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NUMERICAL MODELING OF CONVENTIONAL STEEL, STAINLESS STEEL AND INCONEL ALLOY SOLID ELEMENTS THROUGH THE FINITE ELEMENT METHOD

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Abstract. The work will present the development and implementation of linear numerical models, with the objective of performing mechanical analyzes on solid elements of conventional steel (MS250 and HS350), stainless steel (SS304) and inconel alloy (IA718). For this, we intend to implement computationally, with the aid of the FORTRAN programming language, a mathematical formulation based on the Finite Element Method (FEM), whose purpose will be to obtain the values of deformations and nodal displacements at different points of the parts, in addition to the von Mises stresses in each finite element that compose the structures. For the discretization of solid elements, 4-node tetrahedral (T4) and 8-node hexahedral (H8) finite elements were used. For a greater scope of validation of the implemented module and to prove its efficiency, solids with different geometric and physical characteristics will be analyzed. In order to verify the responses obtained from the implemented computational program, comparisons will be made with results found in the literature.

Keywords: Finite Element Method, Numerical Analysis, Mechanical Analysis, Fortran.

1 Introduction

According to Bath [1], finite element procedures are currently widely used in engineering analysis, in which the procedures are widely used in the analysis of solids and structures, given that, in fact, finite element methods are useful in virtually every field of engineering analysis, and that their use for solving engineering problems began with the advent of the digital computer.

Among the researches developed in the area, we can mention those of the authors Almeida [2] and Maciel [3], since they approached the development of a computer program for the numerical analysis of three-dimensional problems using the finite element method, which obtained satisfactory results in their implementations.

Therefore, this research aims to carry out the study of numerical (mechanical) analyzes of solid elements related to conventional steels, of medium mechanical strength (MS250) and high mechanical strength (HS350); stainless steel (SS304) and inconel alloy (IA718). These analyzes will be carried out through the implementation of a computer program in Fortran language (Chapman [4]), based on the Finite Element Method (FEM), since the 4-node tetrahedral finite element (T4) and the 8-node hexahedral finite element (H8) for problem modeling.

Therefore, the aim of this research is to obtain the values of stresses, strains and displacements. With the purpose of validating the implemented computational program, comparisons of the results will be made with those found in the literature.

2 Formulations

Fig. 1, shows the metre element of the 4-node tetrahedral finite element and the 8-node hexahedral element, respectively.

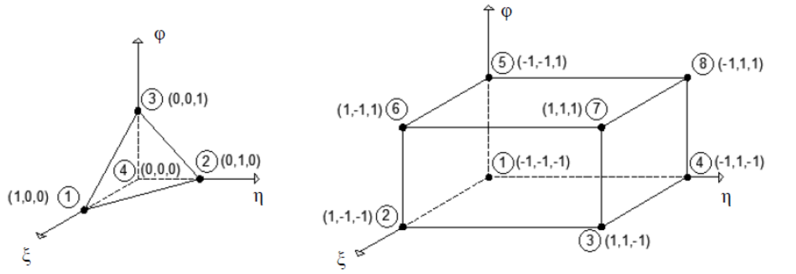

Figure 1. T4 master element

Next, the formulations of the finite elements studied in this research are presented.

2.1 4-node Tetrahedral Finite Element (T4)

The shape functions corresponding to this element are represented by eq. (1) and eq. (2).

$$N_1 = \xi \qquad N_2 = \eta \qquad N_3 = \varphi \tag{1}$$

$$N_1 + N_2 + N_3 + N_4 = 1 \tag{2}$$

For the displacement vector, we have eq. (3).

$$q = [q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5 \quad q_6 \quad q_7 \quad q_8 \quad q_9 \quad q_{10} \quad q_{11} \quad q_{12}] \tag{3}$$

As the relationship between the displacement field vector and the nodal displacement vector, we have:

$$\mathbf{u} = \mathbf{N}\,\mathbf{q} \tag{4}$$

Where N is the matrix that represents the shape functions, given by eq. (5).

$$\mathbf{N} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 \end{bmatrix} \tag{5}$$

Then, with the help of eq. (4) and eq. (5), it is possible to conclude that:

$$u = N_1 q_1 + N_2 q_4 + N_3 q_7 + N_4 q_{10} \tag{6a}$$

$$v = N_1 q_2 + N_2 q_5 + N_3 q_8 + N_4 q_{11} \tag{6b}$$

$$w = N_1 q_3 + N_2 q_6 + N_3 q_9 + N_4 q_{12} \tag{6c}$$

Since the function u depends on x, y and z, and these depend on the natural coordinate's ξ, η and φ, then the function u is also dependent on ξ, η and φ. However, there has been:

$$\begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial u}{\partial \varphi} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \varphi} & \frac{\partial y}{\partial \varphi} & \frac{\partial z}{\partial \varphi} \end{bmatrix} \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial z} \end{Bmatrix} \tag{7}$$

Since the Jacobian matrix is given by eq. (8):

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \varphi} & \frac{\partial y}{\partial \varphi} & \frac{\partial z}{\partial \varphi} \end{bmatrix} \quad (8)$$

Considering that matrix A is the inverse matrix of the Jacobian matrix, one arrives at:

$$\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \end{bmatrix}^T = \mathbf{A} \begin{bmatrix} \frac{\partial u}{\partial \xi} & \frac{\partial u}{\partial \eta} & \frac{\partial u}{\partial \varphi} \end{bmatrix}^T \quad (9)$$

It follows that the relationship between the strain vector and the displacement vector is given by eq. (10):

$$\boldsymbol{\varepsilon} = \mathbf{B}\,\mathbf{q} \quad (10)$$

Knowing that the strain vector is defined by eq. (11):

$$\boldsymbol{\varepsilon} = [\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z \quad \gamma_{zy} \quad \gamma_{zx} \quad \gamma_{yx}]^T \quad (11)$$

After some mathematical manipulations it is possible to conclude that matrix B is equal to:

$$\mathbf{B} = \begin{bmatrix} A_{11} & 0 & 0 & A_{12} & 0 & 0 & A_{13} & 0 & 0 & -\tilde{A}_1 & 0 & 0 \\ 0 & A_{21} & 0 & 0 & A_{22} & 0 & 0 & A_{23} & 0 & 0 & -\tilde{A}_2 & 0 \\ 0 & 0 & A_{31} & 0 & 0 & A_{32} & 0 & 0 & A_{33} & 0 & 0 & -\tilde{A}_3 \\ 0 & A_{31} & A_{21} & 0 & A_{12} & A_{22} & 0 & A_{33} & A_{23} & 0 & -\tilde{A}_3 & -\tilde{A}_2 \\ A_{31} & 0 & A_{11} & A_{32} & 0 & A_{12} & A_{33} & 0 & A_{13} & -\tilde{A}_3 & 0 & -\tilde{A}_1 \\ A_{21} & A_{11} & 0 & A_{22} & A_{12} & 0 & A_{23} & A_{13} & 0 & -\tilde{A}_2 & -\tilde{A}_1 & 0 \end{bmatrix} \quad (12)$$

Given that:

$$-\tilde{A}_1 = [A_{11} + A_{12} + A_{13}] \quad (13a)$$

$$-\tilde{A}_2 = [A_{21} + A_{22} + A_{23}] \quad (13b)$$

$$-\tilde{A}_3 = [A_{31} + A_{32} + A_{33}] \quad (13c)$$

The stiffness of the element can be obtained based on the internal strain energy equation, given by eq. (14):

$$U_e = \frac{1}{2}\,\mathbf{q}^T\,\mathbf{B}^T\,\mathbf{D}\,\mathbf{B}\,\mathbf{q} \int_e dV \quad (14)$$

The calculation of the volume of a tetrahedral, via FEM, is defined by eq. (18):

$$V_e = (\det J) \int_0^1 \int_0^{1-\xi} \int_0^{1-\xi-\eta} d\varphi\, d\eta\, d\xi \quad (16)$$

2.2 8-node hexahedral finite element (H8)

The Lagrangian shape functions are represented by eq. (17):

$$N_i = \frac{1}{8}\,(1+\xi_i\,\xi)(1+\eta_i\,\eta)(1+\varphi_i\,\varphi) \quad (17)$$

In turn, the nodal displacements will be represented by the vector:

$$\mathbf{q} = [q_1, q_2, q_3, \ldots, q_{23}, q_{24}]^T \quad (18)$$

The element stiffness matrix corresponding to the 8-node hexahedral finite element is defined by:

$$\mathbf{k}^e = \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 \mathbf{B}^T\,\mathbf{D}\,\mathbf{B}\,|\det J|\, d\varphi\, d\eta\, d\xi \quad (19)$$

J being the Jacobian matrix with dimension (3x3) and remembering that the integrals will be solved numerically with the aid of the Gauss-Legendre Method (Gauss Quadrature).

The relationships between the partial derivatives of the displacements can be represented in a matrix form, as:

$$\begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial u}{\partial \varphi} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \varphi} & \frac{\partial y}{\partial \varphi} & \frac{\partial z}{\partial \varphi} \end{bmatrix} \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial z} \end{Bmatrix} \tag{20}$$

Considering that the gamma matrix is the inverse matrix of the Jacobian matrix, we have:

$$\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \end{bmatrix}^T = \mathbf{\Gamma} \begin{bmatrix} \frac{\partial u}{\partial \xi} & \frac{\partial u}{\partial \eta} & \frac{\partial u}{\partial \varphi} \end{bmatrix}^T \tag{21}$$

The matrix B corresponding to the 8-node hexahedral finite element is represented by eq. (22):

$$\mathbf{B} = \mathbf{H}\ \mathbf{\Gamma_u}\ \mathbf{DN} \tag{22}$$

From eq. (23) it is possible to conclude that:

$$\begin{bmatrix} \frac{\partial u}{\partial \xi} & \frac{\partial u}{\partial \eta} & \frac{\partial u}{\partial \varphi} & \frac{\partial v}{\partial \xi} & \frac{\partial v}{\partial \eta} & \frac{\partial v}{\partial \varphi} & \frac{\partial w}{\partial \xi} & \frac{\partial w}{\partial \eta} & \frac{\partial w}{\partial \varphi} \end{bmatrix}^T = \mathbf{DN\ q} \tag{23}$$

since the **DN** matrix has 9 lines and 24 columns and will be organized with the use of sub-matrices proposed in the present work and presented as:

$$\mathbf{DN1} = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & 0 & 0 & \frac{\partial N_2}{\partial \xi} & 0 & 0 \\ \frac{\partial N_1}{\partial \eta} & 0 & 0 & \frac{\partial N_2}{\partial \eta} & 0 & 0 \\ \frac{\partial N_1}{\partial \varphi} & 0 & 0 & \frac{\partial N_2}{\partial \varphi} & 0 & 0 \end{bmatrix} \mathbf{DN2} = \begin{bmatrix} \frac{\partial N_3}{\partial \xi} & 0 & 0 & \frac{\partial N_4}{\partial \xi} & 0 & 0 \\ \frac{\partial N_3}{\partial \eta} & 0 & 0 & \frac{\partial N_4}{\partial \eta} & 0 & 0 \\ \frac{\partial N_3}{\partial \varphi} & 0 & 0 & \frac{\partial N_4}{\partial \varphi} & 0 & 0 \end{bmatrix} \mathbf{DN3} = \begin{bmatrix} \frac{\partial N_5}{\partial \xi} & 0 & 0 & \frac{\partial N_6}{\partial \xi} & 0 & 0 \\ \frac{\partial N_5}{\partial \eta} & 0 & 0 & \frac{\partial N_6}{\partial \eta} & 0 & 0 \\ \frac{\partial N_5}{\partial \varphi} & 0 & 0 & \frac{\partial N_6}{\partial \varphi} & 0 & 0 \end{bmatrix} \tag{24.a}$$

$$\mathbf{DN4} = \begin{bmatrix} \frac{\partial N_7}{\partial \xi} & 0 & 0 & \frac{\partial N_8}{\partial \xi} & 0 & 0 \\ \frac{\partial N_7}{\partial \eta} & 0 & 0 & \frac{\partial N_8}{\partial \eta} & 0 & 0 \\ \frac{\partial N_7}{\partial \varphi} & 0 & 0 & \frac{\partial N_8}{\partial \varphi} & 0 & 0 \end{bmatrix} \mathbf{DN5} = \begin{bmatrix} 0 & \frac{\partial N_1}{\partial \xi} & 0 & 0 & \frac{\partial N_2}{\partial \xi} & 0 \\ 0 & \frac{\partial N_1}{\partial \eta} & 0 & 0 & \frac{\partial N_2}{\partial \eta} & 0 \\ 0 & \frac{\partial N_1}{\partial \varphi} & 0 & 0 & \frac{\partial N_2}{\partial \varphi} & 0 \end{bmatrix} \mathbf{DN6} = \begin{bmatrix} 0 & \frac{\partial N_3}{\partial \xi} & 0 & 0 & \frac{\partial N_4}{\partial \xi} & 0 \\ 0 & \frac{\partial N_3}{\partial \eta} & 0 & 0 & \frac{\partial N_4}{\partial \eta} & 0 \\ 0 & \frac{\partial N_3}{\partial \varphi} & 0 & 0 & \frac{\partial N_4}{\partial \varphi} & 0 \end{bmatrix} \tag{24.b}$$

$$\mathbf{DN7} = \begin{bmatrix} 0 & \frac{\partial N_5}{\partial \xi} & 0 & 0 & \frac{\partial N_6}{\partial \xi} & 0 \\ 0 & \frac{\partial N_5}{\partial \eta} & 0 & 0 & \frac{\partial N_6}{\partial \eta} & 0 \\ 0 & \frac{\partial N_5}{\partial \varphi} & 0 & 0 & \frac{\partial N_6}{\partial \varphi} & 0 \end{bmatrix} \mathbf{DN8} = \begin{bmatrix} 0 & \frac{\partial N_7}{\partial \xi} & 0 & 0 & \frac{\partial N_8}{\partial \xi} & 0 \\ 0 & \frac{\partial N_7}{\partial \eta} & 0 & 0 & \frac{\partial N_8}{\partial \eta} & 0 \\ 0 & \frac{\partial N_7}{\partial \varphi} & 0 & 0 & \frac{\partial N_1}{\partial \varphi} & 0 \end{bmatrix} \mathbf{DN9} = \begin{bmatrix} 0 & 0 & \frac{\partial N_1}{\partial \xi} & 0 & 0 & \frac{\partial N_2}{\partial \xi} \\ 0 & 0 & \frac{\partial N_1}{\partial \eta} & 0 & 0 & \frac{\partial N_2}{\partial \eta} \\ 0 & 0 & \frac{\partial N_1}{\partial \varphi} & 0 & 0 & \frac{\partial N_2}{\partial \varphi} \end{bmatrix} \tag{24.c}$$

$$\mathbf{DN10} = \begin{bmatrix} 0 & 0 & \frac{\partial N_3}{\partial \xi} & 0 & 0 & \frac{\partial N_4}{\partial \xi} \\ 0 & 0 & \frac{\partial N_3}{\partial \eta} & 0 & 0 & \frac{\partial N_4}{\partial \eta} \\ 0 & 0 & \frac{\partial N_3}{\partial \varphi} & 0 & 0 & \frac{\partial N_4}{\partial \varphi} \end{bmatrix} \mathbf{DN11} = \begin{bmatrix} 0 & 0 & \frac{\partial N_5}{\partial \xi} & 0 & 0 & \frac{\partial N_6}{\partial \xi} \\ 0 & 0 & \frac{\partial N_5}{\partial \eta} & 0 & 0 & \frac{\partial N_6}{\partial \eta} \\ 0 & 0 & \frac{\partial N_5}{\partial \varphi} & 0 & 0 & \frac{\partial N_6}{\partial \varphi} \end{bmatrix} \mathbf{DN12} = \begin{bmatrix} 0 & 0 & \frac{\partial N_7}{\partial \xi} & 0 & 0 & \frac{\partial N_8}{\partial \xi} \\ 0 & 0 & \frac{\partial N_7}{\partial \eta} & 0 & 0 & \frac{\partial N_8}{\partial \eta} \\ 0 & 0 & \frac{\partial N_7}{\partial \varphi} & 0 & 0 & \frac{\partial N_8}{\partial \varphi} \end{bmatrix} \tag{24.d}$$

or even in a compact form as:

$$\mathbf{DN} = \left[\begin{array}{c:c:c:c} \mathbf{DN1} & \mathbf{DN2} & \mathbf{DN3} & \mathbf{DN4} \\ \hdashline \mathbf{DN5} & \mathbf{DN6} & \mathbf{DN7} & \mathbf{DN8} \\ \hdashline \mathbf{DN9} & \mathbf{DN10} & \mathbf{DN11} & \mathbf{DN12} \end{array}\right] \tag{25}$$

Based on the equation referring to the strain vector, it is possible to conclude that:

$$\boldsymbol{\varepsilon} = \mathbf{H} \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} & \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix}^T \tag{25}$$

where the matrix H will be expressed by eq. (26):

$$\mathbf{H}=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0\end{bmatrix} \tag{26}$$

Hence, from Eq. (27):

$$\begin{bmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} & \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z}\end{bmatrix}^T = \mathbf{\Gamma}_u \begin{bmatrix}\frac{\partial u}{\partial \xi} & \frac{\partial u}{\partial \eta} & \frac{\partial u}{\partial \varphi} & \frac{\partial v}{\partial \xi} & \frac{\partial v}{\partial \eta} & \frac{\partial v}{\partial \varphi} & \frac{\partial w}{\partial \xi} & \frac{\partial w}{\partial \eta} & \frac{\partial w}{\partial \varphi}\end{bmatrix}^T \tag{27}$$

It is possible to arrive at $\mathbf{\Gamma}$u, defined by the relation that follows:

$$\mathbf{\Gamma}_u=\begin{bmatrix}\Gamma(\xi,\eta,\varphi) & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \Gamma(\xi,\eta,\varphi) & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \Gamma(\xi,\eta,\varphi)\end{bmatrix} \tag{28}$$

Since the matrix $\Gamma(\xi,\eta,\varphi)$ has 3 rows and 3 columns.

2.3 Calculation of von Mises Stress

For elements that are subject to the plane stress state, it is possible to inform that the von Mises stress is represented by the following relation:

$$\sigma_{VM}=\sqrt{\sigma_x{}^2+\sigma_y{}^2-\sigma_x\sigma_y+3\tau_{xy}{}^2} \tag{29}$$

3 Results and discussions

For the validation of the implemented computer program, 02 examples will be presented. For the first example, the solid was modeled by the 4-node tetrahedral finite element, while in the second example, the solid was modeled with the 8-node hexahedral finite element.

For the numerical analyses, the mechanical properties were used: for conventional steels of medium and high mechanical strength, the Modulus of Elasticity (E) of 200 GPa and the Poisson's Ratio (υ) of 0.3 were used; for stainless steel, the Modulus of Elasticity (E) of 193 GPa and the Poisson's Ratio (υ) of 0.27 and for the inconel alloy, the Modulus of Elasticity (E) of 206 GPa and the Poisson's Ratio (υ) of 0.28.

For the yield stress of the materials, the following values were used: 250 MPa and 350 MPa for conventional steels of medium and high mechanical strength, respectively; 215 MPa for stainless steel and 820 MPa for inconel alloy.

3.1 Examplo 1

The example shown in Fig. 2, refers to modeling the solid that has been discretized into just a single 4-node tetrahedral finite element.

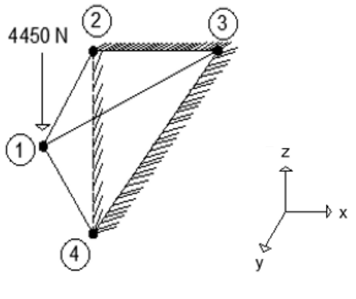

Figure 2 – Examplo 1

The coordinates of the nodes, given in millimeters (mm), were defined as: 1 (0,25,25) 2 (0,0,25) 3 (25,0,25) 4 (0,0,0)

The Tab. 1 shows the comparison of the result obtained by the research and that found in the literature, in addition to the results obtained with the solids under study:

Table 1 – Nodal displacement

Nodes	Displacement in z (mm)						
	Present work	Chandrupatla e Belegundu [5]	Present work / Literature	MS250	HS350	SS304	IA718
1	- 0.01341	- 0.0134	1.0007463	- 0.01388	- 0.01388	- 0.01406	- 0.01327
2	0.0	0.0	0.0	0	0	0	0
3	0.0	0.0	0.0	0	0	0	0
4	0.0	0.0	0.0	0	0	0	0

3.2 Examplo 2

The example shown in Fig. 3, it is the modeling of the solid that was discretized in four 8-node hexahedral finite elements, with a total number of 20 nodes, since the measurements were given in millimeters.

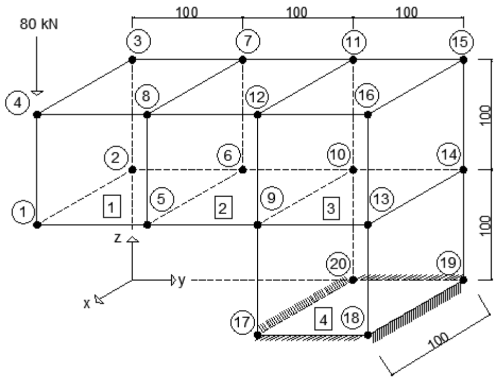

Figure 3 – Examplo 2

In this way, Tab. 2 shows the values obtained by the implemented code for the displacements of nodes 1, 4, 15 and 20, in the z-axis direction.

Table 2 – Comparative z-axis nodal displacement

Nodes	Displacement in z (mm)						
	Present work	Chandrupatla e Belegundu [5]	Present work / Literature	MS250	HS350	SS304	IA718
1	- 0.4098	- 0.409828806	0.9999297	- 0.4098	- 0.4098	- 0.4317	- 0.4025
4	- 0.4278	- 0. 427797766	1.0000052	- 0.4278	- 0.4278	- 0.4507	- 0.4202
15	- 0.0558	- 0. 055798691	1.0000234	- 0.0558	- 0.0558	0.05946	- 0.05524
20	0.0	0.0	1.0	0.0	0.0	0.0	0.0

For the von Mises stress calculated for each element, they are shown in Tab. 3, for element 2, the comparison of the values obtained by the code with the values found in the literature, in addition to those obtained for the solids under study, since the numerical integration occurred with 8 points.

Table 3 – Comparative von Mises Stress

Element	Von Mises Stress (MPa)						
	Present work	Chandrupatla e Belegundu [5]	Present work / Literature	MS250	HS350	SS304	IA718
2	31.42	31.416	1.0001273	31.42	31.42	31.50	31.47
	61.17	61.174	0.9999346	61.17	61.17	61.57	61.44
	28.27	28.272	0.9999293	28.27	28.27	28.52	28.44
	51.61	51.608	1.0000388	51.61	51.61	52.56	52.25
	35.72	35.722	0.9999440	35.72	35.72	36.31	36.12
	35.95	35.948	1.0000556	35.95	35.95	36.79	36.51
	26.19	26.193	0.9998855	26.19	26.19	26.94	26.69
	38.61	38.615	0.9998705	38.61	38.61	38.61	38.61

4 Conclusions

It can be seen that in example 1, the values of the nodal displacements had a percentage difference of 0.07463%, when compared with the values found in the literature. For the materials analyzed in this work, the smallest nodal displacement occurred for the inconel alloy, thus demonstrating the influence of the Modulus of Elasticity and Poisson's ratio on the load-bearing capacity of a solid.

As for example 2, the largest percentage difference between the von Mises stress results obtained via the implemented computer code and the literature results was 0.02697%. As for the nodal displacements, the biggest difference was 0.02086%. The inconel alloy obtained the lowest displacement value, thus demonstrating that the Modulus of Elasticity is inversely proportional to the resistance capacity of a given material.

For both examples presented, there was a convergence between the results obtained between the implemented code and the responses obtained in the literature. It is concluded, therefore, that the implementation developed was satisfactory, contributing with precise values of stress, deformation and displacement in solids submitted to a certain type of loading.

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