the control force to be developed by the controller.

2.3 Input Functions

When subjected to an unit step $Zr(t)$, $K1$ and $C1$ are immediately compressed establishing a new condition for the whole system. As a test input widely used on theory, the unit step is used for the project design and preliminary comparisions. Besides, in order to perform realistic simulations, the equations (7), (8) and (9) will also be tested to mathematically model the displacement of the suspension in contact with a sinusoidal road, along the path of the ½ vehicle model and also for the ¼ model, with the exception that on the quarter-car model tests, only the equation (7) will be applicable, since there is only one axle (ACUNA, 2020):

$$Z_{01}(t) = h\,sen\left\{\frac{2\,\pi}{L}\,[v_x t - (d - a_1)]\right\} \tag{7}$$

$$Z_{02}(t) = h\,sen\left\{\frac{2\,\pi}{L}\,[v_x t - (d + a_2 - b_1)]\right\} \tag{8}$$

$$Z_{02}(t) = h\,sen\left\{\frac{2\,\pi}{L}\,[v_x t - (d + a_2 + b_2)]\right\} \tag{9}$$

Table 1: Road and truck geometric parameters

Road Parameters	
$h = 0{,}3$	bumps height in meters (m);
$L = 7$	road length in meters (m);
$Vx = 10$	longitudinal speed (m/s);
$t = 60$	simulation time in seconds (s);
$d = 50$	road excitation position (m);
a_1	distance from the center of gravity to front axle (presented in table 4);
a_2	distance from the center of gravity to the center of intermediate and back axle (presented in table 4);
b_1	distance from the back suspension (bogie type) anchor point (chassis) to the intermediate axle
b_2	distance from the back suspension (bogie type) anchor point (chassis) to the back axle

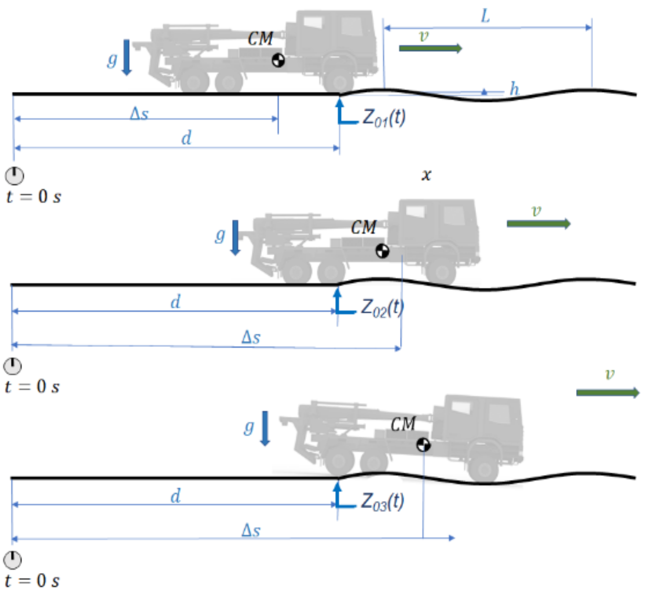

Figure 1: Bumps excitation. Available from: ACUÑA, 2020

3 Quadratic Performance Index

At the optimization process, a performance index widely used in the literature is the quadratic performance index (THOMPSON, 1976), which is presented in (10) below.

$$J = \int_0^{\infty} q1.fa^2(t) + q2.Z2^2(t) + q3.\dot{Z}2^2(t) \tag{10}$$

Where 1 , $q2$ and $q3$ are the penalty constants. With the criteria described above, which considers $x2 = \dot{Z}2(t)$ and $x1 = Z2(t)$ to be optimized, we can obtain the Cost Function J in terms of the state variables $x(t)$ and the control force $fa(t) = u(t)$ on a matrix form.

$$J = \int_0^{\infty} x(t)^T\, Qx(t) + u(t)^T Ru(t) \tag{11}$$

Where the penalty matrices Q and R are taken to be positive semidefinite as follows:

$$Q = \begin{bmatrix} q2 & 0 & 0 & 0 \\ 0 & q3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ R = [q1] \tag{12}$$

4 Controller Project

The controller projects are developed applying two different strategies, in order to evaluate the dynamic response and to compare both techniques. The first one is the Linear Quadratic Regulator (LQR), which consists of minimizing the cost function $\boldsymbol{J}$. The second method is the (GA-LQR), which consists of an association of the LQR method and the Genetic Algorith, where the main goal is to achieve the LQR optimal parameters in a iterative manner, in order to minimize the objective function $\boldsymbol{J}$ considering the project restrictions.

4.1 Linear Quadratic Regulator (LQR)

Optimal controller designs for linear systems using LQR are easily found in the literature (KAILATH, 1979). For a continuous system described as (5), the LQR problem is intended to determine a full state feedback (FSFB) controller that minimizes the Cost Function J. In order to find the minimum, both matrices Q and R are considered the weight matrices, such that $Q = Q' \geq 0$ and $R = R' \geq 0$.

The optimal controller LQR that minimize the Cost Function (9) is given by:

$$u(t) = Kx(t), \text{ when} \tag{13}$$

$K = -R^{-1}B'P,$ where $P = P' \geq 0$, is the solution for the algebraic Ricatti's equation:

$$A'P + PA - PBR^{-1}\, B'P + Q = 0 \tag{14}$$

4.2 Genetic Algorithm (GA-LQR)

Genetic algorithms are a parallel search and optimization technique, inspired by the Darwinian principle of natural selection and genetic reproduction (GOLDBERG). According to C. Darwin's theory, the principle of selection favors the fittest individuals, with greater longevity, and therefore, greater probability of reproduction. In this context, GA looks for a better solution for the optimization problems, through an iterative search process, initiated by generating an initial population, which combined with the best representatives, generates a new one, replacing the previous one. At each new iteration, a new population is generated with individuals that generate the best solution to the optimization problem, culminating in their convergence.

In the GA structure, some terms are used and their definition becomes necessary:

1) Gene: Optimization variable;
2) Chromosome: Set of genes;
3) Initial Population: Randomly generated set of chromosomes;
4) Generations: Genetically modified populations from previous generations through recombination, selection and/or mutation;
5) Recombination: Process of modifying and creating a new chromosome from the combination of 2 or more chromosomes;
6) Mutation: Process of changing an chromosome at random;
7) Fitness Function: Solution function evaluated (Objective Function);
8) Stopping Criteria: End of the iterations, which can be the number of generations and execution time.

4.2.1 GA-LQR Methodology

As a solution to the multi-objective problem presented in equation (10), the GA-LQR appears as an alternative, delivering controller designs with good performance and stability as a result. The GA-LQR model presented in figure 4 performs the search for the state and control weighting matrices Q and R, in order to design a controller that satisfies the physical and design constraints for the dynamic system.

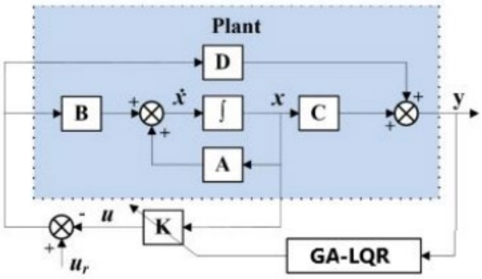

Figure 4: GA-LQR Structure (MORAES, 2007)

As described in 4.2, to start the iteration process, it is necessary to establish the inputs to the optimization problem:

I. QR Population Modeling

A chromosome QR_{nxg} is composed of g genes, depending on the dimension n and the m inputs of the system. Considering the active suspension optimization problem, subject to an input (step/bumps), the chromosome has the following structure:

$$QR_{nxg} = [q1; q2; q3; q4; r1]$$

II. Genetic Operations

- Elitist Selection: At this stage of the process, the n_{ie} best chromosomes are stored for the next generation;
- Crossover Model: The crossover is the recombination operation, which in this model was responsible for combining 2 chromosomes from an initial (previous) population G. The chromosomes QR_{l1}e QR_{l2} from a population QR_{nxG} share genetic information, creating a new one $QR_{G+1,l1}$ as follows:

$$QR_{G+1,l} = [q1_{G,l1}, q2_{G,l1}, q3_{G,l1}, q4_{G,l2}, r1_{G,l2}]$$

- Mutation Model: In the GA-LQR project of the active suspension, the alteration of a gene of the mutated chromosomes was considered for the generation of the population $G + 1$ randomic, using the function *randi*$(5,1, n_{im})$ of MATLAB/Simulink, where n_{im} is the number of chromosomes mutated at the new generation, and 5 is the number of genes of one chromosome QR_{nxG}.

III. Objective Function and Constraints

In the controller design, the objective function to be minimized is characterized by the cost functional J, however, for realistic modeling of the Fitness Function of the optimization problem, the constraints must be considered, so that the controller design proposed by the iterative method GA meets not only the stability and design characteristics of the control system, but also the physical constraints of the dynamic system, which are described below:

- According to the ISO 2631 standards (ISO 2631, 1997), there is a perception of comfort when the RMS acceleration of the suspended mass does not exceed 0.315m/s²:

$$F1 = \ddot{Z}_2 - 0.315 \leq 0 \quad (15)$$

- In order to meet the physical requirements of the suspension, the working space must not exceed 0.127m:

$$F2 = Z_2 - Z_1 - 0.127 \leq 0 \quad (16)$$

- To introduce design requirements, a maximum overshoot $M_p = 25\%$, equivalent to a 15% reduction in relation to the passive system, and a settling time $T_s = 1{,}8\ sec$, equivalent to a 10% of reduction in relation to the passive system. Finally, an steady state error of $e_{ss} = 2\%$, ensuring system stability and imposing the constraints F3 e F4 below:

$$F3 = \zeta\omega_n + 2.31 \leq 0 \tag{17}$$

$$F4 = \frac{y_p}{y_a} - 0.02 \leq 0 \tag{18}$$

IV. Função Fitness

To define the fitness function, the constraints of the problem must be considered, so that they must be incorporated into the objective function through the approach of penalty functions. Modeling the problem with constraints through the introduction of penalty constants makes its application possible, while making the problem unconstrained, and therefore, easier to solve. The choice of penalty parameters must guarantee proportionality to the unconstrained fitness function, so that the algorithm moves in the search direction in the feasible region. In the GA-LQR controller design, constraints F1, F2, F3 and F4 assume the role of auxiliary function, being introduced in the fitness function with their respective penalty parameters $a_1 = 1$, $a_2 = 10^2$, $a_3 = 10$ e $a_4 = 1$. Thus, failures to meet the constraints of the problem incurs an increasemet of the fitness function value, which diverges from the optimal model. In this way, the optimization problem can be summarized as:

$$\boldsymbol{Min}\, F_{obj}(Q,R,X,u) = \int_0^\infty x(t)^T Qx(t) + u(t)^T Ru(t) \tag{19}$$

Subject to

$$\begin{gathered} \ddot{Z}_2 - 0.315 \leq 0 \\ Z_2 - Z_1 - 0.127 \leq 0 \\ \zeta\omega_n + 2.31 \leq 0 \\ \frac{y_p}{y_a} - 0.02 \leq 0 \\ 0 \leq QR_{nxG} \leq 100 \end{gathered} \tag{20}$$

It is worth mentioning that the management of the penalty constants in the objective function can lead to prioritization of one variable to the detriment of another. For example, the association of a penalty constant in one of the constraints that is much higher than the valuation of another, will make the GA look for solutions that are increasingly suitable for the viable region of this constraint, so that the fitness function is minimized. Therefore, in order to meet all requirements and minimize the fitness function satisfying all constraints, it is important to guarantee its proportionality, considering the dimensions of the variables of interest.

V. GA-LQR Model Parameters

Finally, to perform the optimization process satisfactorily and find the convergence, it is essential that the choice of parameters of the genetic algorithm is adequate. The table 2 lists the GA parameters that structurize the iterative process:

Table 2: GA Procedure

GA Parameters	Value
Number of variables (genes)	5
Population Size	100
Selection Model	Elitist – 20%
Mutation	1 Gene – 30%
Crossover (Recombination)	50%
Stopping Criteria	30 Generations

5 Numerical Analysis

Numerical simulations were performed using MATLAB/Simulink, where a full 6x6 SCANIA P 410 CB truck model was simplified to a ¼ vehicle model for analysis of the active front suspension (SCANIA SPECS, 2020). The vehicle is shown in figure 5, and its simplified parameters are summarized in table 3.

Table 3: ¼ Car model Parameters

Suspension Parameters	Valor
Sprunged mass (M2)	4500 kg
Unsprunged mass (M1)	490 kg
Suspension stiffness (K2)	59.600 N/m
Tire stiffness (K1)	770.700 N/m
Damping coefficient - Suspension (C2)	20.000 N s/m
Damping coefficient - Tire (C1)	2.660 N s/m
Control Force $fa(t)$	Optimization
K Gain	Optimization


Figure 5: SCANIA P 410. Adapted from: ACUÑA, 2020

In order to achieve the project requirements mentioned (17), for the development of the LQR controller, the values $q1 = 10^2, q2 = 2\ e\ r1 = 1$ were considered. With these parameters, the closed-loop transfer function designed (21) is found, with its dominant poles at $[-2.18 \pm 2.78\text{i}]$.

$$\frac{24.13\,S^2 + 7062\,S + 20830}{S^4 + 54.06\,S^3 + 1906\,S^2 + 7951\,S + 21040} \tag{21}$$

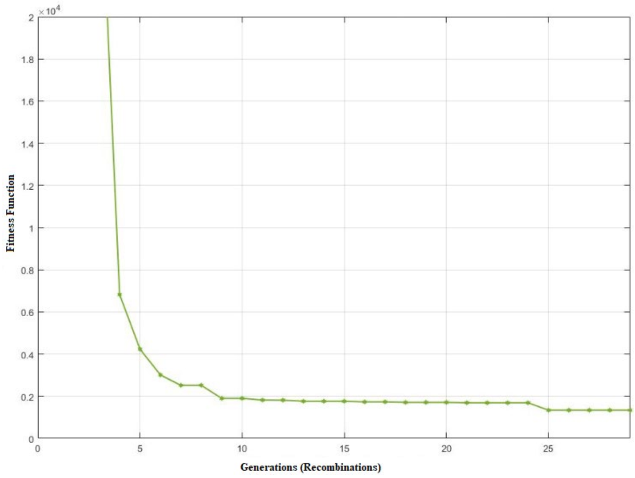

Figure 6: GA- LQR convergence curve [Author]

At the ending of the 30th generation, based on the projects requirements and the constraints imposed, the GA-LQR model reached the values of $QR = [0.72, 0.039, 0.17, 0.016, 76]$, with its dominant poles pair at $[-2.028 \pm 2.69\text{i}]$, culminating in the closed-loop transfer function described in (22) equation.

$$\frac{24.13\,S^2 + 7062\,S + 20830}{S^4 + 61.67\,S^3 + 2088\,S^2 + 2118\,S + 20930} \tag{18}$$

5.1 ¼ Car Model Results

By submitting both quarter-car controllers with 2 DOFs to a unit step-type test input, the values of maximum overshoot M_p and settling time T_s for the system are obtained and shown in figure 7. It is possible to note that the GA-LQR model is able to reduce in 15% the value of the performance indicator J compared to the LQR model, combined with a reduction of e 2% of overshoot M_p. Although the settling time T_s has increased, the controller meets the project requirements of 1,8 sec, greatly reducing peak speed by 12% if also compared to the LQR model. These properties and the system output are also observed in the time response curves of X_1 and X_2, as shown in the figure 7. To sumarize, the tables 4 and 5 describe the controller optimal parameters and the time response comparisions between each active system and the passive suspension for both outputs of interest: X_1 and X_2.

Table 4: Half-car controllers parameters

Suspension Model	Cost Function (J)	Gain K Matrix	% Mp	Ts
Open-loop (Passive)	-	-	29.429	1.9762
LQR (Active)	185	0.009 0.039 0.16 −0.002	22.89	1.3772
GA-LQR (Active)	**161**	0.005 0.04 0.321 -0.007	**22.69**	1.6923

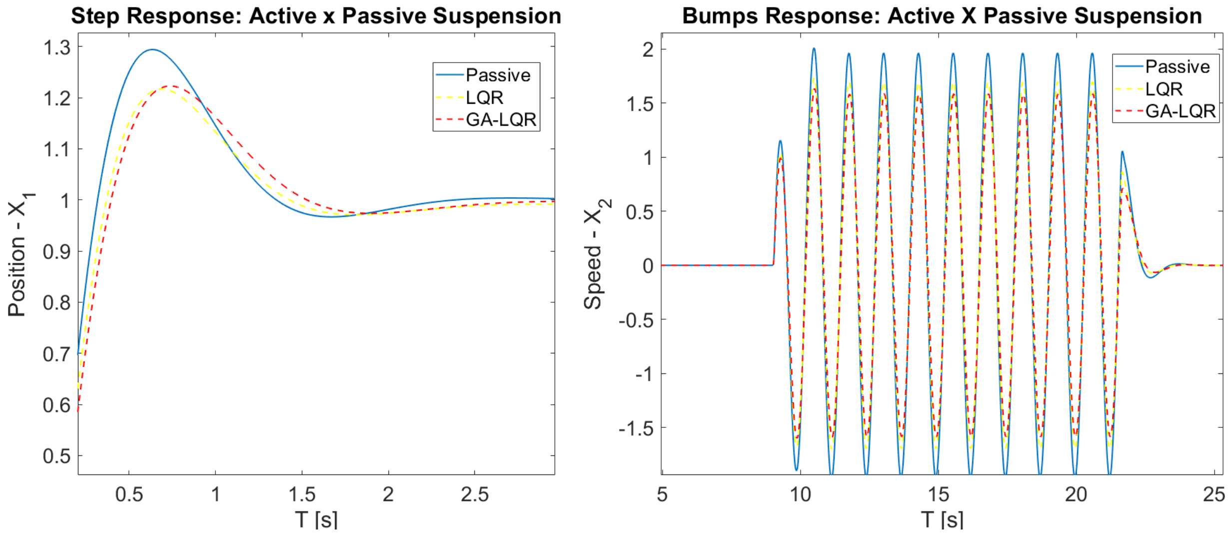

Figure 7: Time Response Curves (Step for X_1 and Bumps X_2). [Author]

Table 5: Numerical Analysis for the quarter-car model

Outputs	Passive	LQR	GA-LQR
X_1	0,404 m	0,372 m	**0,321 m**
X_2	2,01 m/s	1,87 m/s	**1,64 m/s**

5.2 ½ Car Model Results

In this section, all geometric truck parameters and also the back suspension (bogie type) are listed (table 6) and introduced to the model. It is essential to mention that the quarter-car sprunged mass value is calculated from the whole model, taking in consideration the dynamic weight distribution (GILLESPIE, 1992).

For the half-car model dynamic analysis (7 DOFs), the longitudinal displacement was performed with constant speed Vx along the sinusoidal test track described at table 1, where the results of the speed of the center of gravity V_{CG} and pitch θ are shown at figure 9 and resumed at table 7.

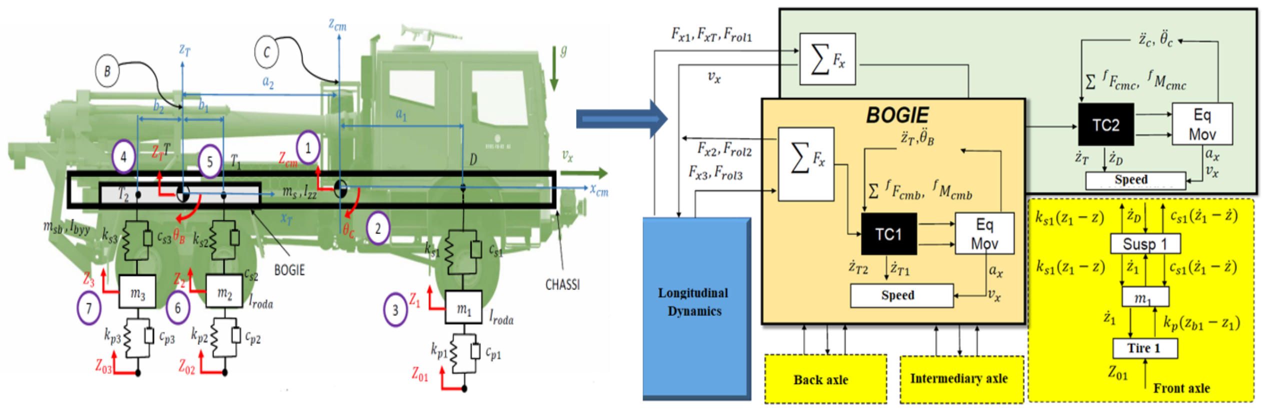

Figure 8: Half-car model. [Author]

Table 6: ½ Car model parameters

Parameters		
Distance from CG to front axle	a_1	2,990 m
Distance from back suspension (bogie) center to CG	a_2	1,988 m
Distance from back suspension (bogie) center to intermediate axle	b_1	0,675 m
Distance from back suspension (bogie) center to back axle	b_2	0,675 m
CG height	h_{CM}	1,359 m
Half-car mass	m_v	22.225 Kg
Chassis mass	m_s	18.135 Kg
Bogie mass	m_{sb}	200 Kg
Front axle unsprunged mass	m_1	490 Kg
Intermediate axle unsprunged mass	m_2	1.700 Kg
Back axle unsprunged mass	m_3	1.700 Kg
Inertia momentum - Chassis x CG (*y-axle)*	I_{yy}	1,040×10^5 Kg m²
Inertia momentum - *Bogie* x CG (*y-axle)*	Ib_{yy}	66,080 Kg m²
Inertia momentum - Vehicle x CG (*z-axle)*	I_{zz}	727,390 Kg m²
Intermediary suspension damping coefficient	c_{s2}	10.000
Intermediary suspension stiffness	k_{s2}	3.220.000
Back Suspension damping coefficient	c_{s3}	8.000
Back Suspension stiffness	k_{s3}	3.220.000

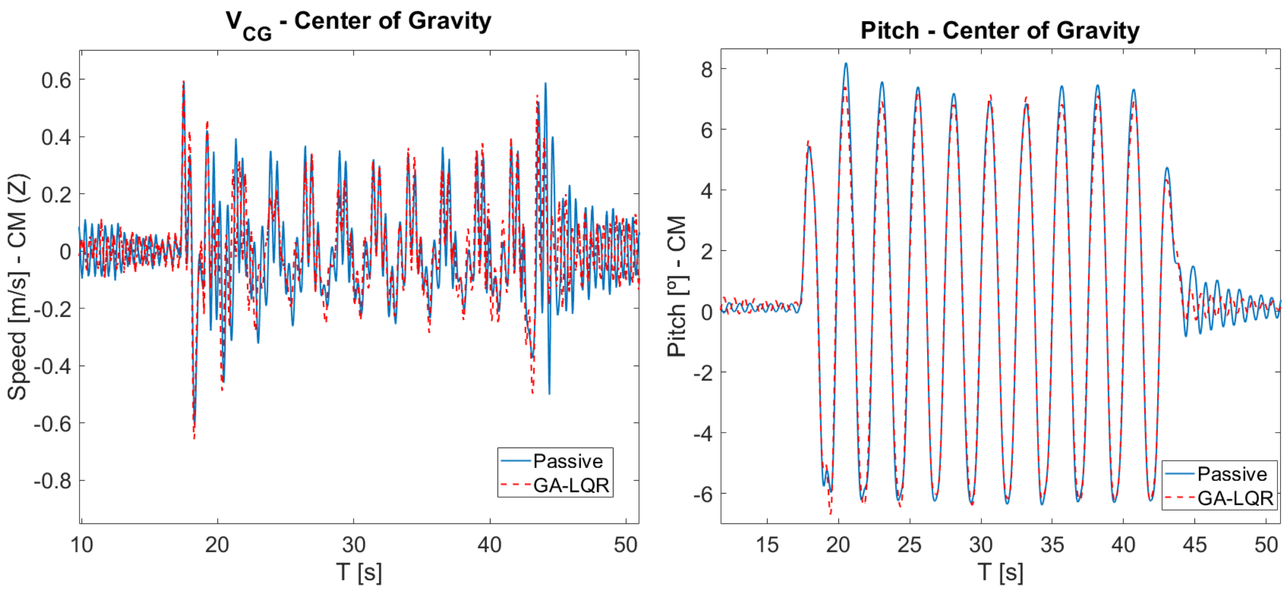

Figure 8: Half-car model analysis. [Author]

Table 7: Numerical Analysis for the half-car model

Outputs	Passive	GA-LQR
V_{CG}	0,61 m/s	**0,59 m/s**
θ	8,2º	**7,3º**

6 Conclusions

The analysis aimed to study the two different control techniques on active suspension system models, in order to compare which strategy would achieve better performance, optimizing the cost function J. First, the passive suspension was described in state-space form and the controllers were designed, so that a control force f_a was applied, in order to improve the behavior of the sprunged mass $M2$.

The numerical results showed that the management of the system eigenvalues, through the modeling of the K gain of the LQR and GA-LQR controllers, allows meeting the design specifications, with a relevant reduction of the variables of interest settling time T_s, maximum overshoot M_p and cost function J, and the GA-LQR methodology that associates the Genetic Algorithm to the LQR allowed to reach more significant reductions in M_p overshoot by additional 2%, 15% in the cost function J and mainly, 8% in the vertical speed X_2 of the sprunged mass $M2$ compared to the LQR model, thus justifying its application.

Finally, studying the half-car model with the parameters of the GA-LQR suspension developed, the pitch and gravity center speed V_{CG} were evaluated. In these aspects, reductions of 2% for the V_{CG} speed and 11% for pitch were identified, and also visually a better accomodation after the sequence of bumps, reinforcing the results of the quarter-car model.

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