Multi-objective structural optimization of a planar truss considering dynamic and global stability aspects

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Abstract. The literature has broadly discussed the development of multi-objective structural optimization problems (MOSOPs) with two objectives: minimizing the structure's weight and the maximum nodal displacement. This paper proposes the formulation of MOSOPs with three objective functions. New objectives are considered, such as the maximization of the first natural frequency of vibration and the maximization of the difference between the second and the first ones, to avoid the superposition of their modes. MOSOPs are also proposed to maximize the first critical load factor related to the structure's global stability and the difference between the second and the first ones. The analyzed structure is a 10-bar truss. The design variables are the cross-sectional areas of the bars. The evolutionary algorithm is the third step of generalized differential evolution (GDE3). The Pareto fronts obtained for the problems are presented. It is possible to observe how the growth of the truss' weight causes increases in the first natural frequency of vibration and the first critical load factor, as well as its influence on the differences between the frequencies of vibration and the critical load factors of the structure. Finally, optimized solutions are extracted from the obtained Pareto fronts.

Keywords: Multi-objective structural optimization, Natural frequencies of vibration, Global stability

1 Introduction

The general design situation in a real-world optimization problem is that a designer or a decision-maker (DM) wants to find a structural configuration that satisfies the requirements set out in a standard or recommended practice. To achieve this purpose, it's necessary to define some parameters, such as the design variables, design constraints and objective functions, leading to an optimization problem. The most common structural optimization problems have a single objective function. However, the literature also widely discusses problems with two or more objective functions. When two objectives are considered, they commonly refer to the minimization of the structure's weight and maximum nodal displacement.

In this work, multi-objective structural optimization problems (MOSOPs) with three objective functions are formulated and solved. In addition to the traditional ones mentioned above, new objectives and constraints related to dynamic and global stability aspects are added.

The analysis of the natural frequencies of vibration is included in the proposed MOSOPs. The importance of this study is to prevent the structures from reaching the frequencies capable of generating the effect of resonance, which can cause severe damage and even the collapse of the structure. On the other hand, the analysis of the critical load factors related to the global stability aims to ensure the integrity of the structure according to the load applied to it.

The structure under study is the well-known 10-bar truss, with the cross-sectional area of these bars as the sizing design variables. Regarding its dynamic aspects, the new objective functions proposed are the maximization of the first natural frequency of vibration and the maximization of the difference between the second and the first ones, intending to move their vibration modes away from each other to avoid their superposition. As for the global stability of the truss, the new objectives are to maximize the first critical load factor and to maximize the difference between the second and the first ones. All these objectives can also be used as constraints according to the formulation of the problems. Therefore, the main objective of this paper is to consider these new objective functions and constraints in the formulations of the structural optimization problems of the proposed truss.

For each MOSOP developed, the Pareto fronts obtained in two or three dimensions are presented. Afterward, a multi-criteria decision-making (MCM) is applied to extract desired solutions according to the DM preferences from the Pareto fronts.

This paper is organized as follows: Section 2 describes the aspects of the multi-objective structural optimization discussed in the paper. The MOSOPs' formulations are presented in Section 3. Section 4 presents and analyzes the Pareto fronts obtained for each MOSOP and the optimized solutions extracted from them. Finally, the conclusions and future works of this research are reported in Section 5.

2 Multi-objective structural optimization

This section presents a theoretical presentation of important concepts covered in this paper, such as the multi-objective structural optimization and the differential evolution algorithm adopted to solve the problems discussed in this paper. It also presents the structural importance of the objective functions used in the MOSOPs and the method applied to extract the optimized solutions.

2.1 Definition and objective functions

Multi-objective structural optimization problems are the ones that present two or more conflicting objective functions to be minimized or maximized simultaneously. As mentioned earlier, each optimization problem in this work has three objective functions.

According to Sommer [1], the condition in which the frequency of the applied load is equal to a natural frequency of vibration of the structural system is called resonance. With the structure working close to the resonance, its displacements, vibrations, and accelerations intensify, shortening the structure's lifespan and causing discomfort to users, material fatigue, corrosion, and even its collapse. Because of these risks, an objective function is adopted to increase the first natural frequency of vibration in the structure, to move it away from the frequencies generated by the loads applied to the structure, avoiding undesirable effects. Another objective function adopted in this paper is the maximization of the difference between the second and the first natural frequencies so that their vibration modes do not overlap, intensifying the resonance and the displacements of the structure.

The global stability of structures indicates their sensitivity to second-order effects, that is, those generated by their displacements. Therefore, verifying global stability concerning the Euler buckling loads is an important requirement in the design of a structure, aiming to guarantee safety in terms of the ultimate limit state of stability. The critical load factor represents the ratio between the estimated critical load at which the structure becomes unstable and the load that is applied to it. In this sense, the objective function of maximizing the first critical load factor aims to find the solution in which the highest possible load can be applied to the structure without causing its instability. Furthermore, the objective of maximizing the difference between the second factor and the first one is to avoid the overlapping of the structure's buckling modes, which would result in the intensification of its instability. Finally, the most usual objective function applied to all MOSOPs in this paper is minimizing the structure's weight, leading to more safety and economical design.

2.2 Differential evolution algorithm and a multi-criteria decision-making

The differential evolution algorithm (DE), introduced by Storn and Price [2], is based on the generation and evolution of a population of candidate solutions with continuous variables. Currently, it is considered one of the most popular meta-heuristics for solving optimization problems. In this paper, the DE used is the third evolution step of generalized differential evolution (GDE3), developed by Kukkonen and Lampinen [3]. The constraint-handling technique adopted in this paper is the APM (Adaptive Penalty Method), proposed by Lemonge and Barbosa [4].

The non-dominated solutions provided by the DE are presented through Pareto fronts. To extract the desired solution from the Pareto front obtained for each MOSOP, a Multicriteria Tournament Decision (MTD), proposed by Parreiras and Vasconcelos [5] is used. This method ranks the best and worst solutions obtained according to the values of their objective functions and the importance (weight) assigned by the DM to each objective function.

3 Formulation of the MOSOPs

The structure under study is the well-known 10-bar truss depicted in Fig. 1. Proposed by Gellatly and Berke [6], the design variables are the cross-sectional areas of the bars, which can assume any value between 6.45 cm^2 and 258.06 cm^2. The vertical downward loads P shown in Fig. 1 are applied at nodes 2 and 4, with a magnitude of 445.9 kN. The material has a specific mass of 2700 kg/m^3 and Young's modulus of 68.95 GPa. A nonstructural mass of 454 kg is applied to each of the truss' free nodes (nodes 1 to 4). Regarding the constraints applied to the problems, the stress in each bar must not exceed ± 172.37 MPa, the maximum displacement is limited to 5.05 cm in all directions, the critical load factor λ_{cr} should be greater than or equal to 1 and the first three natural frequencies of vibration f_1, f_2 and f_3 should be greater than or equal to 7 Hz, 15 Hz and 20 Hz, respectively. Each MOSOP was evaluated through 20 independent runs, each with 200 generations of 50 individuals (10000 number of function evaluations nfe).

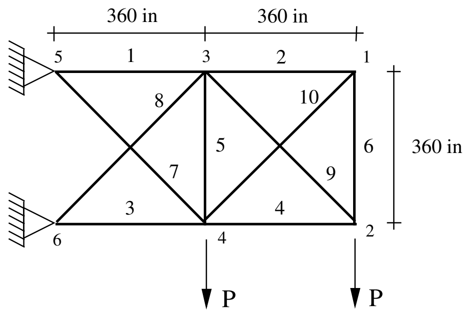

Figure 1. 10-bar truss

Three MOSOPs were developed to evaluate the dynamic and global stability aspects of the truss, as well as its weight. The MOSOP1 has as objective functions the minimization of the truss' weight, the maximization of the first natural frequency of vibration, and the maximization of the first critical load factor related to the structure's global stability. The MOSOP2 aims to minimize the weight and maximize the first natural frequency of vibration and the difference between the second and the first ones. Finally, the MOSOP3 has the objectives of minimizing the structure's weight, maximizing its first critical load factor, and the difference between the second and the first ones.

Since $W(\mathbf{x})$ is the weight of the truss, $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ are the first and second natural frequencies of vibration, $\lambda_1(\mathbf{x})$ and $\lambda_2(\mathbf{x})$ are the first two critical load factors, $\sigma_i(\mathbf{x})$ is the stress in the i^{th} bar, $u_j(\mathbf{x})$ is the displacement of the j^{th} node and **x** represents the design variables (cross-sectional areas of the bars), these optimization problems, with their objective functions and constraints, can be written as follows:

MOSOP1:

$$\begin{aligned} \min \quad W(\mathbf{x}) \quad , \quad \max \quad f_1(\mathbf{x}) \qquad &\text{and} \quad \max \quad \lambda_1(\mathbf{x}) \\ \text{s.t.} \qquad &|\sigma_i(\mathbf{x})| \le 172.37\ MPa \\ &|u_j(\mathbf{x})| \le 5.05\ cm \\ 6.45\ cm^2 \le\ &\mathbf{x} \le 258.06\ cm^2 \end{aligned} \tag{1}$$

MOSOP2:

$$\begin{aligned} \min \quad W(\mathbf{x}) \quad , \quad \max \quad f_1(\mathbf{x}) \qquad &\text{e} \quad \max \quad f_2(\mathbf{x}) - f_1(\mathbf{x}) \\ \text{s.t.} \qquad &|\sigma_i(\mathbf{x})| \le 172.37\ MPa \\ &|u_j(\mathbf{x})| \le 5.05\ cm \\ &\lambda_i(\mathbf{x}) \ge 1 \\ 6.45\ cm^2 \le\ &\mathbf{x} \le 258.06\ cm^2 \end{aligned} \tag{2}$$

MOSOP3:

$$\begin{aligned} \min \quad W(\mathbf{x}) \quad , \quad \max \quad \lambda_1(\mathbf{x}) \quad \text{e} \quad \max \quad \lambda_2(\mathbf{x}) - \lambda_1(\mathbf{x}) \\ \text{s.t.} \qquad |\sigma_i(\mathbf{x})| \leq 172.37\ MPa \\ |u_j(\mathbf{x})| \leq 5.05\ cm \\ f_1(\mathbf{x}) \geq 7\ Hz \\ f_2(\mathbf{x}) \geq 15\ Hz \\ f_3(\mathbf{x}) \geq 20\ Hz \\ 6.45\ cm^2 \leq \mathbf{x} \leq 258.06\ cm^2 \end{aligned} \tag{3}$$

4 Pareto fronts and extracted solutions

This section presents the Pareto fronts obtained by the resolution of each MOSOP in the truss of Fig. 1. Non-dominated solutions are represented by the red circles in the graphs of Figs. 2 to 4. For each MOSOP, the Pareto front is shown in three dimensions (with the three objective functions) and in two dimensions, comparing the results obtained for each objective two by two. Finally, using the MTD method, an optimized solution is extracted from each of the Pareto fronts according to the DM's preferences regarding the analyzed objectives. The defined weights were 0.5 for the structure's weight $W(\mathbf{x})$ and 0.25 for the other objective functions of each problem. The final solution extracted is represented by the blue sphere in the graphs.

4.1 Results obtained for MOSOP1

The MOSOP1 provided the non-dominated solutions in Fig. 2. From the results obtained, the weight of the truss varies from 2395.43 kg to 4514.74 kg, the first natural frequency of vibration presents values from 11.86 Hz to 15.97 Hz, and the first critical load factor related to the truss' global stability are in the range of 168.52 and 1305.5. As the weight of the structure increases, $f_1(\mathbf{x})$ and $\lambda_1(\mathbf{x})$ also grow, as a result of the enhance in the stiffness of the truss. It is also possible to observe a slight trend of growth of $\lambda_1(\mathbf{x})$ while increasing $f_1(\mathbf{x})$. With the objective functions analyzed and the weight assigned to each one of them, the optimized solution extracted from the Pareto front has the values of $W(\mathbf{x})$ = 2920.98 kg, $f_1(\mathbf{x})$ = 13.85 Hz and $\lambda_1(\mathbf{x})$ = 712.98.

4.2 Results obtained for MOSOP2

The non-dominated solutions obtained for MOSOP2 are shown in Fig. 3. The weight of the truss varies between 2386.85 kg and 4493.77 kg, the first natural frequency of vibration has results between 12.01 Hz and 15.96 Hz and the difference between the second and the first ones range between 5.29 Hz and 21.33 Hz. As in the previous example, a clear increase in $f_1(\mathbf{x})$ is observed as the weight of the structure grows. The difference $f_2(\mathbf{x}) - f_1(\mathbf{x})$, with the raise of $W(\mathbf{x})$, presents an initial growth but then stabilizes at the average of 15 Hz, varying for more and for less. A similar behavior is observed in the graph of $f_1(\mathbf{x})$ x ($f_2(\mathbf{x}) - f_1(\mathbf{x})$). Considering the minimization of $W(\mathbf{x})$, the maximization of $f_1(\mathbf{x})$ and the maximization of $f_2(\mathbf{x}) - f_1(\mathbf{x})$, the optimized solution extracted with the given weights was $W(\mathbf{x})$ = 3062.31 kg, $f_1(\mathbf{x})$ = 14.51 Hz and $f_2(\mathbf{x}) - f_1(\mathbf{x})$ = 20.02 Hz.

4.3 Results obtained for MOSOP3.

Figure 4 shows the non-dominated solutions of MOSOP3. The weight of the truss is between 2392.93 kg and 6779.59 kg, the first critical load factor varies from 162.47 to 3322.97, and the difference between the second factor and the first one has a large range from 8.51 to 3950.17. The increases in the weight and, consequently, in the stiffness of the structure significantly increase the values obtained for $\lambda_1(\mathbf{x})$. On the other hand, the graphs involving $\lambda_2(\mathbf{x}) - \lambda_1(\mathbf{x})$ do not show such a clear trend with the growths of $W(\mathbf{x})$ or $\lambda_1(\mathbf{x})$, showing an initial decrease and then varying intensely, mainly in the range between the minimum value obtained and 2000. With the objective functions of this problem and the standardized weights proposed for them, the optimized solution

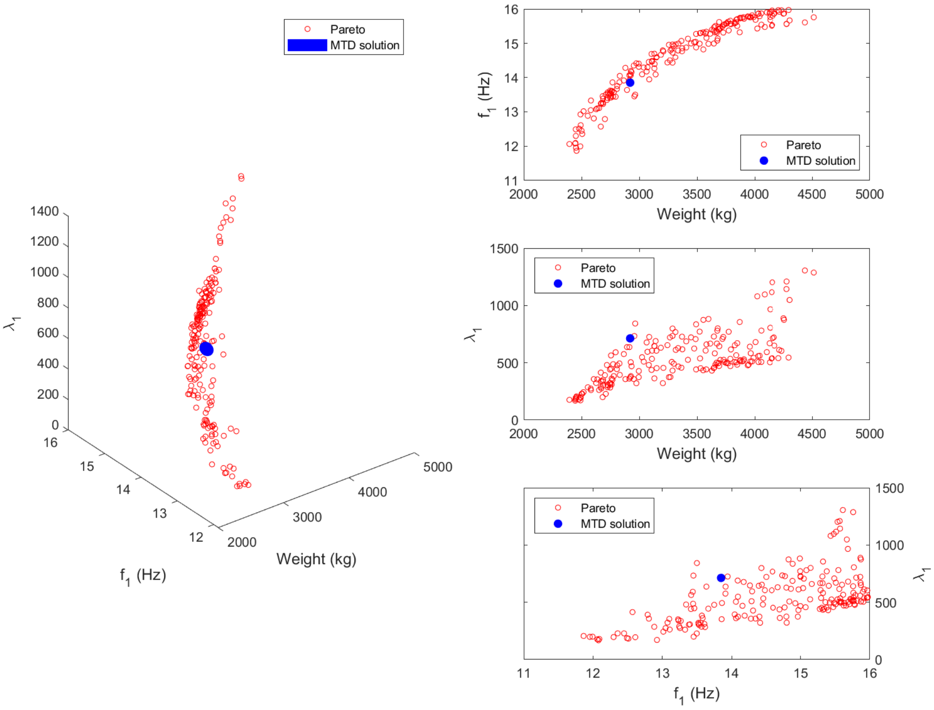

Figure 2. Solutions obtained for MOSOP1.

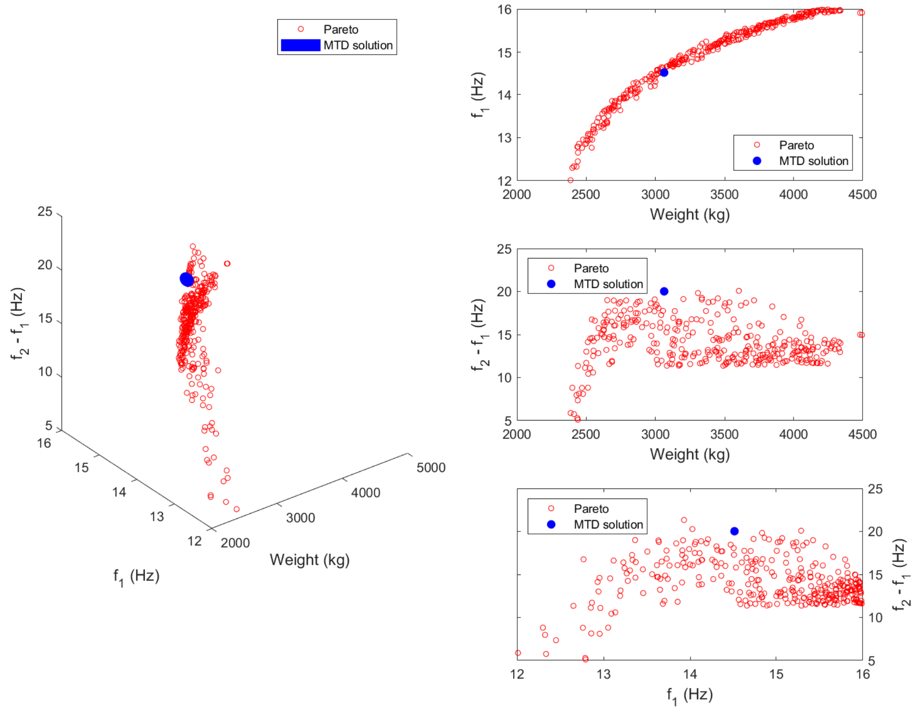

Figure 3. Solutions obtained for MOSOP2.

obtained has the values of $W(\mathbf{x})$ = 3645.26 kg, $\lambda_1(\mathbf{x})$ = 1418.51 and $\lambda_2(\mathbf{x}) - \lambda_1(\mathbf{x})$ = 2957.33.

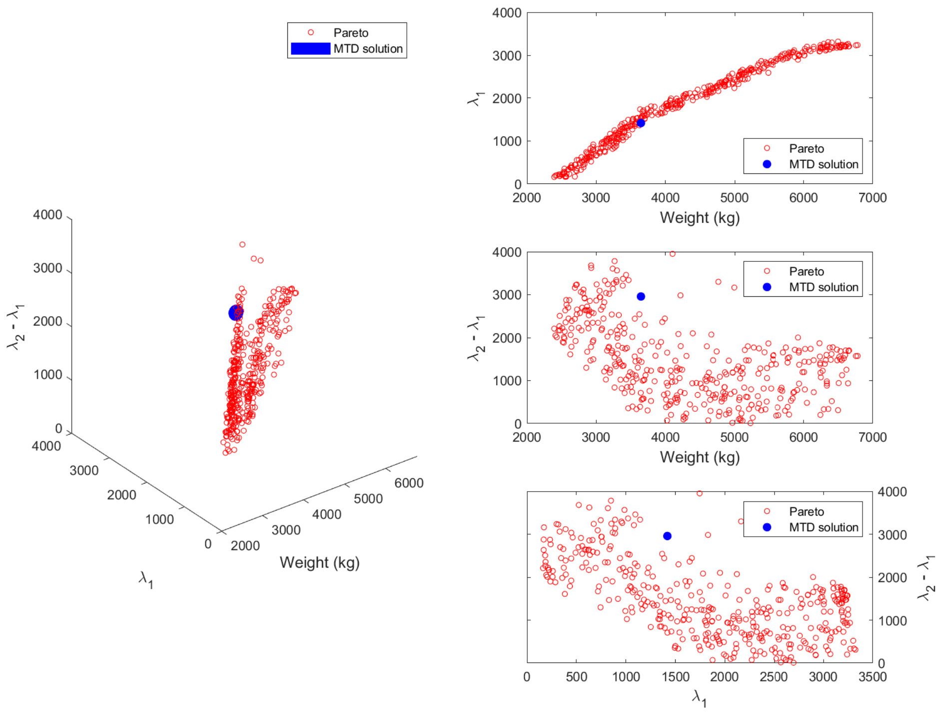

Figure 4. Solutions obtained in MOSOP3.

5 Conclusions

Adding objective functions regarding the natural frequencies of vibration and the critical load factors related to the global stability of the truss is important to avoid the resonance effect and maintain the structure's global stability.

The solutions obtained for the proposed MOSOPs clearly show that minimizing the weight is a conflicting objective concerning the maximization of the first natural frequency of vibration and the first critical load factor. In other words, $f_1(\mathbf{x})$ and $\lambda_1(\mathbf{x})$ grow while increasing the weight and stiffness of the structure and decrease as the truss becomes lighter. Regarding the differences $f_2(\mathbf{x}) - f_1(\mathbf{x})$ and $\lambda_2(\mathbf{x}) - \lambda_1(\mathbf{x})$, their behavior does not show such clear trends as the other functions vary.

Concerning the solutions extracted through the MTD method for each problem, the solution obtained for MOSOP1 has the lowest weight (2920.98 kg). The optimized solution extracted for MOSOP2 has the highest value of $f_1(\mathbf{x})$ (14.51 Hz), softly superior to the frequency obtained for MOSOP1. It is also slightly heavier than the previous problem's solution and the difference $f_2(\mathbf{x}) - f_1(\mathbf{x})$ equals 20.02 Hz. Finally, the MOSOP3 optimized solution presents an ineffective minimization of the truss' weight, but its $\lambda_1(\mathbf{x})$ is equal to 1418.51, a value much higher than the factor obtained in the MOSOP1. The difference $\lambda_2(\mathbf{x}) - \lambda_1(\mathbf{x})$ has shown a wide range of results, while the extracted solution presents the maximized value of 2957.33 for this objective function. Therefore, analyzing the Pareto fronts and the extracted optimized solutions, it can be concluded that the differential evolution algorithm GDE3 was efficient and satisfactory in solving the proposed MOSOPs for this 10-bar truss, providing good and coherent results for the analyzed MOSOPs.

The future works of this research will be the resolution of new MOSOPs with more than three objective functions, as well as the application of these problems in large-scale and more complex trusses, whether planar or spatial.

Acknowledgements. The authors wish to thank the Postgraduate Program in Civil Engineering (UFJF), the Federal

University of Juiz de Fora, FAPEMIG (TEC PPM 174-18), CAPES, and CNPq (grant 308105/2021-4) for their suport.

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References

[1] R. M. R. Sommer. Análise de vibrações em pisos mistos aço/concreto. Master's thesis, Programa de Pós-Graduação em Engenharia de Estruturas, Escola de Engenharia da Universidade Federal de Minas Gerais, 2002.
[2] R. Storn and K. Price. Differential evolution a simple and efficient adaptive scheme for global optimization over continuous spaces. Tech. Rep. 95-012, Univ. of California, Berkeley, CA, 1995.
[3] S. Kukkonen and J. Lampinen. Gde3: The third evolution step of generalized differential evolution. In *IEEE Congress on Evolutionary Computation (CEC 2005)*, pp. 443–450. IEEE, 2005.
[4] A. C. C. Lemonge and H. J. C. Barbosa. An adaptive penalty scheme for genetic algorithms in structural optimization. *International Journal For Numerical Methods In Engineering*, vol. 59, n. 5, pp. 703–736, 2004.
[5] R. Parreiras and J. Vasconcelos. Decision making in multiobjective optimization aided by the multicriteria tournament decision method. *Nonlinear Analysis: Theory, Methods & Applications*, vol. 71, n. 12, pp. e191–e198, 2009.
[6] R. Gellatly and L. Berke. Optimal structural design. Technical Report AFFDL-TR-70-165, Air Force Flight Dynamics Lab., AFFDL, 1971.