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Numerical evaluation of modal indices for damage identification in steel footbridges

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Abstract. Among several damage detection methodologies, the vibration-based damage identification methods (VBDI) are remarkable since the deterioration in structural elements reflects directly on both global and local dynamic response of the structure leading to changes in modal parameters. Different approaches focus mainly on one-dimensional structures, which, in turn, may not represent the actual dynamic behavior of bold structures such as bridges and footbridges. Some modal indices, namely mode shape curvature, modal flexibility, and modal strain energy were evaluated considering a finite element model of a real curved steel footbridge and three-dimensional vibration modes. Moreover, a recently proposed modal index called resultant vector which incorporate three-dimensional modal coordinates is also compared to the aforementioned indices. Results show that the accuracy of the indices is correlated with the region of the structure where damage is found. The influence of damage presence on adjacent girders and how it is reflected in these indices is also discussed.

Keywords: structural health monitoring, damage identification, damage index, footbridge, finite element model

1 Introduction

Structural health monitoring (SHM) represents an extensive and important field of research in engineering with varied applications. Especially non-destructive methods have gained an increasing importance given that structures can be monitored and inspected in service. In most cases, continuous monitoring allows damage detection at an early stage, preventing from potential accidents. Some of these methods are based on the vibration response of the structure, also known as vibration-based damage identification methods (VBDI). The fundamental idea of these methods is that damage-induced changes in the physical properties (mass, damping, and stiffness) will cause detectable changes in modal properties (natural frequencies, modal damping, and mode shapes) [1]. For this reason, several modal damage indices have been studied in the literature with respect to the different levels of damage diagnosis proposed by Rytter [2] like damage detection, damage localization, damage extension and remaining life service. On the other hand, several researches focus on linear structures since most structures can be simplified as a beam or plate-like structures [3,4-6]. However, real structures like bridges and footbridges with non-straight geometry may present three-dimensional mode shapes with torsional, lateral, and vertical behavior.

This work aims to apply some modal indices for damage detection and localization in a real footbridge with complex dynamic behavior in order to evaluate their accuracy to provide more reliable results for structural health monitoring. Moreover, the global influence of damage on a specific girder is examined regarding damage indices results for neighboring girders. Additionally, modal indices are evaluated with respect to damage position in the structure considering 3D modal coordinates with the recently proposed index called Resultant Vector (RV) [6].

2 Theoretical Background

It is know that the direct use of natural frequency and mode shape change as damage index can only roughly localize the damage [1]. However, as mode shapes contain local information of the structure, its derivative damage identification methods are often found in the literature and some of these methods are discussed in this section.

2.1 Mode Shape Curvature (MSC)

Pandey et al. [3] firstly proposed Mode Shape Curvature index (MSC) based on the Euler-Bernoulli beam theory that associates flexural stiffness of a beam cross-section to its curvature. As damage presence in the structure reduces locally its stiffness, the MSC change was proposed as sensitive parameter for damage detection. Curvature at a point can be obtained by eq. (1) using central difference approximation

$$\varphi_i'' = \frac{\varphi_{i+1} - 2\varphi_i + \varphi_{i-1}}{h^2} \tag{1}$$

In which φ_i'' is the curvature at node i, φ_{i+1} and φ_{i-1} are the displacement mode shapes of its forward and backward nodes and h is the length of the elements. The maximum absolute value of MSC variation indicates damaged element and the contribution of all the analyzed mode shapes can be considered as represented by eq. (2) where N is the number of mode shapes analyzed.

$$\Delta\varphi_i'' = \frac{1}{N}\sum_{j=1}^{N} | \varphi_i^{*''} - \varphi_i''| \tag{2}$$

Where $\varphi_i^{*''}$ is the mode shape curvature of the damaged case and φ_i'' is the mode shape curvature of the intact curvature of the structure at node i.

2.2 Modal Flexibility (MF)

As damage presence affects structural stiffness, changes in flexibility matrix of the structure were studied by Pandey and Biswas [4] as a sensitive parameter for damage identification. From the mass normalized mode shapes, modal flexibility can be calculated by the eq. (3)

$$\boldsymbol{F} = \sum_{j=1}^{N} \frac{1}{\omega_j^2}\, \boldsymbol{\varphi_i}\, \boldsymbol{\varphi_i^T} \tag{3}$$

In which F is the modal flexibility, ω_j is the natural frequency for mode j, φ_i is the mass normalized mode shape of the structure, and N is the number of the analyzed mode shapes. If a damage reduces the stiffness of a subregion of a structure, flexibility, in turn, as the inverse of stiffness, increases at this location. Moreover, according to eq. (3), the flexibility of a structure converges rapidly with the increasing values of frequency. For that reason, few mode shapes can provide a good estimate of the flexibility matrix. Damage localization is obtained identifying the nodes of the structure where the absolute variation of modal flexibility is maximum.

2.3 Modal Strain Energy (MSE)

MSE-based methods are widely used due to its sensitive characteristics to damages. Using changes in strain energy of a subregion of a structure as damage index was first proposed by Stubbs et al. [5]. This method relies on the fact that damage changes the strain energy of the damaged element, while the strain energy of other elements remains practically constant after damage occurrence. MSE can be calculated by eq. (4),

$$\beta_j = \frac{\sum_{i=1}^{N}\left(\int_j [\varphi_i^{*''}(x)^2]\, dx + \int_0^L [\varphi_i^{*''}(x)^2]\, dx\right)\int_0^L [\varphi_i''(x)^2]\, dx}{\sum_{i=1}^{N}\left(\int_j [\varphi_i''(x)^2]\, dx + \int_0^L [\varphi_i''(x)^2]\, dx\right)\int_0^L [\varphi_i^{*''}(x)^2]\, dx} \tag{4}$$

In which β_j is the ratio between fractional MSE of the element j and L is the beam element length. The final MSE damage index is normalized as shown in eq. (5),

$$Z_j = (\beta_j - \overline{\beta}) \,/\, \sigma_\beta \tag{5}$$

Where Z_j is the normalized index, $\overline{\beta}$ is the mean and σ_β the standard deviation.

2.4 Resultant Vector (RV)

Recently, a new damage index was proposed by Gonçalves [6] taking into account the three-dimensional dynamic behavior of the structure. RV incorporates on a single index the variation of the three spatial mode shapes coordinates considering that real structures like footbridges may exhibit three-dimensional mode shapes. RV index can be calculated following eq. (6)

$$\boldsymbol{VR'}_{i,j} = \sqrt{\boldsymbol{\varphi}^2_{x,i,j} + \boldsymbol{\varphi}^2_{y,i,j} + \boldsymbol{\varphi}^2_{z,i,j}} \tag{6}$$

Where $\boldsymbol{VR'}_{i,j}$ is RV index for node i and mode j, $\boldsymbol{\varphi}_{x,i,j}$, $\boldsymbol{\varphi}_{y,i,j}$ and $\boldsymbol{\varphi}_{z,i,j}$ are the mode shape displacements in the x, y, z-axes, respectively. Damage localization can be determined obtaining the maximum absolute of the variation between damaged and intact cases of the index as shown by eq. (7) for all N considered modes.

$$\Delta\boldsymbol{VR'}_{i,j} = \sum_{j=1}^{N} |\, \boldsymbol{VR'}^{*}_{i,j} - \boldsymbol{VR'}_{i,j} \,| \tag{7}$$

3 Numerical Application

The aforementioned damage indices are easily found in the literature applied to beam or plate-like structures that may not represent the three-dimensional dynamic behavior of real structures. In order to study the accuracy of theses indices for more complex structures, they are evaluated in the present section. A finite element model developed in Robot Structure Analysis of real footbridge localized in Rio de Janeiro, Brazil, was used to study these damage indices (Fig. 1). Its three girders are made of steel circular cross section that were modeled as 3D beam elements whereas its deck made of a 50-mm concrete layer over a 5-mm steel plate was modeled with single shell elements considering an equivalent cross section area. Footbridge columns bearings were adopted as fixed ended. As the model considered some of most flexible spans of the full footbridge, longitudinal displacements were restricted at the extremities of the model, and the remaining displacements and rotations were not constrained.

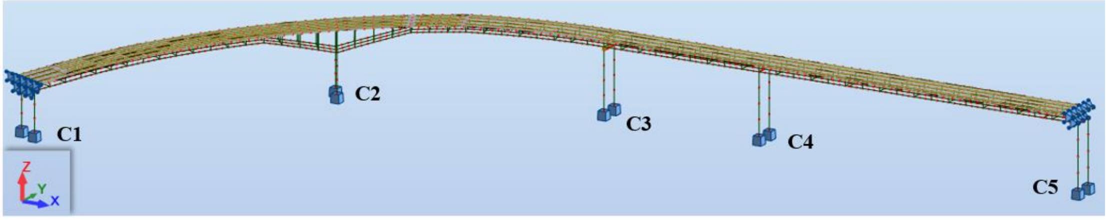
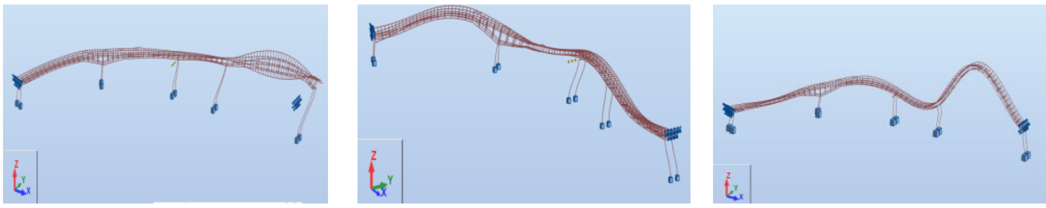


Figure 1. Footbridge Finite Element Model developed in Robot Structural Analysis

This structure was previously studied due to its remarkable flexibility and susceptibility to walking-induced vibration [7] and for an application of artificial neural network to damage identification process using RV index [6]. The footbridge mode shapes have three-dimensional components showing torsional, lateral, and vertical bending modes shapes combined (Tab. 1). Figure 2 illustrates some of the mode shapes among the 8 considered.

Table 1. Natural Frequency of the 8 mode shapes analyzed and its behavior

Mode	Natural Frequency (Hz)	Mode Shape Feature
1	2,56	Lateral bending + torsion
2	3,81	Lateral bending + vertical bending
3	4,14	Vertical bending
4	4,42	Torsion + Vertical bending
5	5,68	Torsion
6	6,67	Vertical bending + Torsion
7	7,01	Torsion
8	8,45	Torsion + Lateral Bending



(a) (b) (c)

Figure 2. Some of the footbridge mode shapes considered in the analysis: (a) mode 1, (b) mode 2 and (c) mode 3.

This work also aims to evaluate the capacity of each index to detect in which footbridge girder the damage is localized since damage location is not known previously in real monitoring cases. Footbridge girders are identified as Girder 1 (G1) referring to the girder on internal girder of the curved footbridge while G2 and G3 refers to the central girder and external girder, respectively (Fig. 3). Each girder beam elements have around 0.50 meters length.

Moreover, damages were introduced in the model by reducing cross-section thickness of beam elements by 50% to simulate the loss of cross-sectional area as the result of corrosion of the steel girders. Two cases were simulated considering rigid (nearby columns) and flexible (mid-span) regions of the structure to evaluate the capacity of each index to detect real damages at the simulated positions. For both damage cases, 8 mode shapes were considered in the analysis.

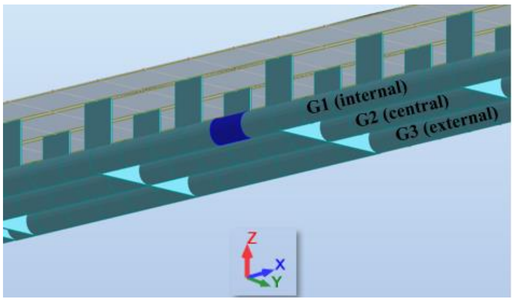
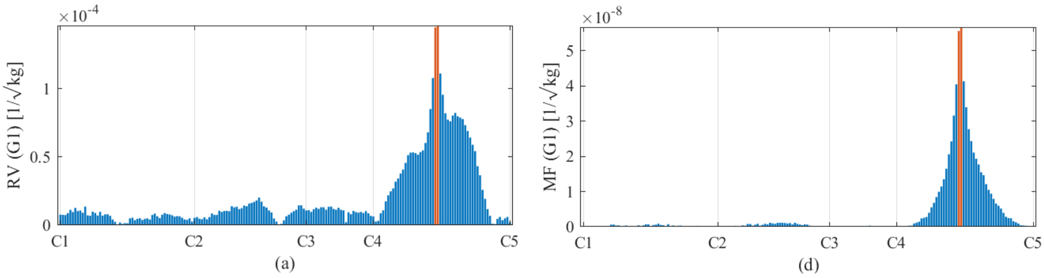


Figure 3. Footbridge bottom view

For the first damage case, damage was introduced in the model in a mid-span element between columns C4 and C5 on girder G1. Expected damage identification is highlighted with orange color bars. Longitudinal dimensions of each girder are represented in horizontal axes identifying footbridges supports by C1, C2, C3, C4 and C5 columns as displayed in the Fig. 1.

Figure 4 shows the results of RV and MF indices for the three footbridge girders. In this case that damage was simulated at a flexible part of the structure, RV index clearly indicates the location of simulated damage in the girder as well as which girder is damaged (Fig. 3a). G2 (fig. 3b) and G3 (fig. 3c) girders are influenced by damage presence in the structure, but the highest values of RV changes are found in the damaged girder at the nodes of the damaged element. MF index exhibits the same pattern of results identifying damaged girder (fig. 3d) and presenting lower peaks on the same region of the structure in G2 (Fig. 3e) and G3 (Fig. 3f) girders. Considering both indices, regardless of the girder, the span with higher peaks indicates clearly which region of the structure requires more attention in maintenance.



(a) (d)

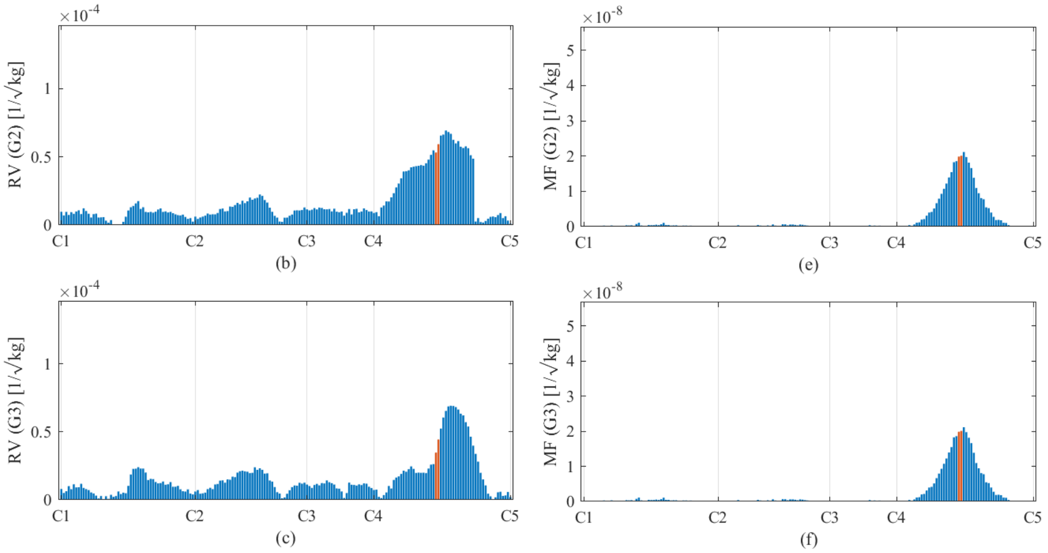


Figure 4. Case 1 results for RV index considering (a) G1, (b) G2, (c) G3 girders, and for MF index for (d) G1, (e) G2, (f) G3 girders. The five footbridge columns are identified as C1, C2, C3, C4 and C5.

Results for MSC and MSE indices for damage case 1 can be found in the Fig. 5. The highest peak in MSC graph correctly identifies the presence as well as the position of damage in the structure (Fig. 5a). Curvature changes in the adjacent girders are not so expressive as in the damaged one as can be seen in Fig. 5b and Fig. 5c. On the other hand, MSE index was not capable of detecting neither the position or the damaged girder in this case. One possible reason is that changes in strain energy in the curved spans of this footbridge were more relevant (see Fig. 5d, e and f). The number of considered mode shapes could also be an important factor.

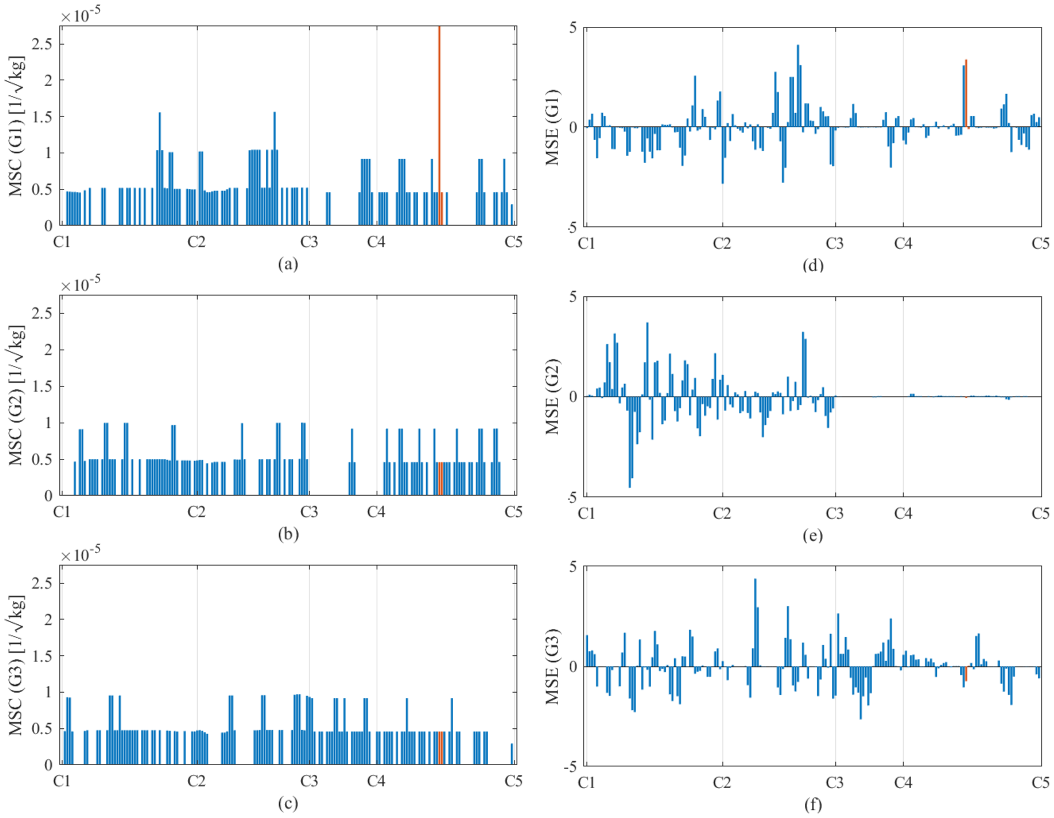


Figure 5. Case 1 results for MSC index considering (a) G1, (b) G2, (c) G3 girders, and for MSE index for (d) G1, (e) G2, (f) G3 girders. The five footbridge columns are identified as C1, C2, C3, C4 and C5.

Damage indices were also studied at rigid regions nearby columns since modal displacements are smaller at the structure bearings. Damage case 2 simulated a damaged beam element close to column C3 in the external girder G3. Results are shown in the Fig. 6.

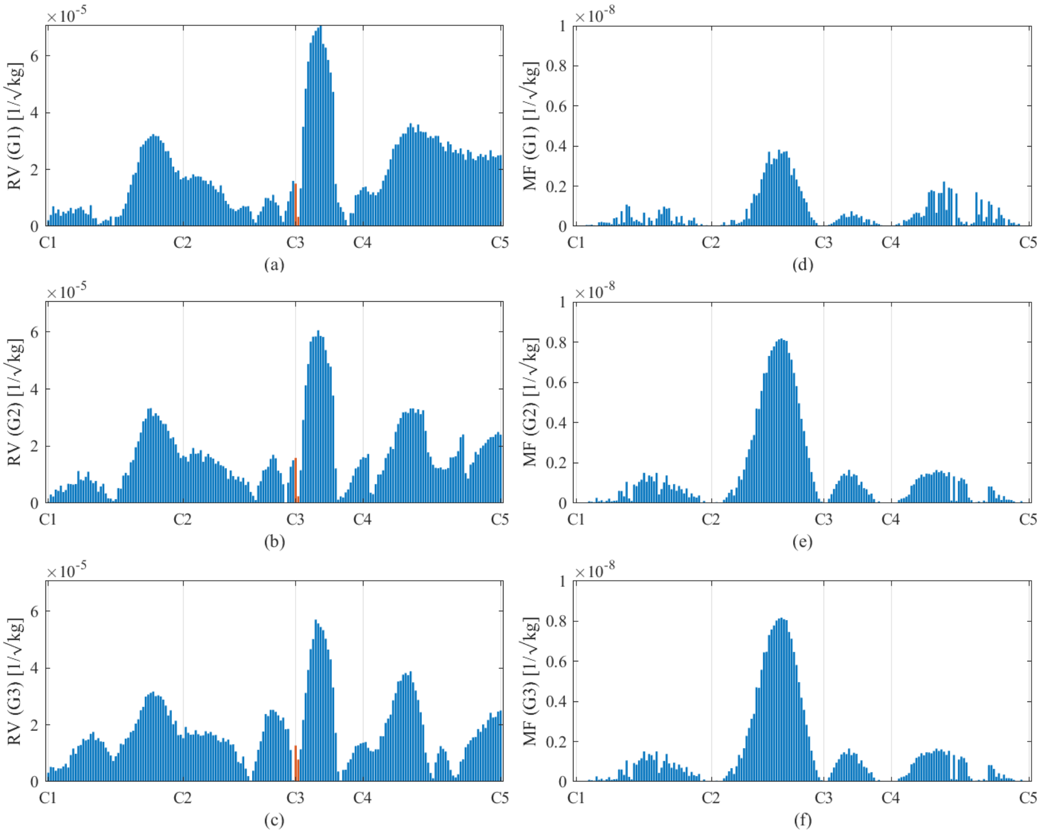
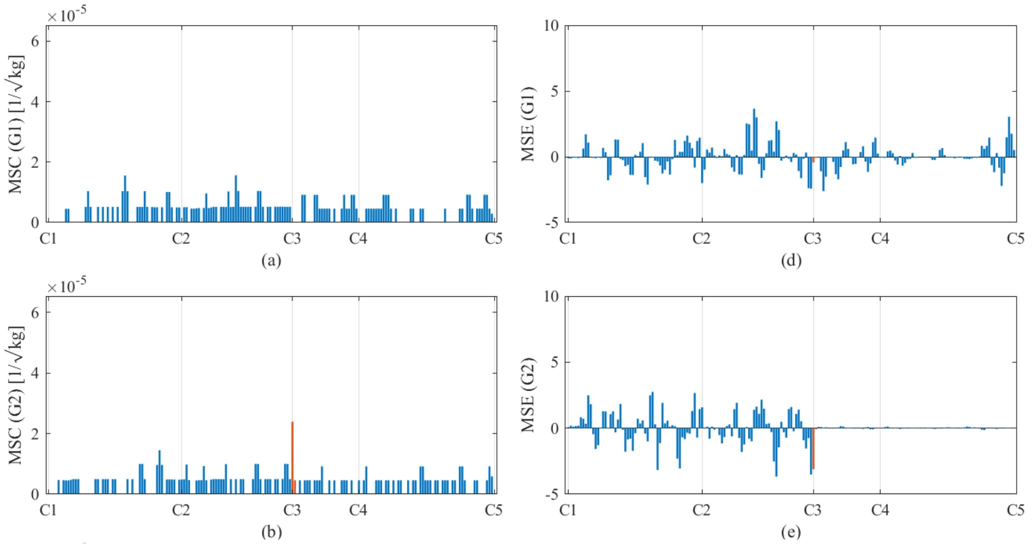


Figure 6. Case 2 results for RV index for girders (a) G1, (b) G2, (c) G3 girders and results for MF index for (d) G1, (e) G2 and (f) G3 girders. The five footbridge columns are identified as C1, C2, C3, C4 and C5.

For rigid regions, RV maximum values detect the presence of damage in the span where it is found, although exhibit a bad performance in terms of precise damage identification. A reason for this behavior is the low contribution of z-modal coordinate that is too small at columns vicinity. Z-modal component has a relevant contribution for all the considered mode shapes and for both damage indices.



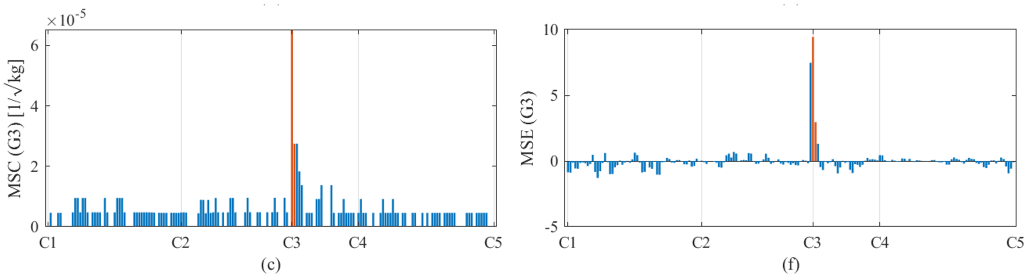


Figure 7. Case 2 results for MSC index for (a) G1, (b) G2, (c) G3 girders, and results for MSE index for (d) G1, (e) G2 and (f) G3 girders. The five footbridge columns are identified as C1, C2, C3, C4 and C5.

Nevertheless, MSC and MSE indices have a better performance in damage detection for this case (Fig. 7). Changes in MSC correctly identified the position and the exact girder where damage was simulated (Fig. 7c). Similarly, an expressive change in MSE index was found for the damaged element and girder (Fig. 7f) identifying damage occurrence. However, non-damaged girders' results for these indices are less conclusive to indicate at least the damaged region of the structure in this case.

4 Conclusions

VBDI methods are valuable tools for damage detection in order to prevent from structural failures. Damage indices must consider all the inherent variables like geometry, dynamic behavior, damage position in structure, etc. This work evaluated the accuracy of some modal indices applied to a real structure with complex geometry and three-dimensional mode shapes. It is shown that some damage indices, namely Resultant Vector (RV) and Modal Flexibility (MF) index, are capable of identifying damage localization correctly in flexible regions whereas the same indices do not work as well for rigid locations of the structure. Mode Shape Curvature (MSC) index proved to be a very good index for both cases presented in this work. Changes in Modal Strain Energy (MSE) were not enough to detect a damage simulated at mid-span element. However, it cannot be rejected since it may work better in other positions of structure like nearby columns. Identifying the impact of damage occurrence in the adjacent girders and how it is reflected in each damage index is also a very important aspect of structural health monitoring since it allows developing an effective monitoring and maintenance program for real structures, optimizing sensor location on the structure and targeting maintenance programs correctly.

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