XLIII CILAMCE
21–25, November, 2022
FOZ DO IGUAÇU – PR – BRAZIL

Numerical Modelling of the Internal Erosion Process in Granular Soils using the Material Point Method

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Abstract. Accidents caused by landslides, landslides, or large soil pits in cities are often seen in the media. These events are usually related to erosion, and there are several causes related to this phenomenon, but without a doubt, the main cause is the runoff of water in direct contact with the soil mass. A specific type of internal erosion called *piping* is the most common cause of failure in dikes and / or earth dams, such as the Aznacollar-Spain dam failure in 1998. Internal erosion can be a natural process, but sometimes they are due to poor decisions or lack of foresight in structural design. The main challenge, then, is to predict with a certain degree of reliability the danger of undermining in this type of structure, subject to large deformations and important changes in its edge conditions. As is well known, one of the great limitations of classical numerical methods, such as the Finite Element Method (FEM), in the simulation of large deformations is the mesh distortion, thus a better alternative should be used. The objective of this work is to present a theoretical/numerical approach to the scour problem using the Material Point Method (MPM), which was developed in order to correct mesh dependence problems with extreme deformations.

Keywords: scour, explicit method, material point method

1 Introduction

The study and prediction of natural disasters as well as man-made disasters are very important issues that have attracted the attention of the scientific community for decades. This interest is mainly due to the well-known relationship between the aforementioned events and illness proliferation, life quality deterioration of urban settlements, and, in some extreme cases, loss of lives.

Experimental modeling of these large scale events (sudden hillslopes failures, seaquake, oil spills, tsunamis, etc.) [1, 2] is almost impossible to perform due to the high costs involved and the several variables that need to be taken into account. For this reason, numerical simulation tools gain importance over experimental techniques, either by the cost of realization or the accuracy of the results.

Regarding the available numerical methods for large deformations in large scale problems, it is well known that particle methods adapt with less difficulty to high deformation rates compared against classical continuous methods, as Finite Element Method (FEM).

Particle methods, which can be derived from a continuum framework, were mostly proposed to model large deformation behavior of granular material using conventional constitutive models. A wide range of meshfree methods are available in the literature and these include Smoothed Particle Hydrodynamics (SPH) Method [3, 4], Material Point Method (MPM) [2, 5], Element Free Galerkin method [6], Particle Finite Element Method (PFEM) [7, 8], etc. Many of these meshfree methods have been applied almost exclusively to computational fluid dynamics (CFD) problems. However, a few years ago some of them were applied to the study of the mechanical behavior of solid continua.

Regarding geotechnical problems, some authors have applied the SPH method to the study of soil slope stability [9, 10], fluid-soil interaction [11], and also combined the SPH method with the Peridynamic method [12–14], while others report solutions with PFEM applied to large deformations of granular materials [15, 16] and effective stress analysis of insertion problems (see Ref. [17]). Nevertheless, probably the most popular meshfree method for large deformation and dynamic model of geotechnical problems is the MPM. In this way, Lei et al. 2020 [18], propose a generalized interpolation material point (GIMP) method for modeling the seepage process in granular materials, other authors [1, 5, 19–21] simulate the slope failure during large deformation Zhang et al. 2009 [22], and Liang et al. 2017 [23], include impact loads on geotechnical problems.

Although PFEM and SPH methods are widely used, they have some disadvantages for geotechnical problems involving boundary interfaces. Also, the loss of accuracy near the boundaries, due to insufficient neighboring particles, result in the loss of consistency, which is a widely investigated topic by the SPH and PFEM research community. Furthermore, numerical oscillations of particles may occur, since SPH does not require the velocity field to be single-valued (i.e. allows particle penetration) [24, 25].

As it is usual in the classical MPM method [5], all the information is attached to material points while the grid does not save permanent information. An important advantage of this method is that the application of boundary conditions is straightforward due to the presence of the background grid and can be applied to grid nodes as in the FEM.

The numerical simulation of continuous media with interstitial pore pressure has also been studied by several authors, some of them present the formulation and numerical implementation for saturated porous media [22, 26–28], partially saturated soils [18, 29] as well as an extension to multiphase flow [30]. Other researchers present applications of the MPM to real-world engineering problems, for example the study of cone penetration [31], hydraulic fracture in excavations [32] as well as structural collapse [26, 33]. In this sense, one of the most popular problems, commonly studied in geotechnical engineering, is the static and dynamic analysis of the coupled soil skeleton/fluids pore pressure mechanical behavior in deformable porous media. This theoretical formulation was mainly studied and implemented in the FEM framework [34–38] in order to model the deformation and pore fluid flow in a time-dependent process for an initially undrained condition [39], followed by a consolidation process.

Since the MPM can be viewed as a special Lagrangian FEM with particle quadrature and continuous mesh-updating with the capability of modeling very large deformations, the theoretical framework of porous media can be implemented in the MPM context by taking into account some special considerations. In this sense, some authors proposed a formulation and numerical implementation of the MPM to simulate the dynamic behavior of porous media, but mainly using a single point for the entire mixture [22, 26, 28, 31, 40]. Other authors proposed one material point for each phase [27, 41, 42]. However, a fully coupled MPM model still requires further development in order to preserve its simplicity and accuracy in the numerical modeling of large deformation of porous media and the prediction of failure shapes of foundations and structures.

Thus, the main aim of this work is to develop a general formulation of multiphase flow in porous media, based on the mixture theory, and its numerical implementation focused on numerical modeling of the internal erosion process granular soils.

Therefore, the proposed formulation is particularly well-adapted for the dynamic analysis of large deformations of biphasic porous media, but also, due to its generic theoretical foundation, this numerical implementation can easily be extended to multiphase flow in porous media [34, 36, 43].

2 Linear momentum balance equation of saturated porous media

Firstly the linear momentum balance equation for the solid phase was obtained considering the total stress tensor and by assuming the superscript s and $\pi = w$ for solid skeleton and water phase, respectively, as:

$$\left(\sigma'_{ij} - (1-n)p^w\delta_{ij}\right)_{,j} + \tilde{\rho}^s g_i - \tilde{\rho}^s a_i^s + nR^w v_i^{ws} = 0 \tag{1}$$

being, $\tilde{\rho}^s = (1-n)\rho^s$ and $\tilde{\rho}^w = n\rho^w$ the average solid and water density, respectively.

Furthermore, the linear momentum balance equation of the water phase can be obtained by considering the water stress tensor, neglecting momentum exchange terms

$$\tilde{\rho}^w a_i^w + \left(np^w\right)_{,i} - \tilde{\rho}^w g_i + nR^w v_i^{ws} = 0 \tag{2}$$

Finally, combining Eqs. (1)-(2) the linear momentum balance equation of the entire mixture can be obtained

$$\tilde{\rho}^s a_i^s + \tilde{\rho}^w a_i^w - \sigma_{ij,j} - \tilde{\rho} b_i = 0 \tag{3}$$

grouping body force terms in b_i. Also, regarding the stress tensor for water saturated porous media, the classical effective stress tensor can be expressed as

$$\sigma'_{ij} = \sigma_{ij} + p^w\delta_{ij} \tag{4}$$

3 Material Point formulation

In this work, the theoretical approach is developed by assuming the porous media as a multiphase heterogeneous material, therefore each constituent should be adequately described. At the best of our knowledge, there are

Figure 1. Representative volume element for fully-saturated porous medium.

Figure 2. Material point domain at different stages.

at least mainly two ways of performing this issue. Although both approaches are based on the classical framework of the mixture theory proposed by Hassanizadeh and Gray Hassanizadeh and Gray [44, 45, 46], there is a fundamental difference between them. The first approaches Lewis and Schrefler [37], Borja and Koliji [38], Beneyto et al. [43], Schreyer-Bennethum [47] assume the validity of the mixture theory at the macroscale, thus, every multiphase constituent occupies, in the same time and space, a certain body subdomain according to each volume fraction. Therefore, just one material point is required to describe the mechanic and kinematic of a piece of porous media. Despite the good efficiency of these models, there is a considerable loss of modeling accuracy when the relative speed between the different phases becomes more important.

In order to overcome the above mentioned issue, a different hypothesis for the material description can be assumed. In the present approach, each constituent of the multiphase porous media is discretized with an individual material point [27, 41, 42]. Therefore, there is no loss of accuracy due to a significant rate of relative velocity between multiphase constituents of the porous media.

3.1 Main hypothesis

As it was mentioned before, the present work is based on the classical continuum mechanics and mixture theory. Therefore, the following assumptions are made for the coupled MPM formulation:

- Each phase (solid and water) is described in a Lagrangian formulation
- Each constituent can be represented as a continuous medium
- Isothermal conditions
- Mass exchange between solid and water phases is neglected
- Variation of water density is not allowed

In the current configuration, the solid skeleton and water phase can be viewed as the superposition of two continuous media. The material point position for both, the solid skeleton and water phase (x^α being $\alpha = s, w$ for solid or water material points, respectively) are located at the current configuration in Ω_α and might have placed at different positions in the reference configuration, Ω_0

3.2 Weak formulation

The weak formulation for the momentum balance equation for the water phase can be derived from by multiplying each component by the test function $\delta\omega_i^w(x_i^w)$ and then integrating over the current water particles domain Ω_w.

$$\int_{\Omega_w} \delta\omega_i^w \left(\tilde{\rho}^w a_i^w + (np^w)_{,i} - \tilde{\rho}^w g_i + nR^w v_i^{ws}\right) d\Omega_w = 0 \tag{5}$$

Then, integrating by parts the term which involves the gradient of pore pressure and using the divergence theorem on the traction boundary condition of

$$\int_{\partial\Omega_w} \delta\omega_i^w (n_i \bar{p}^w - t_i^w)\, d\partial\Omega_w = 0 \tag{6}$$

the weak form of the momentum balance equation for the water phase can be derived

$$\int_{\Omega_w} \delta\omega_i^w \tilde{\rho}^w \bar{a}_i^w d\Omega_w - \int_{\Omega_w} \delta\omega_i^w \tilde{\rho}^w b_i d\Omega + \int_{\Omega_w} \delta\omega_i^w nR^w (v_i^w - v_i^s)\, d\Omega - \int_{\Omega_w} \delta\omega_{i,i}^w n\bar{p}^w d\Omega_w + \int_{\partial\Omega_w} \delta\omega_i^w t_i^w d\partial\Omega_w = 0 \tag{7}$$

Regarding the entire mixture, the weak form of the momentum balance equation can be obtained by multiplying Eq. (3) by the test function $\delta\omega_i^\alpha(x_i^\alpha)$ and then integrating over the current configuration Ω_α, being $\alpha = s, w$. Also, the stress term was integrated by parts and the divergence theorem was considered.

$$\int_{\Omega_\alpha} \delta\omega_i^\alpha \left(\tilde{\rho}^s \bar{a}_i^s + \tilde{\rho}^w \bar{a}_i^w - \sigma_{ij,j} - \tilde{\rho} b_i\right) d\Omega_\alpha = 0 \tag{8}$$

$$\int_{\partial\Omega_s} \delta\omega_i^s \left(\sigma_{ij} n_j - t_i^s\right) d\partial\Omega_s = 0 \tag{9}$$

$$\int_{\partial\Omega_w} \delta\omega_i^w \left(\delta_{ij} n_j \bar{p}^w - t_i^w\right) d\partial\Omega_w = 0 \tag{10}$$

The third term on the left side of Eq. (8) can be rewritten integrating by parts Eq. (9) and Eq. (10), assuming $\delta\omega_i|_{\partial\Omega_\alpha^\alpha} = 0$

$$\int_\Omega \delta\omega_i^s \tilde{\rho}^s \bar{a}_i^s d\Omega + \int_\Omega \delta\omega_i^w \tilde{\rho}^w \bar{a}_i^w d\Omega + \int_\Omega \delta\omega_{i,j}^s \sigma_{ij}' d\Omega - \int_\Omega \delta\omega_{i,j}^w \bar{p}^w \delta_{ij} d\Omega - \int_\Omega \delta\omega_i^s \tilde{\rho}^s b_i d\Omega - \int_\Omega \delta\omega_i^w \tilde{\rho}^w b_i d\Omega$$
$$- \int_{\partial\Omega_s} \delta\omega_i^s t_i^s d\partial\Omega_s - \int_{\partial\Omega_w} \delta\omega_i^w t_i^w d\partial\Omega_w = 0 \tag{11}$$

4 Conclusions

A fully coupled particle method, based on the material point method, for the dynamic analysis of large deformations on saturated porous media, has been presented. A general theoretical framework for multiphase porous media, based on the classical mixture theory, was also presented as well as its particularization to the fully saturated case. The discretization of the mass balance equations for each phase was carried out according to the Garlekin method in order to achieve the discrete equation of motion for the numerical implementation. In this sense, the numerical implementation and solution algorithm was proposed in the material point context.

Acknowledgements. Funding This work was partly funded by the Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET, Argentina) as well as Secretaría de General de Ciencia y Técnica de la Universidad Nacional del Nordeste (UNNE), Argentina (Grants PI 17D002). Also, the authors would like to acknowledge the financial support from the Secretaría de Ciencia y Tecnología de la Universidad Tecnológica Nacional Facultad Regional Resistencia (UTN-FRRe, Argentina, Grant PID 4912TC (2018)) and the Agencia Nacional de Promoción Científica y Tecnológica (ANPCyT, Argentina, Grants PICTO-UNNE-2019-00014 and PICT-2018-01607).
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