XLIII CILAMCE
21–25, November, 2022
FOZ DO IGUAÇU – PR – BRAZIL

Position Guidance and Control for Fully Actuated Multirotor Aerial Vehicles in Dynamic Environments

Jorge A. Ricardo Jr[1], Davi A. Santos[1]

[1]*Division of Mechanical Engineering, Department of Mechatronics, Aeronautics Institute of Technology, São José dos Campos, SP, 12228-900, Brazil,*
jorgejarj@ita.br, davists@ita.br.

Abstract. This paper is concerned with the robust position guidance and control of fully actuated multirotor aerial vehicles subject to velocity constraint in dynamic environments involving external disturbances. The overall method consists of an outer-loop guidance based on an acceleration-velocity-obstacles strategy and an inner stabilizing control loop based on an integral sliding mode policy. The guidance strategy generates velocity commands to reach a target position while avoiding collision with moving obstacles and respecting velocity bounds. On the other hand, the integral sliding mode ensures that the velocity commands are, in theory, exactly tracked all the time. The proposed method is numerically evaluated using a fully actuated octacopter and shows to be effective.

Keywords: Integral sliding mode control, fully actuated UAV, multirotor aerial vehicle, flight control.

1 Introduction

The multirotor aerial vehicles (MAVs) are expected to be extensively used for complex applications, such as aerial manipulation. These applications can benefit from the use of fully actuated MAVs since they can independently control their attitude and position [1, 2]. While performing these tasks, besides being subject to external disturbances, the MAV can encounter static and dynamic obstacles, such as buildings and other aircraft. Aiming at contributing to this topic, this paper establishes a hierarchical position guidance and control scheme for fully actuated MAVs, which treats the guidance and the flight controller separately. The scheme contains an outer guidance loop, which is entrusted with the collision-avoidance functionality, and an inner loop concerned with stabilization.

The robustness requirement for fully actuated MAVs flight control systems against disturbances and uncertainties is well recognized and has effectively been addressed using sliding mode approaches [3–5]. The sliding mode control (SMC) uses a high-frequency switching control to drive the system output to the so-called sliding manifold, where the system ideally becomes insensitive to bounded parametric uncertainties and disturbances of the matched type. In particular, Kotarski *et al.* [3] have adopted a first-order SMC for a fully actuated hexacopter with tilted rotors. Yao *et al.* [5] have presented an integral first-order SMC for the same type of vehicle, but they have used an arctangent approximation of the signum function to smooth the high-frequency control at the cost of losing robustness. In this paper, we design a velocity controller using an integral sliding mode approach [6] and exploit its insensitive property, which holds all the time, as a guidance strategy.

The guidance of MAVs among obstacles has been addressed in the literature using techniques such as model predictive control (MPC) [7, 8], rapidly exploring random trees (RRT) [9], and velocity obstacles (VO) [10]. In particular, the VO approach, introduced by Fiorini and Shiller [11], is a well-defined and simple technique that has been widely used for guiding mobile robots in dynamic and crowded environments [12]. It defines a set of velocity vectors that would result in a collision at some future time if the robot and obstacles kept their velocities constant. Therefore, by continuously selecting a velocity outside this set we can guarantee collision-free guidance since this selected velocity can be instantaneously achieved by the robot. Of course, such condition cannot be achieved by physical robots due to acceleration (or control) bounds. To overcome this drawback, Van Den Berg *et al.* [13] extended the VO concept to the so-called acceleration velocity obstacles (AVO). The AVO assumes that the robot accelerates towards the selected velocity using proportional control, *i.e.*, the acceleration is proportional to the difference between the selected velocity and the current one. In this sense, it defines a set of velocity vectors that would result in a collision at some time if the robot accelerates as assumed and the obstacles accelerate using the same proportional control with the same parameter or, alternatively, if they keep a constant velocity. However, due to disturbances and uncertainties, this condition is not realistic as well.

This paper aims to fill this gap by studying the position guidance and control of fully actuated MAVs with fixed rotors in dynamic environments involving external disturbances. We propose a hierarchical guidance and control scheme that suitably combines the AVO strategy with an integral sliding mode control law. First, we design the velocity controller using a first-order SMC to robustly stabilize the position dynamics. Second, we design a guidance strategy based on the AVO method to generate velocity commands to the stabilizing controller. The smoothness properties of these commands guarantees the existence of an integral sliding mode, which ideally guarantees the exact tracking of the velocity command and the insensitivity of the closed-loop system with respect to bounded uncertainties and disturbances of the matched type during all the time. Consequently, the harmonious combination of the proposed guidance technique and velocity controller robustly guarantee collision-free guidance and constraint fulfillment for fully actuated MAVs subject to model uncertainties and disturbances. We present our formulation for spherical MAVs subject to speed constraint and moving among obstacles in the three-dimensional space. We assume that the MAV exactly observe the obstacles' position and velocity. In summary, to the best of our knowledge, the main contributions of this paper are: 1) a three-dimensional flight control scheme combining a guidance algorithm based on the AVO method with an integral sliding mode velocity controller, and 2) the application of the aforementioned method to fully actuated MAVs with fixed rotors.

The remaining text is organized as follows. Subsection 1.1 presents the notation. Section 2 defines the problem. Section 3 presents the control design. Section 4 evaluates the proposed method using computer simulations. Finally, Section 5 concludes the paper.

1.1 Notation

Matrices and algebraic vectors are denoted, respectively, by uppercase and lowercase boldface letters, while geometric (Euclidian) vectors are denoted as $\vec{a}$. Sets are denoted by uppercase calligraphic letters as $\mathcal{D}$. A Cartesian coordinate system (CCS) is represented as $\mathcal{S}_b \triangleq \{B; \vec{x}_b, \vec{y}_b, \vec{z}_b\}$, with B denoting its origin, and $\vec{x}_b$, $\vec{y}_b$, and $\vec{z}_b$ representing the unit geometric vectors along its orthogonal axes. A null vector and a vector of ones of dimension $n \in \mathbb{Z}_{>0}$ are denoted, respectively, by $\mathbf{0}_n$ and $\mathbf{1}_n$. Consider two arbitrary algebraic vectors $\mathbf{x} = (x_1, \ldots, x_n)$ and $\mathbf{y} = (y_1, \ldots, y_n)$, the vector inequality $\mathbf{x} < \mathbf{y}$ means that $x_i < y_i$, $\forall i \in \{1, \ldots, n\}$. The Euclidean norm and component-wise absolute value of $\mathbf{x}$ are denoted, respectively, by $\|\mathbf{x}\|$ and $|\mathbf{x}|$. Lastly, an open disc of radius $\rho \in \mathbb{R}_{>0}$ centered at $\mathbf{p} \in \mathbb{R}^3$, the Minkowski sum of two sets, and the set subtraction are denoted, respectively, by $\mathcal{D}(\mathbf{p}, \rho) = \left\{\mathbf{x} \in \mathbb{R}^3 \mid \|\mathbf{x} - \mathbf{p}\| < \rho\right\}$, $\mathcal{X} \oplus \mathcal{Y} = \{\mathbf{x} + \mathbf{y} \mid \mathbf{x} \in \mathcal{X},\, \mathbf{y} \in \mathcal{Y}\}$, and $\mathcal{X} \setminus \mathcal{Y} = \{\mathbf{x} \mid \mathbf{x} \in \mathcal{X},\, \mathbf{x} \notin \mathcal{Y}\}$.

2 PROBLEM STATEMENT

Consider a ground reference CCS $\mathcal{S}_r \triangleq \{R; \vec{x}_r, \vec{y}_r, \vec{z}_r\}$ located at a point R on the ground, with $\vec{z}_r$ oriented upwards, parallel to the local vertical. Further, consider a point A coinciding with the MAV center of mass.

The translational kinematics of the MAV w.r.t. $\mathcal{S}_r$ are described by

$$\dot{\mathbf{r}}_{\mathrm{A}} = \mathbf{v}_{\mathrm{A}}, \tag{1}$$

where $\mathbf{r}_{\mathrm{A}} \in \mathbb{R}^3$ and $\mathbf{v}_{\mathrm{A}} \in \mathbb{R}^3$ are, respectively, the position and linear velocity of the MAV.

Using the Newton's second law, the MAV translational dynamics are described by

$$\dot{\mathbf{v}}_{\mathrm{A}} = -g\mathbf{e}_3 + m^{-1}(\mathbf{u} + \mathbf{d}), \tag{2}$$

where $g \in \mathbb{R}_{>0}$ is the gravity acceleration magnitude, $m \in \mathbb{R}_{>0}$ is the total mass of the MAV, $\mathbf{u} \in \mathbb{R}^3$ is the control force, and $\mathbf{d} \in \mathbb{R}^3$ is the disturbance force lumping external disturbances and uncertainties.

It is worth emphasizing that (1)–(2) represents the three-degrees-of-freedom (3-DOF) translational dynamic model of any fully actuated MAV, no matter its rotor configuration (number, type, and relative displacement). However, for simplicity, we are particularly interested in fully actuated MAVs with fixed rotors.

Suppose that the MAV is subject to velocity constraint and moves among dynamic obstacles, *i.e.*, obstacles that can change their velocity. In this sense, the MAV position and velocity must belong to the admissible sets

$$\mathbf{r}_{\mathrm{A}} \in \mathcal{P}(t), \tag{3}$$

$$\mathbf{v}_{\mathrm{A}} \in \mathcal{V}, \tag{4}$$

where $\mathcal{P}(t) \subseteq \mathbb{R}^3$ is the collision-free space, which varies as the obstacles move, and $\mathcal{V} \subseteq \mathbb{R}^3$ is an admissible velocity set.

Denoting a given target position by $\check{\mathbf{r}}_{\mathrm{A}} \in \mathbb{R}^3$, we now define the main problem of this paper.

Problem 1. The guidance problem is to robustly conduct the MAV described by (1)–(2) to the target position $\check{\mathbf{r}}_{\mathrm{A}}$, while respecting the constraints (3)–(4).

3 Hierarchical Control Architecture

To tackle the above problem, this paper adopts a hierarchical control architecture composed of an outer-loop guidance and an inner stabilizing loop, as depicted in Figure 1. The latter has the objective of providing robust stability, and the former is mainly entrusted with the collision-avoidance functionality.

Let us denote a command with an overbar in the corresponding variable. The guidance block receives the target position $\check{\mathbf{r}}_{\mathrm{A}}$ and the obstacles' position and velocity (represented by the set $\mathcal{O}$) as input, the MAV position and velocity as feedback, and the velocity command $\bar{\mathbf{v}}_{\mathrm{A}} \in \mathbb{R}^3$ as output. On the other hand, the velocity controller receives the velocity command as input, the MAV velocity as feedback, and the control force command $\bar{\mathbf{u}}$ as output.

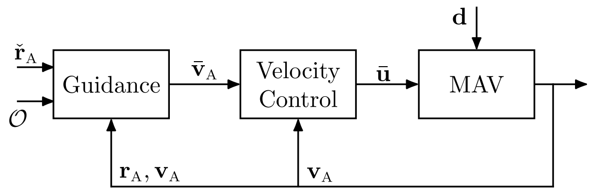

Figure 1. Hierarchical flight control architecture for fully actuated MAVs with fixed rotors.

To design the control strategy, we consider that the rotor dynamics are negligible, which is reasonable in practice since the rotor dynamics are much faster than the position one. As a result, one can consider that the control force $\mathbf{u}$ is equal to its command $\bar{\mathbf{u}}$, and consequently, the velocity controller can be designed considering the translational dynamic model of the MAV described by (1)–(2).

In this paper, the velocity controller is designed using an integral SMC [6] to robustly track the velocity command $\bar{\mathbf{v}}_{\mathrm{A}}$ in the presence of the disturbance $\mathbf{d}$. Conversely, the guidance is based on the AVO method and aims to take the MAV to its target position $\check{\mathbf{r}}_{\mathrm{A}}$ while fulfilling constraints (3)–(4). The strategies are harmoniously combined to ensure the existence of the sliding mode all the time, collision-free guidance, and constraint satisfaction.

3.1 Integral Sliding Mode Velocity Controller

Consider the objective of robustly tracking the time-varying velocity command $\bar{\mathbf{v}}_{\mathrm{A}}(t)$ in the presence of disturbances. To this end, let us define the velocity tracking error $\tilde{\mathbf{v}}_{\mathrm{A}} \triangleq \mathbf{v}_{\mathrm{A}} - \bar{\mathbf{v}}_{\mathrm{A}}$, the sliding variable

$$\mathbf{s}(t) \triangleq \tilde{\mathbf{v}}_{\mathrm{A}}(t), \tag{5}$$

and its corresponding sliding set $\mathcal{S} \triangleq \left\{(\tilde{\mathbf{r}}_{\mathrm{A}}, \tilde{\mathbf{v}}_{\mathrm{A}}) \in \mathbb{R}^6 \mid \mathbf{s} = \mathbf{0}_3\right\}$.

Regarding the disturbance $\mathbf{d}$ and the velocity command $\bar{\mathbf{v}}_{\mathrm{A}}$, consider the following assumptions, respectively:

Assumption 1. The disturbance is bounded according to $|\mathbf{d}| \leq \mathbf{d}^{\max}$, where $\mathbf{d}^{\max} \in \mathbb{R}^3$ is a known vector with positive components.

Assumption 2. The velocity command is continuous and satisfies $\bar{\mathbf{v}}_{\mathrm{A}}(0) = \mathbf{v}_{\mathrm{A}}(0)$.

The boundedness in Assumption 1 is reasonable in practice, although one can rarely obtain a non-conservative estimate of the bound $\mathbf{d}^{\max}$ without using an adaption scheme. Assumption 2 is not restrictive; it just requires the knowledge of the MAV's initial velocity and the use of a continuous command.

The following lemma guarantees the existence of an integral sliding mode of the system (1)–(2) in $\mathcal{S}$. It is worth emphasizing that by integral, we mean that $\mathbf{s}(t) = \mathbf{0}_3$, $\forall t \geq 0$.

Lemma 1. Under Assumptions 1–2, the following control law guarantees the integral sliding mode of the system (1)–(2) in $\mathcal{S}$:

$$\bar{\mathbf{u}} = m\left(g\mathbf{e}_3 + \dot{\bar{\mathbf{v}}}_{\mathrm{A}}\right) - \mathbf{K}\mathrm{sign}(\mathbf{s}), \tag{6}$$

where $\mathbf{K} \in \mathbb{R}^{3\times 3}$ is a positive-definite diagonal matrix satisfying $\mathbf{K}\mathbf{1}_3 = \mathbf{d}^{\max} + \boldsymbol{\epsilon}$, with $\boldsymbol{\epsilon} \in \mathbb{R}^3$ being a vector with positive components.

Proof. Consider the Lyapunov candidate function $V(\mathbf{s}) = \mathbf{s}^{\mathrm{T}}\mathbf{s}/2$. From Assumption 2, the system (1)–(2) starts the movement on the sliding surface. Therefore, to prove the integral sliding mode of the system (1)–(2) in $\mathcal{S}$, it is sufficient to show that $\dot{V}(\mathbf{s}) < 0$, $\forall \mathbf{s} \neq \mathbf{0}_3$. To this end, differentiating $V(\mathbf{s})$ with respect to time, substituting (6) into the resulting expression and choosing $\mathbf{K1}_3 = \mathbf{d}^{\max} + \boldsymbol{\epsilon}$, one can show that

$$\begin{aligned}
\dot{V}(\mathbf{s}) &= -m^{-1}(\mathbf{s})^{\mathrm{T}}\left(\mathbf{K}\mathrm{sign}(\mathbf{s}) - \mathbf{d}\right), \\
&= -m^{-1}\left|(\mathbf{s})^{\mathrm{T}}\right| \mathrm{diag}(\mathrm{sign}(\mathbf{s}))\left(\mathbf{K}\mathrm{sign}(\mathbf{s}) - \mathbf{d}\right), \\
&= -m^{-1}\left|(\mathbf{s})^{\mathrm{T}}\right|\left(\mathbf{K1}_3 - \mathrm{diag}(\mathrm{sign}(\mathbf{s}))\mathbf{d}\right), \\
&\leq -m^{-1}\left|(\mathbf{s})^{\mathrm{T}}\right|\left(\mathbf{d}^{\max} + \boldsymbol{\epsilon} - \mathbf{I}_3\mathbf{d}^{\max}\right) < 0,\ \forall \mathbf{s} \neq \mathbf{0}_3,
\end{aligned}$$

thus completing the proof. □

From the sliding variable definition (5), we can see that in the integral sliding mode the closed-loop system is capable of exactly tracking the velocity command, *i.e.*, $\mathbf{v}_{\mathrm{A}}(t) = \bar{\mathbf{v}}_{\mathrm{A}}(t), \forall t \geq 0$.

Now, to solve Problem 1, the velocity command $\bar{\mathbf{v}}_{\mathrm{A}}$ has to be generated in such a way to fulfill Assumption 2, respect constraints (3)–(4), and guide the MAV to the target position $\check{\mathbf{r}}_{\mathrm{A}}$. These objectives are accomplished by using a guidance strategy based on a three-dimensional formulation of the AVO method.

3.2 Guidance Strategy

Consider a spherical obstacle and a point B representing its center of mass. The obstacle has a radius $\rho_{\mathrm{B}} \in \mathbb{R}_{>0}$, position $\mathbf{r}_{\mathrm{B}} \in \mathbb{R}^3$, and velocity $\mathbf{v}_{\mathrm{B}} \in \mathbb{R}^3$. Moreover, considering that the MAV is enclosed by a sphere of radius $\rho_{\mathrm{A}} \in \mathbb{R}_{>0}$, a collision occurs with the obstacle if

$$\|\mathbf{r}_{\mathrm{A}}(t) - \mathbf{r}_{\mathrm{B}}(t)\| < \rho_{\mathrm{AB}}, \tag{7}$$

where $\rho_{\mathrm{AB}} \triangleq \rho_{\mathrm{A}} + \rho_{\mathrm{B}}$.

The VO method, proposed in Fiorini and Shiller [11], consider that the MAV and obstacle keep their velocities constant, *i.e.*, their position can be written, respectively, as $\mathbf{r}_{\mathrm{A}}(t) = \mathbf{r}_{\mathrm{A}}(t_0) + (t - t_0)\mathbf{v}_{\mathrm{A}}$ and $\mathbf{r}_{\mathrm{B}}(t) = \mathbf{r}_{\mathrm{B}}(t_0) + (t - t_0)\mathbf{v}_{\mathrm{B}}$, where $t_0 \in \mathbb{R}_{>0}$ is an initial time instant. Substituting these expressions in the collision condition (7) and dividing both sides by $(t - t_0)$, where $t \in \mathbb{R}_{\geq t_0}$ is the time, results in

$$\left\|\frac{\mathbf{r}_{\mathrm{AB}}(t_0)}{(t - t_0)} + \mathbf{v}_{\mathrm{AB}}\right\| < \frac{\rho_{\mathrm{AB}}}{(t - t_0)},$$

where $\mathbf{r}_{\mathrm{AB}} \triangleq \mathbf{r}_{\mathrm{A}} - \mathbf{r}_{\mathrm{B}}$ and $\mathbf{v}_{\mathrm{AB}} \triangleq \mathbf{v}_{\mathrm{A}} - \mathbf{v}_{\mathrm{B}}$. In this sense, the VO defines a set of velocities $\mathbf{v}_{\mathrm{A}}$ that will result in a collision with the obstacle inside a time horizon $\tau \in \mathbb{R}_{>0}$ if the MAV and obstacle keep their velocities constant. It is represented by

$$\mathcal{VO}_{\mathrm{AB}}^{\tau} = \bigcup_{t_0 < t \leq t_0 + \tau} \mathcal{D}\left(-\frac{\mathbf{r}_{\mathrm{AB}}(t_0)}{(t - t_0)}, \frac{\rho_{\mathrm{AB}}}{(t - t_0)}\right) \oplus \mathbf{v}_{\mathrm{B}}.$$

The procedure is as follows: selecting a velocity outside $\mathcal{VO}_{\mathrm{AB}}^{\tau}$ guarantees that a collision with the obstacle will not occur within the time horizon τ if the MAV and obstacle keep their velocities constant. Therefore, continually selecting velocities outside $\mathcal{VO}_{\mathrm{AB}}^{\tau}$ guarantees collision-free guidance. However, this guarantee only holds if the MAV instantaneously achieve the selected velocity, which may not be possible due to physical limitations.

To overcome this issue we propose a guidance strategy based on the AVO formulation. The original method, proposed in Van Den Berg *et al.* [13], let the MAV accelerates towards the selected velocity using proportional control, *i.e.*, the acceleration is known and proportional to the difference between the selected velocity and the current one. However, due to disturbances and uncertainties, the acceleration is not precisely known. In this paper, this problem is bypassed by designing a guidance strategy based on the AVO method and combining it with the integral SMC strategy. Instead of designing a proportional control, we design a feed-forward velocity filter to fulfill the smoothness requirement of the velocity command $\bar{\mathbf{v}}_{\mathrm{A}}$ in Assumption 2, and consequently guarantee the existence of the integral sliding mode. The filter receives as input a target velocity selected by the AVO method, defined by $\check{\mathbf{v}}_{\mathrm{A}} \in \mathbb{R}^3$, and outputs the velocity command $\bar{\mathbf{v}}_{\mathrm{A}}$ to the velocity controller, as depicted in Figure 2.

We design the velocity reference filter similarly to the proportional control of Van Den Berg *et al.* [13], *i.e.*,

$$\dot{\bar{\mathbf{v}}}_{\mathrm{A}} = \delta^{-1}\left(\check{\mathbf{v}}_{\mathrm{A}} - \bar{\mathbf{v}}_{\mathrm{A}}\right), \tag{8}$$

where $\delta \in \mathbb{R}_{>0}$ is a parameter.

From (8) it can be seen that the velocity command is minimally continuous, *i.e.*, when $\check{\mathbf{v}}_{\mathrm{A}}$ is a discontinuous function then $\dot{\bar{\mathbf{v}}}_{\mathrm{A}}$ is also discontinuous, and consequently $\bar{\mathbf{v}}_{\mathrm{A}}$ is continuous. Therefore, by choosing $\bar{\mathbf{v}}_{\mathrm{A}}(0) = \mathbf{v}_{\mathrm{A}}(0)$ Assumption 2 is fulfilled. Regarding the time response of $\bar{\mathbf{v}}_{\mathrm{A}}$ consider the following remark:

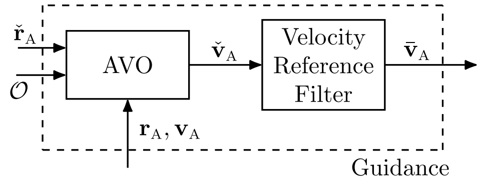

Figure 2. Block diagram of the proposed guidance method.

Remark 1. Considering $\check{\mathbf{v}}_{\mathrm{A}}$ as a constant input and integrating (8), it can be seen that $\bar{\mathbf{v}}_{\mathrm{A}}(t) \to \check{\mathbf{v}}_{\mathrm{A}}$ asymptotically and $|\bar{\mathbf{v}}_{\mathrm{A}}(t)| \leq |\check{\mathbf{v}}_{\mathrm{A}}|, \forall t > 0$.

Substituting (8) and (6) into the position dynamics (2), and integrating the resulting expression two times with respect to time considering $\check{\mathbf{v}}_{\mathrm{A}}$ as a constant input, we obtain the following expression for the MAV position:

$$\mathbf{r}_{\mathrm{A}}(t) = \mathbf{r}_{\mathrm{A}}(t_0) + (t - t_0)\check{\mathbf{v}}_{\mathrm{A}} + \delta \left(e^{-(t-t_0)/\delta} - 1\right) \left(\check{\mathbf{v}}_{\mathrm{A}} - \bar{\mathbf{v}}_{\mathrm{A}}(t_0)\right). \tag{9}$$

Considering that the obstacle keeps a constant velocity, its position can be written as $\mathbf{r}_{\mathrm{B}}(t) = \mathbf{r}_{\mathrm{B}}(t_0) + (t - t_0)\mathbf{v}_{\mathrm{B}}$. Substituting $\mathbf{r}_{\mathrm{B}}(t)$ and (9) into the collision condition (7), we obtain

$$\|\mathbf{r}_{\mathrm{AB}}(t_0) + (t - t_0)\left(\check{\mathbf{v}}_{\mathrm{A}} - \mathbf{v}_{\mathrm{B}}\right) + g(t)\left(\check{\mathbf{v}}_{\mathrm{A}} - \bar{\mathbf{v}}_{\mathrm{A}}(t_0)\right)\| < \rho_{\mathrm{AB}}, \tag{10}$$

where $g(t) \triangleq \delta\left(e^{-(t-t_0)/\delta} - 1\right)$. Dividing both sides of (10) by $t - t_0 + g(t)$ it becomes

$$\left\|\check{\mathbf{v}}_{\mathrm{A}} + \frac{\mathbf{r}_{\mathrm{AB}}(t_0) - (t - t_0)\mathbf{v}_{\mathrm{B}} - g(t)\bar{\mathbf{v}}_{\mathrm{A}}(t_0)}{t - t_0 + g(t)}\right\| < \frac{\rho_{\mathrm{AB}}}{t - t_0 + g(t)}. \tag{11}$$

Similarly to the VO method, the AVO defines a set of target velocities $\check{\mathbf{v}}_{\mathrm{A}}$ that will result in a collision with the obstacle within the time horizon τ, if the obstacle's velocity remains constant. It is represented by

$$\mathcal{AVO}^{\tau}_{\mathrm{AB}} = \bigcup_{t_0 < t \leq t_0 + \tau} D\left(\frac{(t - t_0)\mathbf{v}_{\mathrm{B}} + g(t)\bar{\mathbf{v}}_{\mathrm{A}}(0) - \mathbf{r}_{\mathrm{AB}}(t_0)}{t - t_0 + g(t)}, \frac{\rho_{\mathrm{AB}}}{t - t_0 + g(t)}\right).$$

The procedure is analogous to the VO method: selecting a velocity outside $\mathcal{AVO}^{\tau}_{\mathrm{AB}}$ guarantees that the MAV and the obstacle will not collide within the time horizon τ if the obstacle velocity remains constant. Continually selecting velocities outside $\mathcal{AVO}^{\tau}_{\mathrm{AB}}$ guarantees collision-free guidance for the MAV.

Now, to account for the velocity constraint (4), it has to be written in terms of the target velocity $\check{\mathbf{v}}_{\mathrm{A}}$. To this end, it can be concluded from Remark 1 and the integral sliding mode existence ($\mathbf{v}_{\mathrm{A}}(t) = \bar{\mathbf{v}}_{\mathrm{A}}(t)$), that if $\check{\mathbf{v}}_{\mathrm{A}} \in \mathcal{V}$, then $\mathbf{v}_{\mathrm{A}}(t) \in \mathcal{V}$, $\forall t \geq 0$, and consequently (4) is satisfied. In this sense, we can define a set of admissible target avoidance velocities as $\mathcal{V}_{\mathrm{AV}} = \mathcal{V} \backslash \mathcal{AVO}^{\tau}_{\mathrm{A}}$, where $\mathcal{AVO}^{\tau}_{\mathrm{A}}$ is the union of all the acceleration velocity obstacles of the MAV. Note that, continually selecting $\check{\mathbf{v}}_{\mathrm{A}}$ inside $\mathcal{V}_{\mathrm{AV}}$ robustly guarantees collision-free guidance and fulfillment of the velocity constraint (4). We highlight that he above robustness is a consequence of the existence of the integral sliding mode, being the AVO method itself not robust.

Lastly, to completely solve Problem 1, we calculate $\check{\mathbf{v}}_{\mathrm{A}}$ according to the following minimization that aims to guide the MAV to its target position $\check{\mathbf{r}}_{\mathrm{A}}$:

$$\check{\mathbf{v}}_{\mathrm{A}} = \min_{\mathbf{v} \in \mathcal{V}_{\mathrm{AV}}} \|\mathbf{v}^p - \mathbf{v}\|, \tag{12}$$

where $\mathbf{v}^p \in \mathbb{R}^3$ is a preferred velocity. This vector has a magnitude equal to the maximum admissible speed in the direction of the target position. hen the MAV is near its target, the magnitude of $\mathbf{v}^p$ is gradually decreased according to the remaining distance. We highlight that $\mathbf{v}^p$ can be designed using different strategies [11, 14].

4 Computational Implementation

To evaluate the proposed method, we simulate a scenario containing one spherical fully actuated octacopter flying among forty five obstacles also represented by spherical fully actuated octacopters. The MAV has a radius of 0.5 m, a mass of 1 kg, an initial position $(0, 0, 5)$ m, and is required to move to the target position $(22, 0, 5)$ m. Moreover, it is subject to the periodic disturbance $\mathbf{d}(t) = [0.6, -0.6, 0.6]^{\mathrm{T}} \sin(0.2t)$, and its speed is restricted to $\|\mathbf{v}_{\mathrm{A}}\| \leq v^{\max}$, where $v^{\max} = 1.5$ m/s. The obstacles have a radius of 0.5 m, and five of them are static.

The simulation is coded in MATLAB using a time step of 0.01 s and the explicit first-order Euler integration method. The proposed controller is compared to the original AVO method [13], which is implemented by using the control law $\mathbf{u} = mg\mathbf{e}_3 + m\left(\check{\mathbf{v}}_{\mathrm{A}} - \mathbf{v}_{\mathrm{A}}\right)/\delta$, and letting the AVO method select the target velocity $\check{\mathbf{v}}_{\mathrm{A}}$. For this simulation, we have set $\mathbf{K} = \mathbf{I}_3$, $\delta = 0.2$, $\tau = 15$ s, and approximated the minimum of (12) by uniformly distributing samples inside $\mathcal{V}_{\mathrm{AV}}$. A video containing the simulation of both methods is available at https://youtu.be/OInNCSwpT3E. It shows that the proposed method robustly guarantees collision-free guidance for the MAV, while the original AVO method failed to achieve the same objective since collision occurs.

Figures 3–4 present, respectively, the simulation results for the proposed methodology and the original AVO method. Figures 3(a) and 4(a) show the components of the MAV's velocity and their respective commands. It can be seen that the proposed controller performs an accurate tracking all the time. Figure 3(b) presents the Euclidian norm of the sliding variable and shows that it is restricted to a small neighborhood of the sliding surface all the time, thus confirming the existence of an integral practical sliding mode, and that chattering has a maximum amplitude of 0.055 m/s. Figures 3(c) and 4(b) show the control inputs for both methods. Figures 3(d) and 4(c) show the norm of the MAV velocity and the target velocity, respectively, for the proposed method and the AVO method. On one hand it can be seen that the proposed method is effective in respecting the speed bounds, and on the other hand, it can be seen that despite the original AVO method selecting target speeds inside the bounds, the MAV speed actually violates them. Lastly, Figures 3(e) and 3(d) present the time evolution of the distance from the MAV to the obstacles. Under the proposed method, the MAV is safely guided to the target position, while under the original AVO method, the MAV collides with one obstacle, as it can be seen in Figure 4(e).

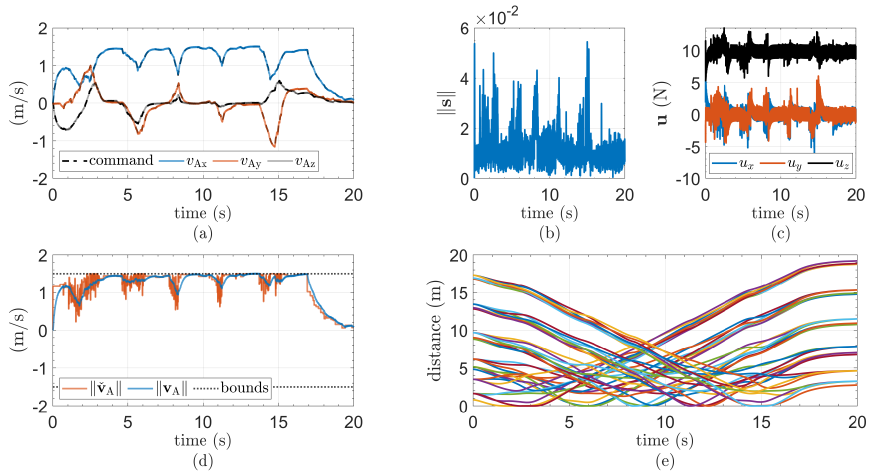

Figure 3. Results for the proposed method. (a): Velocity components and its commands. (b): Euclidian norm of the sliding variable. (c): Control components. (d): Euclidian norm of the MAV velocity and the target one, and the speed bounds. (e): Distance from the MAV to obstacles.

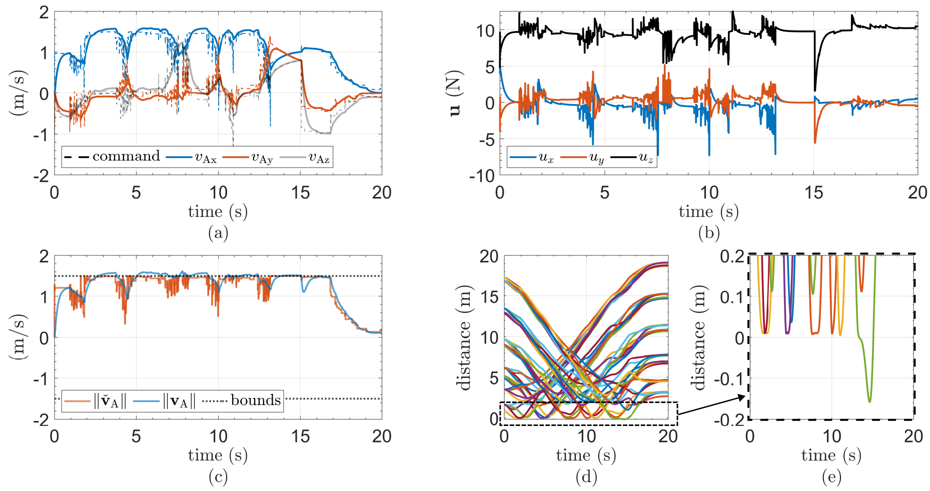

Figure 4. Results for the AVO method. (a): Velocity components and its commands. (b): Control components. (c): Euclidian norm of the MAV velocity and the target one, and the speed bounds. (d): Distance from the MAV to obstacles. (e): Zoom in on plot (d).

5 Conclusions

This paper studied the robust guidance and control of fully actuated MAVs subject to velocity constraints in dynamic environments involving external disturbances. The overall method consists of an outer-loop guidance based on the AVO method and a stabilizing control loop designed using an integral SMC. These methods are harmoniously combined to ensure the existence of the integral sliding mode, where, in theory, the velocity commands generated by the guidance are exactly tracked all the time. Due to such exact tracking, the AVO method in fact allows collision-free guidance and guarantees the fulfillment of the constraints. The proposed method is numerically illustrated on a fully actuated octacopter flying among forty-five obstacles. It has shown to be effective in robustly guiding the MAV to its target while avoiding collision with the obstacles and respecting the constraints. In future works, the proposed method can be extended to reciprocal collision avoidance.

Acknowledgments. The authors would like to thank FAPESP and FINEP for the financial support under grants 2019/05334-0 and 01.22.0069.00, respectively. The first author is grateful to the Coordination of Superior Level Staff Improvement (CAPES), EMBRAER S.A., and the Aeronautics Institute of Technology (ITA), for the doctorate scholarship under the Academic-Industrial Graduate Program (DAI). The second author is also grateful for the support of the CNPq, under grant 304300/2021-7.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] D. Brescianini and R. D'Andrea. Design, modeling and control of an omni-directional aerial vehicle. *IEEE International Conference on Robotics and Automation (ICRA)*, vol. , pp. 3261–3266, 2016.

[2] J. A. Ricardo Jr and D. A. Santos. Smooth second-order sliding mode control for fully actuated multirotor aerial vehicles. *ISA Transactions*, 2022.

[3] D. Kotarski, P. Piljek, H. Brezak, and J. Kasac. Design of a fully actuated passively tilted multirotor uav with decoupling control system. *International Conference on Mechanical and Aerospace Engineering*, pp. 385–390, 2017.

[4] D. Kotarski, P. Piljek, H. Brezak, and J. Kasac. Chattering-free tracking control of a fully actuated multirotor with passively tilted rotors. *Transactions of FAMENA*, vol. 42, pp. 1–14, 2018.

[5] C. Yao, J. Krieglstein, and K. Janschek. Modeling and sliding mode control of a fully-actuated multirotor with tilted propellers. *IFAC-PapersOnLine*, vol. 51, pp. 115–120, 2018.

[6] V. Utkin and J. Shi. Integral sliding mode in systems operating under uncertainty conditions. In *Proceedings of 35th IEEE Conference on Decision and Control*, volume 4, pp. 4591–4596, 1996.

[7] D. A. Santos and C. M. Lagoa. Wayset-based guidance of multirotor aerial vehicles using robust tube-based model predictive control. *ISA Transactions*, 2021.

[8] J. C. Pereira, V. J. S. Leite, and G. V. Raffo. An ellipsoidal-polytopic based approach for aggressive navigation using nonlinear model predictive control. In *2021 International Conference on Unmanned Aircraft Systems (ICUAS)*, pp. 827–835, 2021.

[9] Y. Bouzid, Y. Bestaoui, and H. Siguerdidjane. Quadrotor-uav optimal coverage path planning in cluttered environment with a limited onboard energy. In *2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pp. 979–984, 2017.

[10] P. Conroy, D. Bareiss, M. Beall, and van den J. P. Berg. 3-d reciprocal collision avoidance on physical quadrotor helicopters with on-board sensing for relative positioning. *CoRR*, vol. abs/1411.3794, 2014.

[11] P. Fiorini and Z. Shiller. Motion planning in dynamic environments using velocity obstacles. *International Journal of Robotics Research*, vol. 17, n. 7, pp. 760–772, 1998.

[12] S. J. Guyy, J. Chhugani, C. Kim, N. Satish, M. Lin, D. Manocha, and P. Dubey. ClearPath: Highly parallel collision avoidance for multi-agent simulation. *Computer Animation, Conference Proceedings*, pp. 177–187, 2009.

[13] J. Van Den Berg, J. Snape, S. J. Guy, and D. Manocha. Reciprocal collision avoidance with acceleration-velocity obstacles. *Proceedings - IEEE International Conference on Robotics and Automation*, pp. 3475–3482, 2011.

[14] J. Van Den Berg, M. Lin, and D. Manocha. Reciprocal Velocity Obstacles for Real-Time Multi-Agent Navigation. *IEEE Transactions on Robotics*, vol. 23, n. 4, pp. 834–834, 2007.