Risk optimization of a RC frame under column loss scenario

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Abstract. The sudden loss of a single supporting element in a RC frame may lead to the disproportionate partial or total structural collapse if its design fails to confine the initial damage through resisting mechanisms, like compressive arch action, Vierendeel action, and catenary action. Since uncertainties related to material properties and geometrical parameters plays a major role in the behavior of these resisting mechanisms, and consequences are highly significant for such failure events, the risk optimization is a very convenient approach to optimize the balance between economy and safety. This is shown herein by the optimization of a RC frame, considering the cross sections and the steel rebar areas of the beam and columns as design variables. Failure consequences are considered for serviceability, beam bending, shear failure, flexo-axial compression of the columns, and steel rupture at and before catenary action. A physical and geometrical nonlinear static analysis is employed, in which the sample points are submitted to pushdown analysis. Material behavior is represented by an elastoplastic model with isotropic hardening for the steel rebars, and by combination of Mazars μ model with the modified Park-Kent model for the confined concrete. Failure probabilities are evaluated by the Weighted Average Simulation Method, and the Risk optimization is done by the Firefly Algorithm. In order to reduce the computational cost due to the nonlinearities involved and the high number of sample points required, Kriging is used to generate a sufficiently accurate metamodel for the limit states and reliability indexes. It is shown that the adopted formulation leads to more allocation of material when a column loss scenario starts to be significant in terms of safety x economy.

Keywords: kriging, progressive collapse, reinforced concrete, reliability analysis, risk optimization

1 Introduction

Progressive collapse happens when an initial member failure triggers the failure of the adjacent elements, in resemblance to a cascade effect, leading to a final failure with a disproportionate higher severity in relation to the initial event. When under multiple hazards, the probability of structural collapse $P[C]$ is given as:

$$P[C] = \sum_{H} \sum_{LD} P[C|LD, H]\, P[LD|H]\, P[H] \tag{1}$$

where $P[H]$ is the probability of hazard occurrence; $P[LD|H]$ is the conditional probability of local damage for a given hazard H; and $P[C|LD, H]$ is the conditional probability of collapse for a given LD and H.

Beck et al. [1, 2] uses this formulation considering $P[LD|H]\, P[H]$ as the probability of local damage P_{LD}, combining column loss and intact structure scenarios in a single objective function in order to study the cost-benefit of considering column loss situations for usual civil engineering structures. Aiming to expand this study to usual reinforced concrete (RC) structures while considering the realistic nonlinear structural behavior, this manuscript uses this approach to study the cost-benefit of considering a column removal to design a RC frame.

2 Formulation and implementation

The RC frame considered herein is shown in Figure 1. Its beams have a span of 4.00 m, cross section width of 20 cm, concrete cover of 2.5 cm, and stirrups with a diameter of 6 mm, while each column span is 3.00 m long and have cross section width of 20 cm, concrete cover of 2.5 cm, and stirrups with diameter of 6 mm spaced by 10 cm. Every member has a concrete strength of f_c' = 45 MPa with modulus of elasticity E_c = 35.5 GPa and tensile strength f_{ctm} 3.33 MPa (E_c and f_{ctm} inferred via Model Code [3]). Similarly, each member has longitudinal rebars with yielding strength of 511 MPa and modulus of elasticity E_S = 212 GPa. Both dead load and live load are 7.0 kN/m², and an additional 2.0 kN/m due to non-structural components over the beams is considered. Since the floors are one-directional, this leads to a nominal dead load D_n and live load L_n of 16 kN/m and 14 kN/m, respectively. The design parameters to be optimized are the mean values of: the beams cross section height, top and bottom beam rebar areas, stirrups spacing in the beams, longitudinal rebar area at the columns, and column cross section height. Hence, every design variable is a random variable. No discontinuities are considered along the elements, and the same optimal design for beam and column is attributed to every beam and column, respectively.

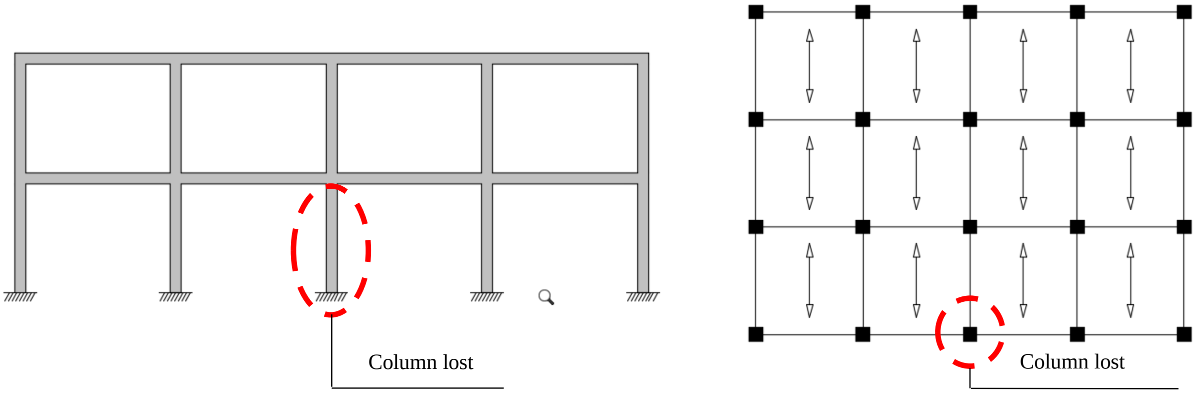

Figure 1. Studied frame

2.1 Risk optimization

The risk optimization problem follows the formulation proposed by Beck et al. [1, 2] with a total expected cost C_{TE} adapted to the RC frame studied herein:

$$C_{TE} = C_M + C^I_{ef,S} + C^I_{ef,B} + C^I_{ef,V} + C^I_{ef,FA} + C^{CL}_{ef,CA} + C^{CL}_{ef,Snap} + C^{CL}_{ef,V} + C^{CL}_{ef,FA} \quad (2)$$

where C_M is the manufacture cost; the superscripts I and CL stands for intact and column loss scenarios, respectively; $C^I_{ef,S}$ is the expected cost of serviceability failure; $C^I_{ef,B}$ is the expected cost of beam bending failure; $C^{CL}_{ef,CA}$ is the expected cost of rebar rupture at catenary action; $C^{CL}_{ef,Snap}$ is the expected cost of rebar rupture before catenary action (non-ductile); $C^I_{ef,V}$ and $C^{CL}_{ef,V}$ are the expected costs of shear failure at the beams for the intact and column loss scenarios, respectively; and $C^I_{ef,FA}$ and $C^{CL}_{ef,FA}$ are the expected costs of flexo-axial compression failure at the columns for the intact and column loss scenarios, respectively.

The SINAPI database is adopted to estimate C_M in R$, where unencumbered prices for Rio de Janeiro regarding the period of June 2021 are considered. Hence, C_M is composed by cost of formwork, obtainment of concrete, pouring of concrete, obtainment of steel rebars, and placing of steel rebars. For a given failure mode, the expected cost of failure C_{ef} is given by the product of a cost multiplier k times C_M times the probability P_f that the considered failure mode occurs. Thus, for CL the probability of local damage P_{LD} also multiplies $k \times C_M \times P_f$. The multipliers k are chosen according to the order of severity of each failure mode and regarding the real life ratio between the cost of the building and the cost of reconstruction after failure [1, 2]. Therefore, k is assumed equal to 5 for serviceability failure, 20 for beam bending failure, 30 for steel rupture at catenary action, and 60 for each fragile and severe failure mode: shear, flexo-axial compression, and steel rupture before catenary action.

2.2 Reliability analysis

The Weighted Average Simulation Method (WASM), proposed by Rashki et al. [4], is used herein to estimate the failure probabilities P_f required to compute the C_{TE}. This technique is appropriate for optimization problems involving random design variables since the estimation of P_f depends only on the index function $I(\boldsymbol{x})$ and the weight index $W(\boldsymbol{x})$ of the n_{sp} sample points, with $\boldsymbol{x}$ being the random variable vector. Therefore, changing the mean value of the candidate for optimal design only requires the re-evaluation of the weight index $W(\boldsymbol{x})$.

$$P_f = \frac{\sum_{k=1}^{n_{sp}} I(\boldsymbol{x}_k)\, W(\boldsymbol{x}_k)}{\sum_{k=1}^{n_{sp}} W(\boldsymbol{x}_k)} \quad (3)$$

The uncertainties adopted in this work are addressed in Table 1. A total of 7 million sample points are used to estimate every P_f for 2000 optimal candidates, which are generated via Latin Hypercube Sampling over the design domain. These optimal candidates are used to elaborate a metamodel for every $\hat{\beta} = -\Phi^{-1}(\hat{P}_f)$, reducing even further the computational cost to compose C_{TE}.

Table 1. Uncertainties considered

Variable	Distribution	Mean (μ)	Standard deviation (σ)	Coefficient of variation (δ)	Reference
Beams cross section height (h_V)	Normal	To be optimized*	1 mm	-	[5]
Bottom rebar area (A_{si})	Normal	To be optimized*	-	0.05	[5, 6]
Top rebar area (A_{ss})	Normal	To be optimized*	-	0.05	[5, 6]
Spacing between the beam's stirrups (s_t)	Normal	To be optimized*	-	0.05	Assumed
Rebar area of the columns (A_{sp})	Normal	To be optimized*	-	0.05	[5, 6]
Columns cross section height (h_V)	Normal	To be optimized*	1 mm	-	[5]
Concrete resistance (f_c')	Lognormal	45 MPa	-	0.12	[7, 8]
Yielding strength (f_y)	Normal	511 MPa	-	0.05	[6, 8]
Concrete's self-weight (γ_c)	Normal	25 kN/m³	-	0.05	Assumed
Ultimate steel strain (ε_{su})	Normal	0.13	-	0.14	[9]
Dead load (D)	Normal	1.05 D_n	-	0.10	[10]
50-year live load (L_{50})	Gumbel	1.00 L_n	-	0.25	[10]
Arbitrary point in time live load (L_{apt})	Gamma	0.25 L_n	-	0.55	[10]
Model error (E_M)	Lognormal	1.107	-	0.229	Obtained

2.3 Structural analysis

In order to estimate the probabilities of failure, a metamodel via kriging is employed [11], which requires the evaluation of a sufficient number of support points by an accurate model of structural analysis. The finite element method based on positions proposed by Coda [12] is used herein, where layered 2D beam elements are adopted. Each beam is discretized into 15 finite elements with a fifth-degree of approximation, and each column into 3 finite elements with the same degree. A total of 15 layers with 1 integration point each is used to discretize the cross-sections, being 13 layers for the concrete core and one for each steel reinforcement. An uniaxial model with isotropic hardening is used to represent the elastoplastic behavior of the longitudinal rebars, while μ-Model [13] is used to represent the damage evolution and the unilateral behavior of the concrete. Stirrups cannot be explicitly considered, but its influence on the ductility of the confined concrete is regarded by considering the resulting uniaxial curve from the Modified Park-Kent Model [14] to calibrate the parameters of the μ-Model.

Two structural analysis are carried out for every sample point: one for the intact structure (I), where an increasing uniform load is applied over each beam, and one for the column loss scenario (CL), where the uniform load is increased only over the beams directly affect by the column removal. The first scenario aims to obtain the uniform load q_{δ}^{I} that leads to a maximum mid-span displacement of L/600; the uniform load q_{M}^{I} that leads to the maximum beam bending moment allowed; the greatest observed shear force V_{B}^{I} until q_{M}^{I} at the beams; and the maximum acting axial force N_{C}^{I} and bending moment M_{C}^{I} until q_{M}^{I} at the columns. For the column loss scenario it is obtained the uniform load q_{CA} that leads to the first longitudinal rebar rupture; the greatest uniform load q_{CAA} observed during the compressed arch action (CAA) action stage; the greatest observed shear force V_{B}^{CL} at the beams until q_{CA}; and the maximum axial force N_{C}^{CL} and bending moment M_{C}^{CL} at the columns for q_{CA}. Due to the symmetry in the structural geometry and loading conditions, only half of the structure is modelled for both scenarios.

2.4 Kriging

Kriging is used to estimate a simplified, yet accurate, model of the limit states and the system reliability indexes in order to allow the realization of the risk optimization, otherwise it would be unviable due to a high computational cost. The choice of this metamodeling technique is due to its high efficiency and robustness for structural reliability problems, besides the fact of having great performance for multi-dimensional analysis [11].

A sufficient number of support points n_S is required in order to make the estimated model accurate relative to the original model. The base of functions chosen to generate the simplified model is a cubic polynomial with all the possible crossed terms. Also, the hyperparameters $\boldsymbol{\theta}$ are considered non-isotropic, being calibrated by the minimization of the function proposed by Dubourg [15] (eq. (4)) via Firefly Algorithm [16].

$$\boldsymbol{\theta} = \arg\min_{\boldsymbol{\theta} \in n\theta} \mathcal{L}(\boldsymbol{\theta}) = \sigma^2(\boldsymbol{\theta})\, |R(\boldsymbol{\theta})|^{1/ns} \tag{4}$$

where $n\theta$ is the number of hyperparameters coordinates to be evaluated, $\sigma^2(\boldsymbol{\theta})$ is the metamodel variance, and $R(\boldsymbol{\theta})$ is the matrix containing the correlation between pairs of support points.

3 Results

The following results were obtained considering 2000 support points for metamodeling the structural response (thus the limit states), which allowed the obtainment of 7 million sample points via kriging to guarantee the estimative of additional 2000 support points for metamodeling the reliability indexes of each failure mode considered. Firefly algorithm [16] was used for the risk optimization (20 fireflies, 50 iterations + auxiliary extensive search), calibration of hyperparameters (10 fireflies, 20 iterations + auxiliary extensive search), and calibration of the physical non-linear parameters of μ Model (50 fireflies and 100 iterations).

Figure 2 shows the optimal design for every value of P_{LD} ranging from P_{LD}^{min} = 5E-6 [1, 2] until P_{LD} = 1.0. From P_{LD}^{min} to P_{LD} = 1E-2 the optimal design remains practically constant, with beam cross section height of ~35 cm, bottom rebar area of ~2ϕ13, top rebar area of ~2ϕ12, spacing between the beam's stirrups of ~26 cm, longitudinal rebar area at the columns of ~4ϕ12, and column cross section height of ~25 cm. As shown in Figure 2, these optimal values ensure optimal reliability indexes of 3.20, 3.60, 4.10 and 5.20 for serviceability, beam bending, shear, and flexo-axial compression failure, respectively. These optimal reliability indexes reflects the chosen values of k, which guided the optimization process in order to guarantee higher safety margins against the most severe failure modes for the intact structure.

However, as P_{LD} > 1E-2, the optimal design changes drastically, leading to beam cross section height of 40 cm, bottom rebar area of ~3ϕ17, top rebar area of ~2ϕ16, spacing between the beam's stirrups of 10 cm, longitudinal rebar area at the columns of ~4ϕ13, and column cross section height of ~27 cm. This sudden change happens because of the greater expected costs of rebar rupture at catenary action due to the increasing P_{LD} (Figure 4), making necessary to ensure a higher safety margin against this failure mode as well in order to minimize C_{TE}.

It should be noticed that guaranteeing safety against this failure mode reduces the safety margins of the other failure modes of the column loss scenarios, mainly the shear failure. However, this optimal configuration allowed to reduce the expected cost of rebar failure at catenary action, which was growing much faster than the other failure modes with an increasing P_{LD}.

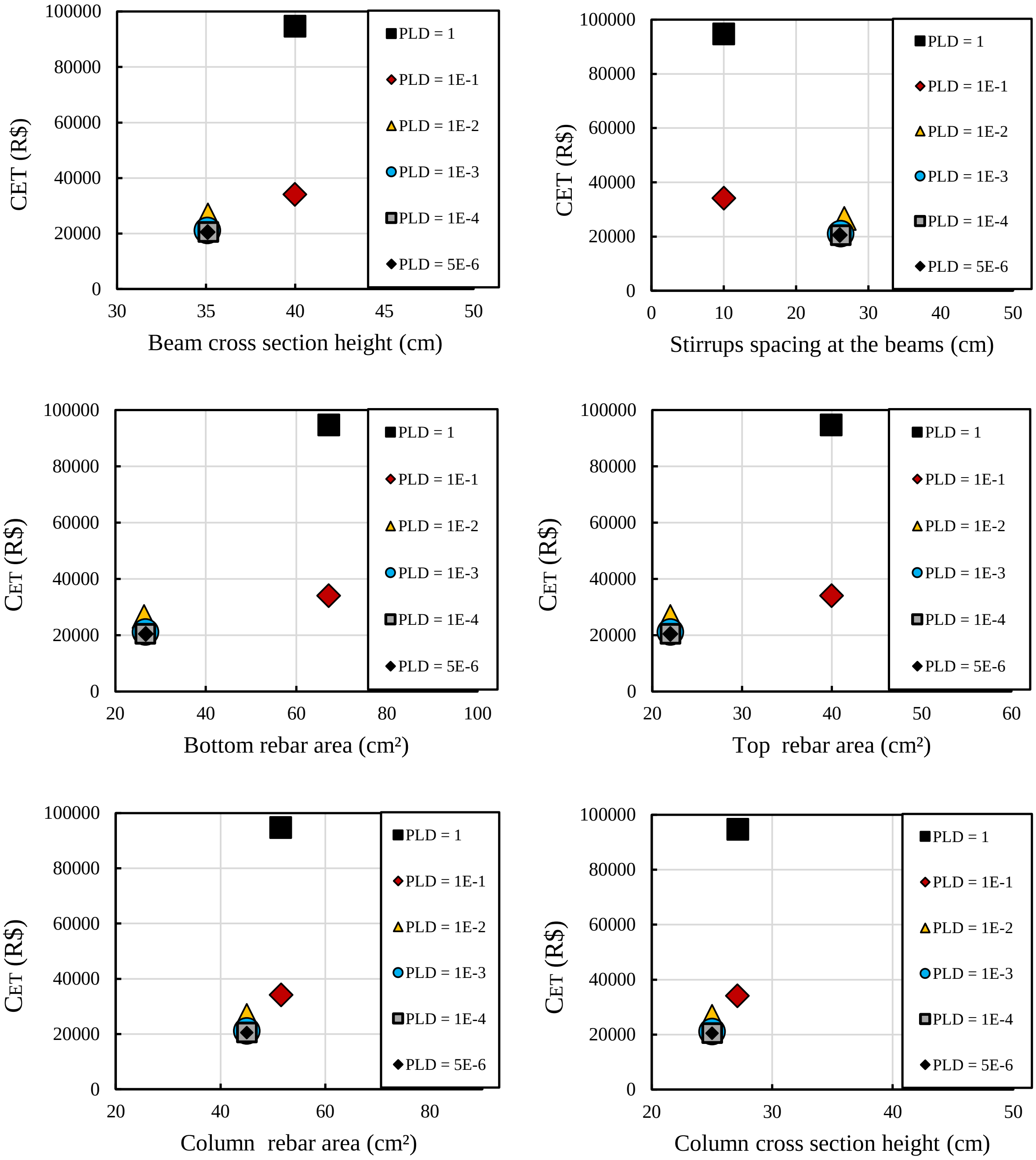

Figure 2. Optimal values for each design variable according to P_{LD}

Following Beck et al. [1, 2], P_{LD} = 1E-2 represents the threshold local damage probability P_{LD}^{th}, since after this value the cost-benefit of considering a column loss scenario in the design is positive. Keeping the previous optimal design for $P_{LD} > P_{LD}^{th}$ would lead to a much larger total expected cost, even though the manufactural cost was smaller. Hence, this procedure leads to more allocation of material in order to reduce the expected costs of failure due to the column loss scenario, guaranteeing the best balance between safety x economy. Thus, the new optimal design after P_{LD}^{th} leads to optimal reliability indexes even higher for the failures modes of the intact structure, showing that the design for column loss scenario also provides more safety against a normal load condition.

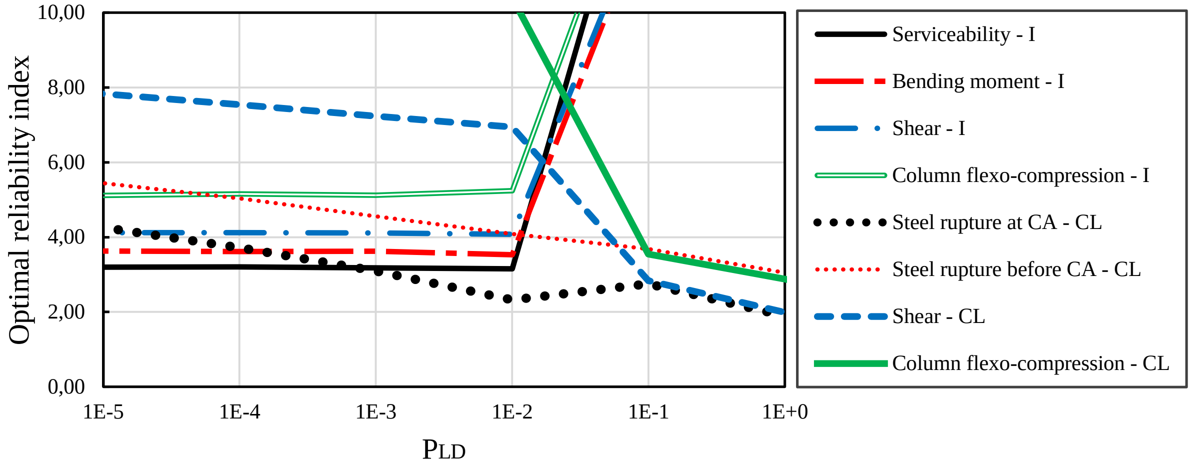

Figure 3. Optimal reliability indexes x P_{LD}

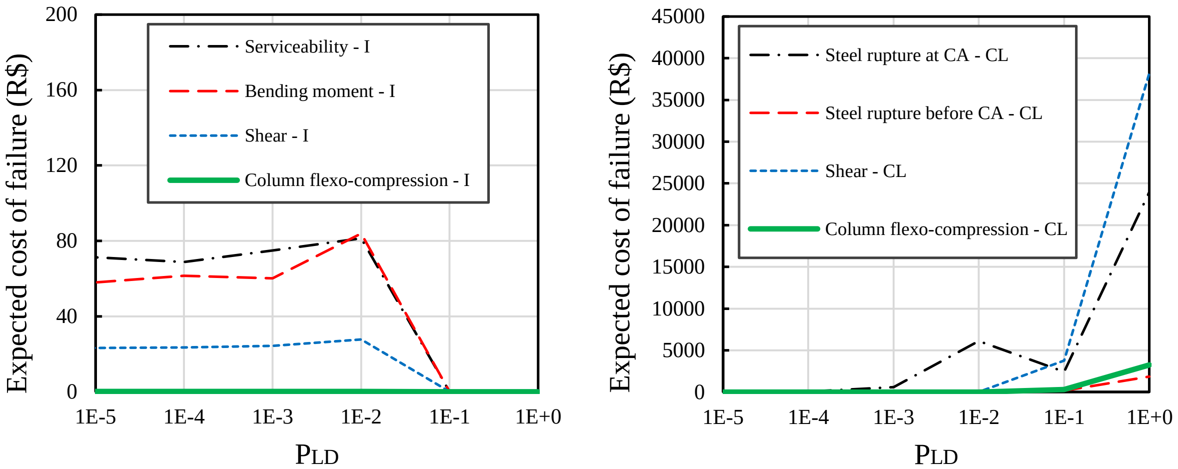

Figure 4. Expected costs of failure x P_{LD}

4 Conclusions

This manuscript shows how the behavior of the optimal design for the considered RC frame suddenly changes after the consideration of a column loss scenario starts to have a positive cost-benefit. The optimal beam height, top rebar area, bottom rebar area, column rebar area and column cross section height increases after P_{LD}^{th} in order to ensure safety against the failure modes of the column loss scenario, mainly steel rupture at catenary action. The increase of the cross sections means an increased resistance against shear failure for the beams and flexo-axial compression failure for the columns, while the increase of the rebar areas of the beams allows safety against the beam bending failure and the premature rebar rupture (before catenary action). The spacing between the beam's stirrups is the only variable that decreases, which happens in order to guarantee ductility for the concrete after the compressive arch action and safety against shear failure for $P_{LD} > P_{LD}^{th}$. Thus, the new optimal design after P_{LD}^{th} leads to higher safety margins for the failures modes of the intact structure, showing that designing for a single column loss scenario also provides more safety for a normal load condition. In view of this, the adopted formulation leads to more allocation of material when a column loss scenario starts to be significant in terms of safety x economy, leading to optimal designs that are robust, realistic and behaving close to the expected.

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