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Fully Implicit Algebraic Dynamic Multilevel and Multiscale Method with Non-Uniform Resolution for the Simulation of Two-Phase Flow in Highly Heterogeneous Porous

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Abstract. Large reservoir flow models today can contain more than a billion unknowns. Simulation of these models is not feasible even with the most powerful parallel computers. Generally, upscaling techniques are used to define coarser, i.e., smaller, models that can be processed with reasonable computer resources and in a reasonable amount of time. These techniques consist in a kind of homogenization of the models to obtain representative properties. Such procedures, of course, lead to loss of information. In the last decades, Multiscale Finite Volume (MsFV) methods have been developed to solve these problems. These techniques, in which operators are responsible for transferring information between the fine and coarse scales, provide more accurate solutions than upscaled models at an acceptable CPU cost. Several authors have developed different strategies to obtain accurate solutions using multilevel or multiscale strategies, such as: the i-MsFV, an iterative procedure to smooth the multiscale solution with an efficiency comparable to the original MsFV; the Algebraic Multiscale Solver (AMS), as a preconditioner; the Two-stage AMS (TAMS), which applies an algebraic variant of the MsFV; the monotone Multiscale Finite Volume, with a selective coarse-scale stencil fixing, where either the RBC is replaced by linear boundary conditions (LBC) or "large" non-physical terms are recalculated with upscaling; the zonal MsFV (zMsFV), which splits the domain of interest into a classical MsFV or with additional zonal functions; and the Adaptive Algebraic Dynamic Multilevel (A-ADM), which solves flow problems in highly heterogeneous petroleum reservoirs using non-uniform levels. In this work, we have implemented the Algebraic Dynamic Multilevel with Non-Uniform Resolution (NU-ADM) using a finite volume formulation of the Two Point Flux Approximation (TPFA) for fully implicit simulation of two-phase flows in highly heterogeneous porous media. The NU-ADM operators are based on the Algebraic Multi Scale (AMS) operators. This method provides adaptive multiscale resolution based on the contributions of each volume to the non-physical terms on the coarse scale matrix. We use mesh adaptivity to control these terms. Our parameters are based on the pressure-pressure terms of the Jacobian matrix and the saturation field and takes place during the Newton-Raphson procedure that is used to solve the nonlinear system.

Keywords: Non-Uniform Algebraic Dynamic Multilevel (NU-ADM) Method, Reservoir Simulation, Finite Volume, Heterogeneous Porous Media, Fully Implicit Simulation.

1 Introduction

Numerical simulation of fluid flow in petroleum reservoirs is an important tool for obtaining information that will allow optimal ultimate oil recovery. The current industry standard for geocelular (static models) have a size of up to 10^9 blocks, while simulation models (dynamic meshes) are only up to 10^7 blocks in size. Therefore, the simulation of flows on high-resolution grids is possible only through distributed computer systems [1]. In general, homogenization techniques, such as upscaling, are used to obtain approximate solutions using lower resolution

grids, resulting in a loss of information and accuracy. More recently, multiscale finite volume (MsFV) methods have been developed as they provide more accurate solutions than the upscaling techniques by transferring information between the fine and coarse scales using data transfer operators (restriction and prolongation), at a lower CPU cost compared to fine scale simulations. The prolongation operator is obtained through local solutions of elliptic problems with reduced boundary conditions (RBCs) [2]. It is well known in the literature that although RBCs provide fine-scale decoupling in local distributable problems, they can lead to strongly non-monotonic pressure solutions, especially for very heterogeneous reservoirs [3]. In order to control the pressure oscillations, some strategies may be employed as the Non-Uniform Algebraic Dynamic Multilevel and Multiscale (NU-ADM) method [4], an extension of the ADM framework [5] that can handle non-uniform levels in any coarse-scale control volume. This method uses algebraic criteria that identify volumes that must be maintained at the fine scale in order to keep the non-physical terms in the NU-ADM system below a user-defined value. These criteria analyze the multiscale prolongation operator and the fine-scale transmissibility matrix of each primal coarse-scale control volume. This method was employed to work over an Implicit Pressure Explicit Saturation (IMPES) scheme

In this work, we extended the NU-ADM to work with the Fully Implicit (FIM) scheme in which the pressure and saturation fields are determined nonlinearly by implicitly solving the pressure and saturation fields on the FIM-NU-ADM resolution. The proposed method was applied to the simulation of two-phase flows in petroleum reservoirs with highly heterogeneous permeability fields and demonstrates an effective accurate way to get an approximated solution to the cases under study.

2 Mathematical model

In this paper, we assume immiscible, isothermal, and incompressible fluid flow of oil and water, neglecting capillarity, gravity, and adsorption, through a non-deformable porous medium and use a fully implicit (FIM) strategy to solve the mass conservation equation. Under these hypotheses, the mass conservation equation can be written as:

$$\phi \rho_i \frac{\partial S_i}{\partial t} = -\rho_i \vec{\nabla} \cdot \vec{v_i} + q_i \,, \tag{1}$$

Where, S_i, ρ_i and $\vec{v_i}$ are the saturation, density and velocity of the phase $i = o$,w, q_i represents the source/sink terms. The velocity is calculated through the Darcy's law, as:

$$\vec{v_i} = -\frac{k_{ri}}{\mu_i} K \vec{\nabla} p, \tag{2}$$

With, μ_i and k_{ri} representing the phase viscosity and relative permeability, respectively. The latter calculated using the Brooks and Corey model. K and p represent the rock porosity tensor and the pressure, respectively. In addition, we consider the reservoir rock fully saturated by the fluids, i.e., $S_o + S_w = 1$.

3 Numerical approximation

In this work, we have applied the backward Euler's method for time differentiation which led to a non-linear scheme with coupled pressure and saturation fields. By applying the finite volume method, we discretize Eqs (1) and (2). To discretize the diffusive terms of these equations we use the Two Point Flux Approximation (TPFA) method and mobilities are approximated by the First Order Upwind (FOU) method, leading to a non-linear system of equations that we solve by the Newton-Raphson method, that can be written as:

$$\begin{cases} J(x^k)\Delta x^{k+1} = -F(x^k) \\ x^{k+1} = x^k + \Delta x^{k+1} \end{cases}, \tag{3}$$

Here, J represents the Jacobian matrix of the F, this last represents the mass balance on each volume of the computational domain, J is stated as the expression on Eq. (4).

$$\begin{bmatrix} \frac{\partial f_{o_1}^{n+1,k}}{\partial p_1^{n+1,k}} & \cdots & \frac{\partial f_{o_1}^{n+1,k}}{\partial p_N^{n+1,k}} & \frac{\partial f_{o_1}^{n+1,k}}{\partial s_1^{n+1,k}} & \cdots & \frac{\partial f_{o_1}^{n+1,k}}{\partial s_1^{n+1,k}} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_{o_N}^{n+1,k}}{\partial p_1^{n+1,k}} & \cdots & \frac{\partial f_{o_N}^{n+1,k}}{\partial p_N^{n+1,k}} & \frac{\partial f_{o_1}^{n+1,k}}{\partial s_1^{n+1,k}} & \cdots & \frac{\partial f_{o_1}^{n+1,k}}{\partial s_1^{n+1,k}} \\ \frac{\partial f_{w_1}^{n+1,k}}{\partial p_1^{n+1,k}} & \cdots & \frac{\partial f_{w_1}^{n+1,k}}{\partial p_1^{n+1,k}} & \frac{\partial f_{w_1}^{n+1,k}}{\partial s_1^{n+1,k}} & \cdots & \frac{\partial f_{w_1}^{n+1,k}}{\partial s_N^{n+1,k}} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_{w_N}^{n+1,k}}{\partial p_1^{n+1,k}} & \cdots & \frac{\partial f_{w_1}^{n+1,k}}{\partial p_1^{n+1,k}} & \frac{\partial f_{w_N}^{n+1,k}}{\partial s_1^{n+1,k}} & \cdots & \frac{\partial f_{w_N}^{n+1,k}}{\partial s_N^{n+1,k}} \end{bmatrix} \begin{bmatrix} \Delta p_1^{n+1,k} \\ \vdots \\ \Delta p_N^{n+1,k} \\ \Delta S_1^{n+1,k} \\ \vdots \\ \Delta S_N^{n+1,k} \end{bmatrix} = - \begin{bmatrix} f_{o_1}^{n+1,k}\left(p_1^{n+1,k}, S_1^{n+1,k}, \cdots, p_N^{n+1,k}, S_N^{n+1,k}\right) \\ \vdots \\ f_{o_N}^{n+1,k}\left(p_1^{n+1,k}, S_1^{n+1,k}, \cdots, p_N^{n+1,k}, S_N^{n+1,k}\right) \\ f_{w_1}^{n+1,k}\left(p_1^{n+1,k}, S_1^{n+1,k}, \cdots, p_N^{n+1,k}, S_N^{n+1,k}\right) \\ \vdots \\ f_{w_1}^{n+1,k}\left(p_1^{n+1,k}, S_1^{n+1,k}, \cdots, p_N^{n+1,k}, S_N^{n+1,k}\right) \end{bmatrix}, \quad (4)$$

We organize the terms of Eq. (4) using the following submatrices:

$$\begin{bmatrix} J_{op} & J_{os} \\ J_{wp} & J_{ws} \end{bmatrix} \begin{bmatrix} \Delta x_p \\ \Delta x_s \end{bmatrix} = - \begin{bmatrix} r_o \\ r_w \end{bmatrix}, \quad (5)$$

3.1 Extension of NU-ADM to fully implicit

The NU-ADM applies three algebraic parameters to define the volumes to be maintained on the fine scale. The first is the front tracking criteria, $\delta_{ij} = abs(S_i - S_j)$, we maintain on the fine scale the volumes where the value is bigger than a user defined value, $\delta^{\lim}$. The second measures the contribution of a volume on the fine scale to the non-physical terms on the coarse scale, $\eta_i = máx\left(J_{op}P\right)_{i,K\neq I}/t_{II}^{C}\right)$, the volume i is maintained on fine scale if η_i is bigger than η^{lim}. The third parameter groups volumes with $\beta_i = (1 - \phi_{I(i)})/\phi_{I(i)}$ bigger than β^{lim}. More details about this parameters can be found on [1].

Our method uses operators to transfer information between the scales, these operators are based on the work of Santos, 2022 [1] and can be written as:

$$R_{FIM} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}, P_{FIM} = \begin{bmatrix} P & 0 \\ 0 & R^T \end{bmatrix}, \quad (6)$$

Where, R and P are the NU-ADM restriction and prolongation operators, R^T is the transpose of the NU-ADM restriction operator.

The coarse-scale system is constructed as:

$$R_{FIM}J^k P_{FIM}\Delta x^{k+1} = -R_{FIM}r^k, \quad (7)$$

We use the FIM-NU-ADM with to solve the linear equations given by the Newton-Raphson's method to approximate the solution of the system given in Eq. (5) through the algorithm show in Figure 1.

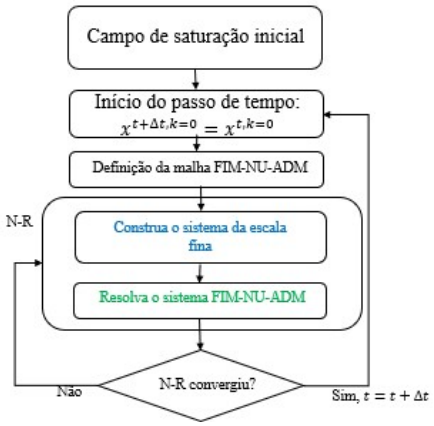

Figure 1: FIM-NU-ADM Algorithm.

4 Results

In this work, we have implemented the proposed methods in Python, based on the basic libraries of this language, e.g. NumPy, SciPy, SymPy for algebraic and symbolic computations, and PyVista and Matplotlob for visualization and graphics. At this point, we do not have a massively parallel implementation. Only some parts of the code run in a parallel shared memory implementation and we have not done yet an extensive study of computation time.

In this section, we present the results, $x^{FIM-NU-AD}$, obtained by applying the fully implicit extension of the NU-ADM framework to a set of test cases and compare to those obtained solution on the fine-scale mesh, x^{fs} ($x = p$, for the pressure and $x = s$, for the saturation). The error norm is defined as,

$$||e_x|| = \left|\left|x^{FIM-NU-ADM} - x^{fs}\right|\right| / \left|\left|x^{fs}\right|\right|. \tag{8}$$

In the example we have used a fixed coarsening ratio CR = (5, 5). The permeability field and the wells positions are shown in Fig. 3. As initial condition we set a constant water saturation of Sw = 0.0 equal to the irreducible, the residual oil saturation was set as 0.0. The viscosity is 0.3 cP for the water and 3cP for the oil, in addition null flux was set in all external boundaries. The permeability field is shown on Figure 2, we used the following NU-ADM parameters, $\delta^{lim} = 0.2$, $\eta^{lim} = 1$ and $\beta^{lim} = 3$. The definition of these parameters is available on [1].

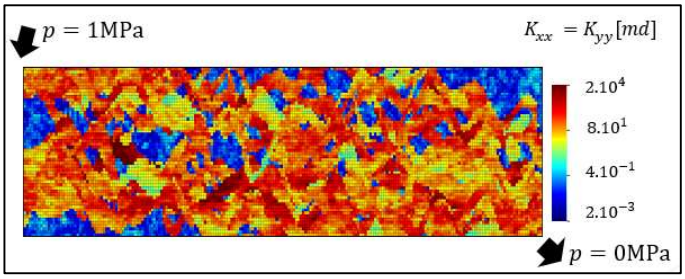

Figure 2. Permeability field and well's location of the 85th layer of the SPE-10.

The simulation was performed until we reach 70% of porous volume injected (PVI) and the pressure and the saturation error norms are presented on Fig. 3.

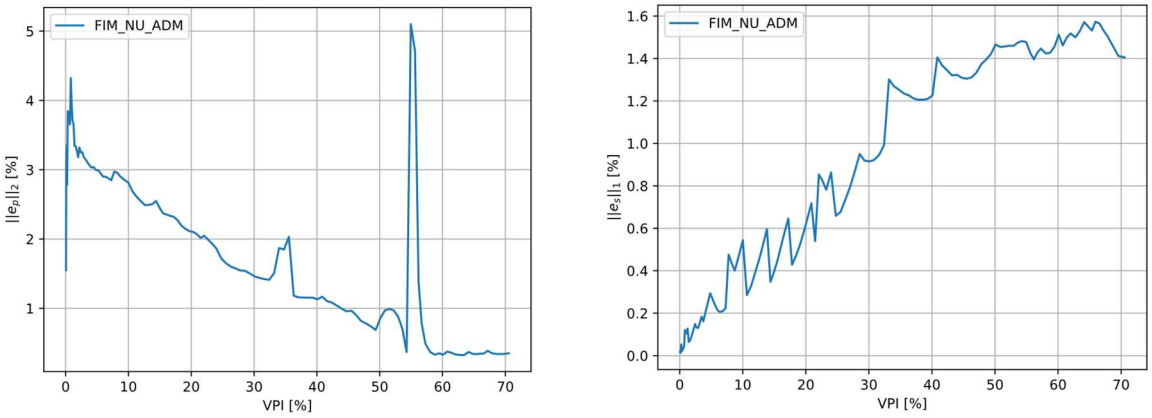

Figure 3. Pressure error norm, L_2 (left) and saturation error norm, L_1 (right)

The production curve and the percent of active volume and the production curve are shown on Fig. 4 where we can see that our methodology was capable of reproduce with a reasonable accuracy the production curve while using a fraction of volumes on the fine scale.

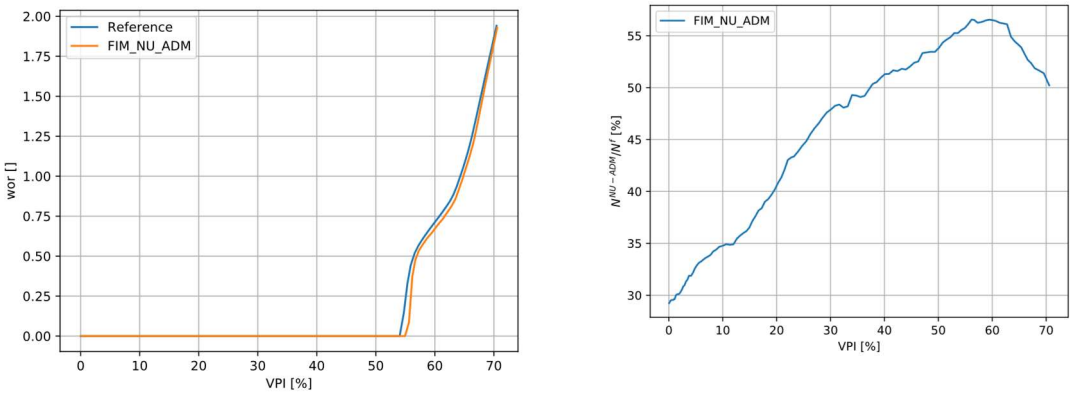

Figure 4: Water/Oil ratio (left) and percent of active volumes (right).

The saturation field on the time corresponding to 57% of porous volume injected is shown on Fig. 5, where we can see the reference solution obtained with the direct simulation on the fine scale and the solution obtained with the FIM-NU-ADM strategy with the volumes on the fine scale presented on the bottom figure.

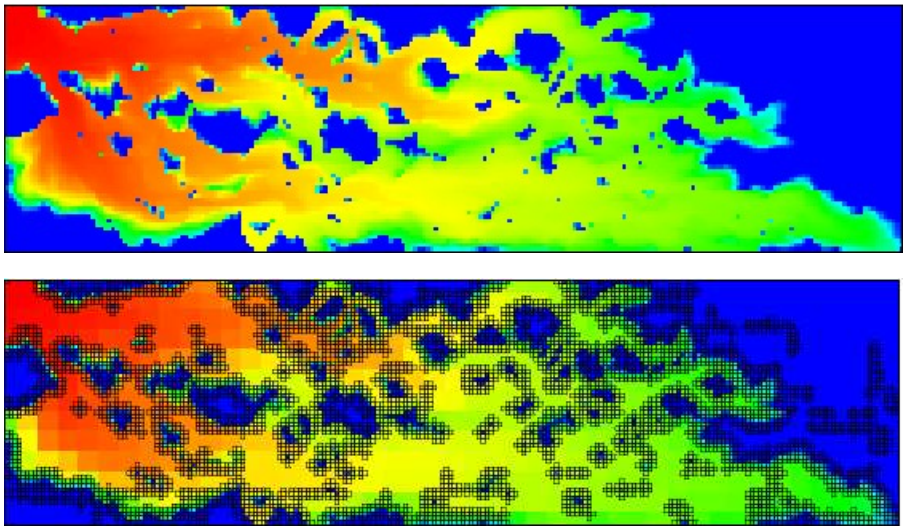

Figure 5: Saturation fields, obtained by solution on fine-scale (top) and NU-ADM (bottom)

5 Conclusions

In this paper, we have extended the Algebraic Dynamic Multilevel method with Non-Uniform levels to deal with fully implicit schemes. This method applies the level definition criteria, in a non-uniform mesh resolution to control the non-physical terms that may induce non-monotone solutions at the NU-ADM transmissibility matrix throughout the simulation. The results obtained for the example presented here show that the NU-ADM framework is perfectly extensible to the fully implicit simulation and is capable of producing accurate results for a challenging problem (bottom layer of SPE-10) using only a fraction of the volumes of the original fine-scale problem.

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