XLIII CILAMCE
21–25, November, 2022
FOZ DO IGUAÇU – PR – BRAZIL

A Continuum Damage Micromechanical-Based Model for Fracture Propagation in Viscoelastic Material

Cássio B. Aguiar[1], Samir Maghous[2]

[1]*Coordination of Civil Engineering, Federal Technological University of Parana*
R. Marcílio Dias, 635, 86812-460, Apucarana/PR, Brazil
Cassio.barros.aguiar@gmail.com
[2]*Department of Civil Engineering, Federal University of Rio Grande do Sul*
Av. Osvaldo Aranha, 99, 90035-190, Porto Alegre/RS, Brazil
samir.maghous@ufrgs.br

Abstract. The paper aims to model via micromechanics and macroscopic thermodynamic concepts a propagation law for high densely fractured materials, formulated through a continuous damage variable that account for incorporates the delayed viscoelastic material behavior. The first step of the approach is intended to present the effective behavior of the viscoelastic fractured material and the damage activation criterion. The Mori-Tanaka elastic homogenization scheme and the Laplace-Carson elastic/viscoelastic correspondence principle are combined with an appropriate thermodynamic approach for formulating the damage propagation criterion. This approach allows for analytical description of the macroscopic damage law, which can be handled numerically in the time domain. Due to the damage variable evolution, it is necessary to formulate a non-linear viscoelastic behavior law. The damage evolution rate is assessed by means of a reasoning inspired from plasticity theory. The damage evolution algorithm relies upon the time integration of the damage evolution through an incremental implicit scheme. The numerical implementation and application of the model emphasize the time-dependent effects on damage propagation.

Keywords: damage propagation, viscoelasticity, micromechanics, homogenization.

1 Introduction

Most of engineering materials and particularly geomaterials such as rocks, concrete, masonry or asphalt pavements, exhibit at different scales discontinuity surfaces with various sizes and orientations. Generally referred to as fractures, these discontinuities are zones of small thickness along which the physical and mechanical properties of the intact matrix are degraded, significantly impairing the overall material behavior. When the medium contains a high density of fractures, most theoretical and computational analyzes resort to phenomenological theories based on continuous damage, focusing on instantaneous responses [1-4]. However, phenomenological laws do not usually isolate the effects of fracture propagation, indirectly including other phenomena, like as plasticity or other non-linearities. In contrast, laws formulated via micromechanics theory provide direct interpretation of the fracture density as damage variable at macroscopic scale [5-8]. Referring to the situation of long-lasting loading analyses, damage propagation laws (phenomenological or micromechanics) based on instantaneous responses are not appropriate when the delayed deformation is a fundamental component of the material behavior. In this context, the association between viscoelasticity and damage mechanics produces a robust tool capable of dealing with deferred behavior and inferring the degradation of material properties. However, most of the models developed in this way have a phenomenological origin [9-11], having difficulties to dissociate the propagation of microfractures from other degrading effects. Viscoelastic micromechanical models capable of dealing with the propagation of microfractures on the microscopic scale, reinterpreting them as damage on the macroscopic scale, are relatively new in the literature [12-14]. At the material scale, these approaches define a macroscopic parameter that describes the average behavior of surrounding microfractures.

At the structure scale, this parameter can be described by a damage field, mapping the average behavior of microfractures along the entire structure. In this paper we aim to present a damage evolution law at the material scale, using micromechanical models to describe the evolution of microfractures in viscoelastic solids through a macroscopic damage variable.

2 Modeling Framework

The upscaling procedure developed in the subsequent analysis is based on micromechanical concepts, relying upon the concept of a representative elementary volume (REV) for densely fractured mediums. In this context, we assume it is possible to define a REV for the studied materials with randomly distributed fractures. In the subsequent analysis Ω denotes the REV composed by the solid matrix and fractures, $\omega = \bigcup_{j=1}^{N} \omega_j$ is the volume occupied by all fractures and $\Omega \backslash \omega$ stands for the homogeneous solid matrix phase, corresponding to the REV without the fractures. Each fracture is modeled as an interface geometrically described by a surface ω_i, related to its normal orientation vector $\underline{n}_i$. At a smaller scale however, the fracture would be represented by a finite-thickness volume delimited by distinct upper ω_i^+ and lower ω_i^- boundaries (see Figure 1). Making an analogy with elastic fracture behavior [15, 16], the viscoelastic fracture behavior may be assessed by correlating stress forces $\underline{T} = \underline{\underline{\sigma}} \cdot \underline{n}$ (acting in the discontinuity interface) and displacements jumps $[\underline{\xi}] = \underline{\xi}^+ - \underline{\xi}^-$ (along the discontinuity length) by the fracture relaxation tensor $\underline{\underline{k}}^v$

$$\underline{T} = \underline{\underline{k}}^v \circ [\underline{\xi}] = \underline{\underline{k}}^v(t,t) \cdot [\underline{\xi}](t) - \int_0^t \frac{\partial \underline{\underline{k}}^v(t,\tau)}{\partial \tau} \cdot [\underline{\xi}](\tau) d\tau \tag{1}$$

where the operator $\circ$ stands for the Boltzmann hereditary integral, $\underline{\xi}^+$ refers to the fracture upper boundary displacement and $\underline{\xi}^-$ is the lower boundary displacement. Referring to the local frame $(\underline{n}, \underline{t}, \underline{t}')$, $\underline{\underline{k}}^v$ commonly takes the form:

$$\underline{\underline{k}}^v = k_n^v \underline{n} \otimes \underline{n} + k_t^v (\underline{t} \otimes \underline{t} + \underline{t}' \otimes \underline{t}') \tag{2}$$

where k_n^v and k_t^v are respectively the normal and tangential fracture relaxation functions. These components are classically evaluated from laboratory test performed on material specimen with a single fracture. On the other hand, the viscoelastic solid matrix behavior is modeled as a 3D continuum relating strains $\underline{\underline{\varepsilon}}$ and stresses $\underline{\underline{\sigma}}$ by the fourth-order relaxation tensor $\mathbb{R}^s$:

$$\underline{\underline{\sigma}} = \mathbb{R}^s \circ \underline{\underline{\varepsilon}} = \mathbb{R}^s(t,t) : \underline{\underline{\varepsilon}}(t) - \int_0^t \frac{\partial \mathbb{R}^s(t,\tau)}{\partial \tau} : \underline{\underline{\varepsilon}}(\tau) d\tau \tag{3}$$

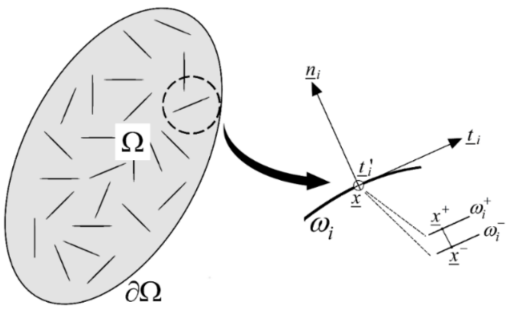


Figure 1: REV and fracture representation.

Based on the micromechanical theory and the microscopic behaviors described above, Aguiar and Maghous [14] formulated the equivalent macroscopic behavior for the fractured viscoelastic material introducing the damage parameter ε originally defined by Budiansky and O'Connell [17], which macroscopically describes the microfractures. This approach can provide a micromechanical (non-phenomenological) interpretation for the damage, isolating the effect of microfractures on material degradation. Furthermore, Aguiar and Maghous [14] showed that regardless of the rheological model of the solid matrix and fractures, the macroscopic behavior can always be described by a generalized Maxwell model (see Figure 2). The homogenized mechanical behavior (at macroscopic scale) for the particular case of fractures isotropically distributed within the material is given by

$$\underline{\underline{\Sigma}} = \mathbb{R}^{hom} \circ \underline{\underline{E}}$$
$$\mathbb{R}^{hom} = 3k^{hom}\mathbb{J} + 2\mu^{hom}\mathbb{K} \tag{4}$$

where $\underline{\underline{\Sigma}} = \langle \underline{\underline{\sigma}} \rangle$ is the macroscopic stress tensor, standing for the average of the microscopic stress field over the REV. $\underline{\underline{E}} = \langle \underline{\underline{\varepsilon}} \rangle$ is the macroscopic strain tensor, matching the average of the microscopic strain field over the REV. $\mathbb{R}^{hom}$ is the homogenized relaxation tensor, which is the equivalent behavior of the damaged material. $\mathbb{J}$ and $\mathbb{K}$ are respectively the spherical and deviatoric four-order projectors. k^{hom} and μ^{hom} are respectively the bulk and shear homogenized moduli, which mathematical description is given on Laplace-Carson domain (see [14]) by

$$k^{hom*} = \frac{k^{S*}}{1+\varepsilon M_k^*} \quad and \quad \mu^{hom*} = \frac{\mu^{S*}}{1+\varepsilon M_\mu^*}$$
$$M_k^* = \frac{\frac{\frac{4}{3}\pi k^{S*}}{\mu^{S*}}}{\pi\kappa_1^* + \frac{a k_n^*}{\mu^{S*}}} \quad and \quad M_\mu^* = \frac{16\pi\ \kappa_4^*}{15}\ \frac{6\kappa_2^*+4\kappa_3^*+9\pi\kappa_4^*(3\kappa_1^*+\kappa_4^*)}{(3\pi\kappa_1^*\kappa_4^*+\kappa_2^*)\left(4\kappa_3^*+9\pi\kappa_4^*(\kappa_1^*+\kappa_4^*)\right)} \tag{5}$$
$$\kappa_1^* = \frac{3k^{S*}+\mu^{S*}}{3k^{S*}+4\mu^{S*}} \quad ; \quad \kappa_2^* = \frac{3k_n^* a}{3k^{S*}+4\mu^{S*}} \quad ; \quad \kappa_3^* = \frac{3k_t^* a}{3k^{S*}+4\mu^{S*}} \quad ; \quad \kappa_4^* = \frac{\mu^{S*}}{3k^{S*}+4\mu^{S*}}$$

Parameters k^{S*} and μ^{S*} stand respectively for the apparent bulk and shear moduli of isotropic solid matrix defined in Laplace-Carson domain, whereas k_n^* and k_t^* are respectively the apparent normal and tangential fracture relaxation components. $a = \sqrt[3]{\varepsilon/\mathcal{N}}$ is the average microfracture radius and $\mathcal{N}$ is the number of fractures by volume unity.

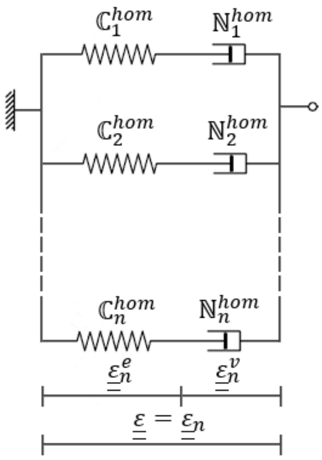


Figure 2: Generalized Maxwell model of the homogenized viscoelastic damaged material.

3 Damage Propagation

The behavior of the homogenized material, presented above, is governed by the presence of microfractures and is expected to statically describe the damaged properties at macroscopic scale. The first step to evaluate damage evolution is to assess the conditions for damage increase (i.e., microfractures propagation). Such a condition can be formulated resorting to an energetic approach based on thermodynamics concepts, similar to that described by Griffith [18]. Aguiar and Maghous [19] formulated Helmholtz's free energy of damaged viscoelastic material directly at the macroscopic scale, from which they applied thermodynamic concepts to formulate a damage propagation condition. The free energy expression can be written as

$$\Psi = \frac{1}{2}\sum_i \underline{\underline{\Sigma}}_i : \underline{\underline{E}}_i^e = \frac{1}{2}\underline{\underline{\Sigma}} : \underline{\underline{E}} - \frac{1}{2}\sum_i \underline{\underline{\Sigma}}_i : \underline{\underline{E}}_i^v \tag{6}$$

where $\underline{\underline{E}}_i^e$ (and $\underline{\underline{E}}_i^v = \underline{\underline{E}} - \underline{\underline{E}}_i^e$) stands for the elastic (viscous) strain associated in the ith branch of generalized Maxwell with the spring (dashpot) element (Figure 2). Tensor $\underline{\underline{\Sigma}}_i = \mathbb{C}_i^{hom} : \underline{\underline{E}}_i^e$ corresponds to the stress within the ith branch of generalized maxwell.

The propagation condition arises from the first and second thermodynamic laws. The expression for the energy dissipation for the damaged viscoelastic material takes the following form

$$\Phi = \frac{1}{2}\sum_i \underline{\underline{\Sigma}}_i : \dot{\underline{\underline{E}}}_i^v - \frac{1}{2}\sum_i \underline{\underline{E}}_i^e : \frac{\partial \mathbb{C}_i^{hom}}{\partial \varepsilon} : \underline{\underline{E}}_i^e \, \dot{\varepsilon} \geq 0 \tag{7}$$

note that the formulation is being directly established on the macroscopic scale.

Aguiar and Maghous [19] observed that if the damage is not propagating (i.e., $\dot{\varepsilon} = 0$), the energy dissipation reduces to the viscous effects, matching with the first right hand side term on Eq. (7). In that case, the viscous dissipation will not directly contribute to fracture propagation. In contrast, damage propagation $\dot{\varepsilon} > 0$ induces energy dissipation Φ_f due the increase in fracture density parameter. In a similar approach that Griffith [18] we assume that the energy dissipation by fracture propagation ($\dot{\varepsilon} > 0$) is controlled by a threshold $\mathcal{F}_v$such that $\Phi_f = \mathcal{F}_v \dot{\varepsilon} > 0$. It should be noted that the threshold $\mathcal{F}_v$ defined in the context of a micromechanical damage approach has a different interpretation than the critical energy $\mathcal{F}_c$ classically introduced in discrete fracture approaches related to linear fracture mechanic. The reason for this difference is that $\mathcal{F}_v$ fundamentally depends on the damage parameter and it is not therefore a material parameter. Within the framework of analogy to linear elastic fracture mechanics, the reasoning developed in Dormieux et al. [20] considers that to the total energy dissipation Φ_f associated with damage evolution is the sum of the contribution of each fracture in the REV, leading to the following relationship between $\mathcal{F}_v$ and $\mathcal{F}_c$:

$$\mathcal{F}_v = \frac{2\pi}{3}\left(\frac{\mathcal{N}}{\varepsilon}\right)^{1/3} \mathcal{F}_c \tag{8}$$

Based on Eq. (7) and in the energy dissipation threshold Φ_f, we can define the driving force associated with the propagation process as the energy release rate $\mathcal{F}$:

$$\mathcal{F} = -\frac{1}{2}\sum_i \underline{\underline{E}}_i^e : \frac{\partial \mathbb{C}_i^{hom}}{\partial \varepsilon} : \underline{\underline{E}}_i^e \tag{9}$$

According to the present theoretical framework, the criterion for damage propagation in viscoelasticity may be written by

$$\mathcal{F} - \mathcal{F}_v \leq 0 \quad ; \quad (\mathcal{F} - \mathcal{F}_v)\dot{\varepsilon} = 0 \quad ; \quad \dot{\varepsilon} \geq 0 \quad ; \quad \dot{\varepsilon} > 0 \Rightarrow \mathcal{F} = \mathcal{F}_v \tag{10}$$

The energy release rate $\mathcal{F} = \mathcal{F}(\varepsilon)$ is a damage-dependent function in the sense it is closely related to the variable responsible for the fracture propagation. Note $\mathcal{F}$ is a time-dependent function, being the main difference when compared to the elastic reasoning presented by Griffith.

The application of the propagation criterion cannot be directly associated with the homogenized behavior described above since the latter was formulated considering linear viscoelasticity (fixed value for the damage parameter ε). In order to proceed with the propagation analysis, it is necessary to replace the linear homogenized behavior with a non-linear viscoelastic formulation that converges to the linear one when the damage parameter is kept constant. Aguiar [21] developed two non-linear viscoelastic formulations for this purpose. In view of the limited physical space of this paper, only the simpler expression is presented

$$\underline{\underline{\sigma}} = \int_{-\infty}^{t} \mathbb{R}^{hom}(t,\tau,\varepsilon(t)) : \frac{\partial \underline{\underline{E}}(\tau)}{\partial \tau} d\tau = \mathbb{R}^{hom}(t,t,\varepsilon(t)) : \underline{\underline{E}}(t) - \int_{-\infty}^{t} \frac{\partial \mathbb{R}^{hom}(t,\tau,\varepsilon(t))}{\partial \tau} : \underline{\underline{E}}(\tau) d\tau \tag{11}$$

Note that the formulation is very similar to the Boltzmann hereditary integral shown in Eq. (3), except for considering the nonlinearities induced by the variations of damage parameter ε within the homogenized relaxation tensor.

To predict the evolution of damage, it is necessary to introduce the concept of "load-damage" surface $F(\mathcal{F},\varepsilon) = 0$, which delimits the region where the damage does not propagate in the space of the thermodynamic force $\mathcal{F}$. The mathematical formulation of the propagation criterion can be expressed as

$$F(\mathcal{F},\varepsilon) = \mathcal{F} - \mathcal{F}_v(\varepsilon) \leq 0 \tag{12}$$

Condition $F(\mathcal{F},\varepsilon) < 0$ indicates a regime with no damage propagation, whereas $F(\mathcal{F},\varepsilon) = 0$ refers to the state in which damage evolves. Note, however, that the "damage-surface" $\mathrm{F}(\mathrm{F},\varepsilon) = 0$ involves a single damage parameters, thus leading to a scalar formulation. According to the reasoning developed in Pavan et. al. [22, 23], when the damage process is active, i.e. $F(\mathcal{F},\varepsilon) = 0$, the damage potential $G(\mathcal{F},\varepsilon)$ that controls the damage evolution allows expressing

$$\dot{\varepsilon} = \dot{\lambda}\frac{\partial G}{\partial \mathcal{F}} \tag{13}$$

where the damage multiplier $\dot{\lambda}$ must meet the Kuhn-Tucker conditions [21].

The damage multiplier rate $\dot{\lambda}$ can evaluated from the consistency condition $\dot{F} = 0$, which leads to:

$$\dot{F} = \frac{\partial F}{\partial \mathcal{F}}\dot{\mathcal{F}} + \frac{\partial F}{\partial \varepsilon}\dot{\varepsilon} = 0 \quad \text{and} \quad \dot{\lambda} = \frac{\dot{\mathcal{F}}}{\frac{\partial \mathcal{F}_v}{\partial \varepsilon}\frac{\partial G}{\partial \mathcal{F}}} \tag{14}$$

The mathematical development of Eq. (14) considering the state variables $\left(\varepsilon, \underline{\underline{E}}, \underline{\underline{E}}_i^v\right)$ on the determination of $\dot{\mathcal{F}}$, leads to

$$\text{and} \quad \dot{\lambda} = \frac{\frac{\partial \mathcal{F}}{\partial \underline{\underline{E}}}:\dot{\underline{\underline{E}}} + \sum_i \frac{\partial \mathcal{F}}{\partial \underline{\underline{E}}_i^v}:\dot{\underline{\underline{E}}}_i^v}{\left(\frac{\partial \mathcal{F}_v}{\partial \varepsilon} - \frac{\partial \mathcal{F}}{\partial \varepsilon}\right)\frac{\partial G}{\partial \mathcal{F}}} \tag{15}$$

Introducing Eq. (15) in Eq. (13) allows the determination of $\dot{\varepsilon}$ by the equation

$$\dot{\varepsilon} = -\frac{\sum_i\left(\underline{\underline{E}} - \underline{\underline{E}}_i^v\right):\frac{\partial \mathbb{C}_i^{hom}(\varepsilon)}{\partial \varepsilon}:\left(\dot{\underline{\underline{E}}} - \dot{\underline{\underline{E}}}_i^v\right)}{\frac{\partial \mathcal{F}_v}{\partial \varepsilon} + \frac{1}{2}\sum_i\left(\underline{\underline{E}} - \underline{\underline{E}}_i^v\right):\frac{\partial^2 \mathbb{C}_i^{hom}(\varepsilon)}{\partial \varepsilon^2}:\left(\underline{\underline{E}} - \underline{\underline{E}}_i^v\right)} \tag{16}$$

The evaluation of the damage function history, represented by $\varepsilon(t)$, is determined by the numerical integration of $\dot{\varepsilon}$ over time. The integration process developed in this paper may be simplified by adopting a constant Δt time step. At each time step j, all the variables ($\underline{\underline{E}}_j, \underline{\underline{\Sigma}}_j, \underline{\underline{E}}_{i,j}^v, \mathbb{C}_{i,j}^{hom}, \mathcal{F}_{v,j}, \mathcal{F}_j$,) are evaluated based on the value ε_{j-1} computed previously for damage parameter, thus allowing for a sequential determination of the rate of damage function $\dot{\varepsilon}_j$ over each time step. The variation of the damage function is obtained using $\Delta\varepsilon_j = \dot{\varepsilon}_j \cdot \Delta t$, which is added to the value of the damage function ε_{j-1} from the previous step to evaluate the current value of the damage function by $\varepsilon_j = \varepsilon_{j-1} + \Delta\varepsilon_j$. In this paper, the integration is explicitly performed (an implicit solution can be found in Aguiar [21]).

4 Numerical Applications

Arguments developed by Aguiar and Maghous [14] demonstrate that constant loadings applied in steps tend to mobilize elastic energy discontinuities at loading instants, which inevitably leads this instant to be the most critical to the propagation condition. This type of loading leads to a similar reasoning to elasticity. Alternatively, the application of loading at constant rates mobilizes an evolution of the free energy that can lead to horizontal asymptotes, depending on the rheological model of the constituents at the microscopic scale. As a consequence, the propagation condition is highly dependent on the loading rate. This effect can only be described through viscoelastic approaches. To exemplify the damage model, the strain history is defined by an isotropic extension rate defined by

$$\underline{\underline{E}} = \dot{E}tH(t)\underline{\underline{1}} \tag{17}$$

where $H(t)$ stands for the Heaviside function and $\dot{E}$ is the strain rate.

Following the procedure described in section 2, the macroscopic behavior can be determined based on the microscopic properties. For this application, the mechanical properties attributed to the solid matrix [14] are described by the Burgers model and conferred on the concrete. To the fractures [14], the mechanical behavior is described by the Maxwell model

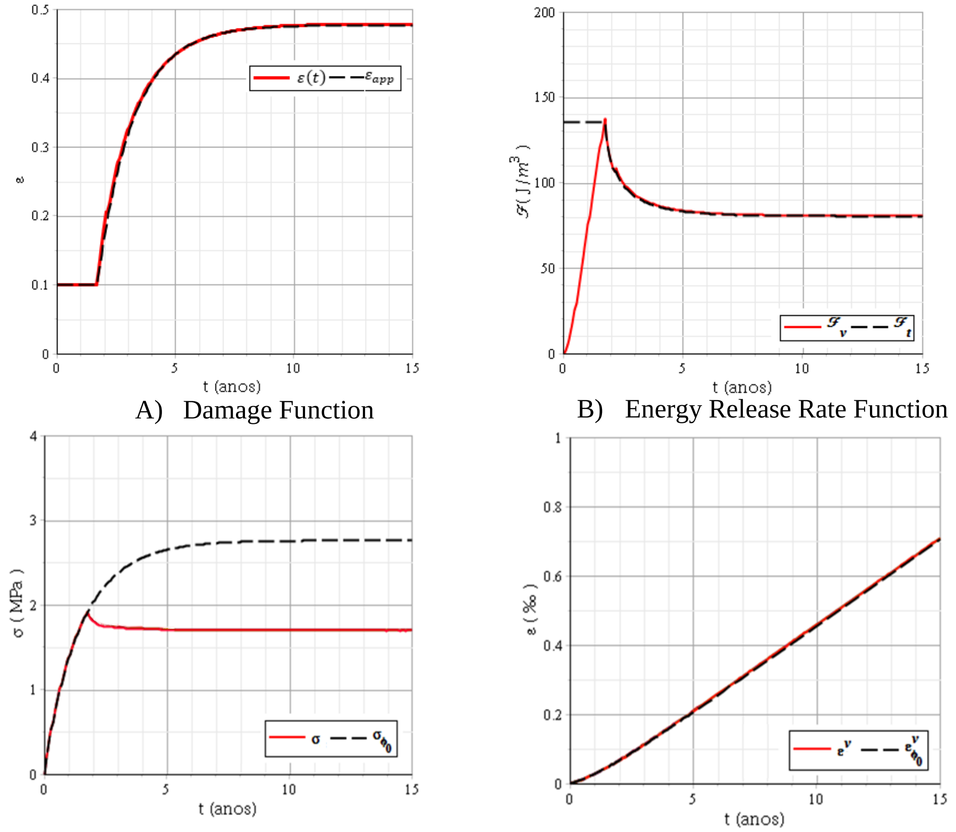
$$\begin{array}{c} k_{e,M}^s = 24.4 GPa \quad ; \quad k_{v,M}^s = 23.2 GPa.yr \quad ; \quad k_{e,K}^s = 39.3 GPa \quad ; \quad k_{v,K}^s = 1.6 GPa.yr \\ \mu_{e,M}^s = 13.3 GPa \quad ; \quad \mu_{v,M}^s = 12.3 GPa.yr \quad ; \quad \mu_{e,K}^s = 14.1 GPa \quad ; \quad \mu_{v,K}^s = 0.4 GPa.yr \\ k_n^e = 42.2 GPa/m \quad ; \quad k_n^v = 22.2 GPa.yr/m \quad ; \quad k_t^e = 16.9 GPa/m \quad ; \quad k_t^v = 8.9 GPa.yr/m \end{array} \tag{18}$$

where the index M and K stands for the Maxwell and Kelvin part from Burgers model, "e" and "v" corresponds to the spring and dashpot element (see Figure 7 of [14]).

Due to the hypothesis of isotropic evolution of the damage, the developed stress tensor maintains the isotropic form imposed on the load:

$$\underline{\underline{\Sigma}} = \Sigma(t)\underline{\underline{1}} \tag{19}$$

The integration of Eq. (16) can be directly performed within the concept described in section 3, obtaining the damage history and other associated variables



A) Damage Function
B) Energy Release Rate Function
C) Stress Function
D) Viscous Strain Function

Figure 3: Temporal evolution of damage, energy release rate, stress, and viscous strain functions.

$$\mathcal{F}_c = 30\, J/m^2, \mathcal{N} = 1, \dot{E} = 1‰$$

Based on Figure 3.A (evolution of the damage parameter) it is verified that the start of propagation does not occur at t=0, when the load starts to be applied. This can be understood based on Figure 3.B, where the energy release rate $\mathcal{F}$ has not yet reached the critical energy $\mathcal{F}_v$. Aguiar [21] showed that, for the conditions imposed in this problem, the evolution of the damage parameter follows an evolution described by the form

$$\varepsilon_{app}(t) = \varepsilon_\infty - (\varepsilon_\infty - \varepsilon_0)e^{-a(t-t_{ini})} \tag{20}$$

where the parameters ε_∞ (final damage parameter), a and t_{ini} (instant of propagation start) may be determined regardless of the damage propagation analysis (see [21]).

Figure 3.B shows the evolution of the energy release rate over time. From the moment that $\mathcal{F} = \mathcal{F}_v$ the propagation starts, verifying the reduction of the critical energy due to the increase of the damage (see Eq. (8)) and of the energy release rate, which follows the critical energy value. Stabilization at an asymptotic level is mainly due to the homogenized behavior and the damage evolution history.

Figure 3.C depicts the evolution of stresses in the material. Dashed line indicates the strain function with no damage evolution. This behavior under stress is experimentally expected when damage propagation begins, providing evidence that the model is consistent with observed phenomena. Note, however, that the model analyzed here corresponds to the viscoelasticity, not having plastic components that are frequent in ductile materials and, therefore, better representing the effect of propagation in fragile materials.

Lastly, according to Figure 3.D the damage propagation does not seem to affect significantly the viscous strains when compared to the dashed line, which indicates the stress function with no damage evolution. This behavior is one of the hypotheses raised about the state variables and has been numerically confirmed in this figure. This stems from the idea that the viscous and propagation dissipations can be disassociated.

5 Conclusion

The paper presented a formulation to the damage evolution resulting from the propagation of microfractures in viscoelastic materials, through an approach that coupled the constituent (solid matrix and fractures) micromechanical behavior with a macroscopic thermodynamic approach. The homogenized mechanical behavior was initially presented, having been developed through the combination between the Mori-Tanaka estimate and the Laplace-Carson correspondence principle. Based on the observation that this behavior can be rheologically represented through the generalized Maxwell model, a specific formulation for damage propagation at the macroscopic scale was developed. Such reasoning was developed based on energetic principles and consists of a simple formulation to deal numerically and analytically with deferred effects arising from the characteristics of viscoelastic materials. Subsequently, a non-linear viscoelastic formulation was introduced in the macroscopic behavior of the material, allowing the consideration of the damage evolution that was until then treated as constant. This approach allowed us to formulate, by analogy with plasticity, the rate of damage function. The numerical integration of the latter one allows to evaluate at each instant of time all the variables involved in the problem, establishing the history of the material state over the time of analysis. Applications under constant strain rates showed that the evolution of the damage function over time has a fundamentally asymptotic form, which may or may not verify the failure of the material, depending mainly on the applied loading levels.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] Singh, B. (1973) “Continuum characterization of jointed rock mass”. Int. J. Rock Mech Min Sci & Geomech Abstr. 1973;10:311-335.
[2] Løland, K. E. (1980). “Continuous damage model for load-response estimation of concrete”. Cement and Concrete Research, v.10(3), p. 395-402.
[3] Lemaitre, J. (1985). “A Continuous Damage Mechanics Model for Ductile Fracture”. Journal of Engineering
[4] Chaboche, J. L. (1992). ‘‘Damage induced anisotropy: On the difficulties associated with the active/passive unilateral condition’’, International Journal of Damage Mechanics, v.1, p. 148–171
[5] Kachanov, M. (1982). ‘‘A microcrack model of rock inelasticity–Part I: Frictional sliding on microcracks–Part II: Propagation of microcracks.’’ Mechanic of Materials, v.1, p.19–41.
[6] Pensée, V.; Kondo, D.; Dormieux, L. (2002). “Micromechanical Analysis of Anisotropic Damage in Brittle Materials”, Journal of Engineering Mechanics, v.128(8), p.889-897.
[7] Pavan, R.C., Creus, G.J., Maghous, S. (2009). “A simplified approach to continuous damage of composite materials and micromechanical analysis”, Composite Structures, v. 91, p. 84-94.
[8] Maghous, S., Dormieux, L., Kondo, D., Shao, J. F. (2011). “Micromechanics approach to poroelastic behavior of a jointed rock”, International Journal for Numerical and Analytical Methods in Geomechanics, 37, 111-129.
[9] Weitsman, Y. (1988). “A Continuum Damage Model for Viscoelastic Materials”, Journal of Applied Mechanics, v.55, p. 773-780.
[10] Lee, H.-J.; Kim, Y. R. (1998). “Viscoelastic Continuum Damage Model of Asphalt Concrete with Healing”, Journal of Engineering Mechanics, v.124(11), p. 1224-1232.
[11] Kutay, M.E.; Lanotte, M. (2017). “Viscoelastic continuum damage (VECD) models for cracking problems in asphalt mixtures”, International Journal of Pavement Engineering”, v.19(3), p.231-242.
[12] Nguyen, S. T., Dormieux, L., Le Pape, Y., Sanhuja, J. (2010). “Crack propagation in viscoelastic structures: Theoretical and numerical analyses.” Computational Materials Sciences, 50, 83–91.
[13] Nguyen, S. T., Dormieux, L. (2014). “Propagation of micro-cracks in viscoelastic materials: Analytical and numerical methods.” International Journal of Damage Mechanics, 24(4), 562–581.
[14] Aguiar, C. B., Maghous, S. (2018). “Micromechanical approach to effective viscoelastic properties of micro-fractured geomaterials”. International Journal for Numerical and Analytical Methods in Geomechanics, v. 42, i. 16, pp. 1-29.
[15] Goodman, R. E. (1976). Methods of geological engineering in discontinuous rocks. St Paul, Minnesota: West Publishers.
[16] Bandis, C. S.; Lumsden, A. C.; Barton, N. R. (1983). “Fundamentals of rock joint deformation”, International Journal of Rock Mechanics and Mining Science & Geomechanics Abstracts, v.20, p.249-268.
[17] Budiansky, B., O’Connell, R. J. (1976). “Elastic moduli of a cracked solid.” International Jounal of Solids Structures, 12, 81-97.
[18] Griffth, A. A. (1921), “The phenomenon of rupture and flow in solids.” Philosophical Transactions of the Royal Society of London. A 221, 163-198.
[19] Aguiar, C. B., Maghous, S. (2018). “A Simplified Micromechanics-based Approach to Damage Propagation Criterion in Viscoelastic Fractured Materials Regarded as Homogenized Media”. International Journal of Solid and Structures, (submitted for publication).
[20] Dormieux, L., Kondo, D. (2016). “Micromechanics of Fracture and Damage”. John Wiley & Sons, USA.
[21] Aguiar, C. B. (2021). “Micromechanical modeling of effective properties and damage propagation in fractured viscoelastic media”. PhD thesis, Federal University of Rio Grande do Sul, Brazil ‘in portugues’.
[22] Pavan,R.C.; Oliveira, B.F.; Maghous, S.; Creus, G.J. (2010). “A model for anisotropic viscoelastic damage in composites”. Composite Structures. v. 92, p. 1223-1228.
[23] Pavan,R.C.; Oliveira, B.F.; Creus, G.J. (2012). “Buckling analyses of viscoelastic structures considering ageing and damage effects”. Composite Structures. v. 94, p.1406-1412.